The latest (surely not the last!) crisis on the Cosmological Constant Problem

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GUIDELINES

- Definition of Λ and brief history.
- Cosmologies with $\Lambda \neq 0$.
- Supernovae and the value of Λ.
- The Standad Model prediction for $< V >$: the Vacuum Energy.
- Trying to solve the Problem: SUSY, Anthropic Principle, Baby Universes....; Dynamical Adjustment, Quintessence...
- Extra dimensions: the Brane World
- RG scaling of Λ: Neutrinos, Cosmons and new (submillimeter!) macroscopic forces?
- Conclusions?...

Definition of Λ .Brief history.

- Gravitational field equations:

$$G_{\mu\nu} + \lambda\, g_{\mu\nu} = -8\,\pi\; G_N\, T_{\mu\nu}$$

$$(G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu}) \quad ([\lambda] = E^2)$$

Einstein in 1915 ($\lambda = 0$!!). They are 10 PDE's for 10 dynamical variables $g_{\mu\nu}$. But there are 4 "Bianchi identities" of $G_{\mu\nu}$ associated to local energy-momentum conservation $\Rightarrow$ only 6 variables $g_{\mu\nu}$ are determined. The remaining gauge invariance of $g_{\mu\nu}$ is a reflect of local diff-invariance:

$$x^\mu \to x^{\mu\prime} = x^{\mu\prime}(x^\mu)$$

- Hilbert-Einstein variational principle:

$$\mathcal{L}_g = \int d^4x\sqrt{-g}\left(\frac{1}{16\pi\, G_N} R - \Lambda\right)$$

$$\Lambda \equiv \frac{\lambda}{8\,\pi\, G_N} \equiv \rho_v \Rightarrow [\Lambda] = E^4$$

FUNDAMENTAL OBSERVATION

Total curvature of spacetime:

$$R = 4\lambda + 8\,\pi\,G_N\,T^{\mu}_{\mu} = 4\lambda + 8\,\pi\,G_N\,(\rho_m - 3\,p)$$

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For $\lambda \neq 0$, flat (Minkowskian) spacetime can never be a solution!!. Deviations cannot be tolerated if λ is big...and least of all if it is VERY big!!... as it is apparently the case in ALL of our most cherished QFT's.

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$$R \sim \lambda \lesssim 10^{-84}\,GeV^2$$

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Ultimately, the Λ-Problem is the problem of understanding why do we have to fine-tune the parameters of the Diff. group of General Covariance in order that the Lorenz subgroup survives as a symmetry group !!.

First time Einstein introduced Λ: static universe!!

A. Einstein, *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie*, Sitzungsberichte der Preussischen Akademie der Wissenchaften, 142-152, Berlin 1917.

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Hubble's law was discovered in 1929

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Einstein: "The biggest blunder of my life!"

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Einstein's new standpoint on Λ:

A. Einstein, *Zum Kosmologischen Problem der allgemeinen Relativitätstheorie*, Sitzungsber. Preuss. Akad. Wiss. 235-237, Berlin 1931

[Sometimes quoted as (!!??): A. Einstein and Preuss. S.B. (1931) Akad. Wiss...]

Einstein's Universe: a model for (old) Western's Philosophy

◇ All in all (and in spite of being excluded!!), it is perhaps the simplemost way to understand Λ as a naturally small parameter constructed out of a fundamental length: the radius of the Universe (a)!:

$$\lambda = \frac{1}{a^2}$$

◇ Solve Einstein eqs. for a static and (spatially) closed Universe filled with "dust" (galaxies):

$$ds^2 = dt^2 - a^2 \left[\frac{dr^2}{1-r^2} + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2) \right]$$

$$T_{00} = \rho_m \quad (\text{non} - \text{vacuum energy density})$$

$$T_{11} = T_{22} = T_{33} = 0 \quad (\text{zero pressure})$$

$$T_{ij} = 0 \, (i \neq j) \quad (\text{homogeneous and isotropic})$$

◇ Geometry?: A 3-sphere S^3 of radius a. Check it! : $r = \sin\chi \quad \Rightarrow$:

$$dl^2_{(3)} = a^2 \left[d\chi^2 + \sin^2\chi \, (d\theta^2 + \sin^2\theta \, d\phi^2) \right]$$

$$dl^2_{(2)} = a^2 \left(d\chi^2 + \sin^2\chi \, d\phi^2 \right)$$

◇ Bianchi identities $\nabla_\mu G^{\mu\nu} = 0$, local conservation laws $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu g^{\mu\nu} = 0$ (covariant constancy of the metric) $\Rightarrow$

$$\nabla_\mu \lambda = \partial_\mu \lambda = 0 \;\rightarrow\; \boxed{\lambda\text{=const}}$$

◇ $\neq 0$ components of $G_{\mu\nu}$ for Einstein 's Universe:

$$G_{00} = -\frac{3}{a^2} \;;\quad G_{11} = G_{22} = G_{33} = \frac{1}{a^2}$$

◇ Field equations:

$$\lambda - \frac{3}{a^2} = -8\,\pi\,G_N\,\rho_m \;;\quad \lambda - \frac{1}{a^2} = 0$$

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$$\boxed{\lambda = \tfrac{1}{a^2} = 4\,\pi\,G_N\,\rho_m}$$

◇ Experiment $\rightarrow \rho_m \simeq 10^{-30} g/cm^3 \simeq 10^{-47} GeV^4$ ($\sim 1\, hadron/m^3$); $\rho_v = \Lambda = \frac{\lambda}{8\,\pi\,G_N}$

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$$\boxed{\rho_v = \tfrac{\rho_m}{2} \lesssim 10^{-47} GeV^4 \Leftrightarrow \;\; a > 10^{23}\, Km \sim 10^{10}\, lyr}$$

Λ as a New Force

$\diamond$ Newtonian limit of Einstein's eqs. with $\Lambda \neq 0$ for a static field produced by a non-relativistic mass density ρ_m.
From $g_{00} = \eta_{00} + h_{00} = 1 + 2\Phi \Rightarrow$

$$\nabla^2 \Phi(r) = 4\,\pi\,G_N \left(\rho_m + \frac{\lambda}{8\,\pi\,G_N}\right) = 4\,\pi\,G_N\,(\rho_m + \Lambda)$$

$\diamond$ Effective macroscopic force and gravitational potential:

$$\Phi(r) = -\frac{G_N\,M}{r} + \frac{\lambda\,r^2}{12}$$

$$F(r) = -\frac{G_N\,M}{r^2} - \boxed{\frac{\lambda\,r}{6}}$$

$\lambda > 0 \Rightarrow$ atractive?.. How could Einstein's Universe exists?. The overall-effect cannot be seen from the G_{00} eq. alone. Needs also G_{ii} and $\lambda\,g_{ii}$-term $\rightarrow$ additional negative pressure!!.

$\diamond$ Energy-momentum tensor for a perfect fluid in the presence of $\Lambda \neq 0$:

$$\bar{T}_{\mu\nu} = -\bar{p}\,g_{\mu\nu} + (\bar{\rho} + \bar{p})\,U^\mu\,U^\nu$$

$$\boxed{\bar{\rho} = \rho_m + \Lambda} \qquad \boxed{\bar{p} = p - \Lambda}$$

- For Einstein's Universe (filled with dust!!) $p = 0$. Effective pressure $\Rightarrow$

$$\bar{p} = -\lambda < 0$$

Compensates total gravitational attractive force.

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- True effect:
$\Lambda > 0 \Rightarrow$ Repulsive linear force!!

- In Einstein's Universe, "Λ-force" is responsible for the static equilibrium of the world $\Rightarrow$ Global effect is very important in spite of the fact that, locally, it's "irrelevant".

- What about cosmological models in evolution?. What can be the effect of a $\Lambda \neq 0$?.

The de Sitter solution: an anti-Machian Universe

◇ Historically, the first "dramatic" example of non-static solution is a Universe whose cosmic evolution is totally impelled by $\lambda \neq 0$ (de Sitter, 1917). Same year that Einstein introduced $\lambda \neq 0$ but opposite philosophy: matter not needed to produce inertia!!

Setting $T_{\mu\nu} = 0 \rightarrow \bar{T}_{\mu\nu} = \Lambda\, g_{\mu\nu} \rightarrow \boxed{\bar{p} = -\bar{\rho}}$.

$$G_{\mu\nu} + \lambda\, g_{\mu\nu} = 0$$

$$\frac{\dot{a}}{a} = H = const. \quad \rightarrow \boxed{a(t) = a_0 \exp{(H\, t)}}$$

Assumes "Hubble's constant H" to be a indeed a constant!!

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$$

Empty Minkowski-type spacetime ($k = 0$) whose spatial part inflates exponentially!!

◇ Driving force of de Sitter's Universe?:

$$\boxed{H = \sqrt{\frac{\lambda}{3}}}\ !!$$

◇ Useless spacetime? Only of academic interest?... No!!. Real cosmic evolution at very early times could have gone through a transient de Sitter phase (vacuum phase)

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INFLATIONARY EPOCH

(useful to solve many cosmological problems of the Standard Cosmological Model)

◇ "Curiosity": What if it all would have started with a Sitter phase?... De Sitter spacetime has no singularity ($a(t) \neq 0, \infty$) and it is unbounded in time (doesn't require a birth or a death). For a Universe that started with a vacuum-dominated phase $\rightarrow$ the time back to the singularity is infinite!!.

◇ Problem?. It's an empty Universe!!. And there is no dynamics (no potential, no dynamical field) that stops inflation. Nothing drives evolution towards a stable vacuum state where matter could be created (reheated) out of λ.

Mach's Principle is not equivalent to General Relativity.
More sophisticated example: Gödel's Universe (1949). $g_{0i} \neq 0 \rightarrow$ There is no universal time coordinate.

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It's a rotating Universe with negative curvature. Possible only if there is a negative Cosmological Constant!!:

$$R = -\alpha^2 = -8\,\pi\,G_N \rho_m\,; \qquad \omega = \frac{\alpha}{\sqrt{2}}$$

and

$$\boxed{\lambda = -\frac{\alpha^2}{2} < 0}$$

Same incoherent matter

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different inertia!!

TGR+BC's $\Leftrightarrow$ Mach's Principle

Friedmann-Lemaître-Robertson-Walker-Models with $\Lambda \neq 0$: Basic facts

- Friedmann (1922). Sistematic treatment of homogeneous and isotropic cosmological models in evolution. Einstein and de Sitter models become particular cases.

- Singles out a (Gaussian) coordinate sytem ("co-moving" frame) with a global time-coordinate ("cosmic time"). Any two points in a $3D$-slice satisfy Cosmological Principle. Loss of general covariance (price to pay for simplicity, as in gauge fixing).

- Line element similar to Einstein's Universe, but: i) Cosmic scale factor ("radius" of the Universe) $a = a(t)$ evolves with time; ii) spatial curvature signature: $k = -1, 0, 1$ (hyperbolic, flat Euclidian, elliptical) 3-spaces.

◇ Solving Einstein's eqs. with or without λ gives a rich variety of cosmologies. Some $\lambda \neq 0$ cases exhibit no singularity!!.

Fundamental eqs. of cosmology: ◇ From the interplay between geometry and source:

$$\boxed{H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\,\pi\,G_N\,\rho_m}{3} - \frac{k}{a^2} + \frac{\lambda}{3}}$$

$$2\left(\frac{\ddot{a}}{a}\right) + H^2 + \frac{k}{a^2} = \lambda - 8\,\pi\,G_N\,p$$

◇ From automatic conservation of the source (Bianchi):

$$\nabla_\mu T^{\mu\nu} = 0 \rightarrow \boxed{\dot{\rho}_m + 3H(\rho_m + p) = 0}$$

or $-d(\rho_m\,a^3) = p\,da^3$. ◇ To complete the fundamental eqs. of cosmology we must provide an equation of state:

$$\boxed{p = p(\rho_m)}$$

e.g. $p = (1/3)\rho_m$ for a free radiation gas.

◇ Combining first and second eqs.

$$\boxed{\ddot{a} = \frac{1}{3}\,\lambda\,a - \frac{4\,\pi\,G_N}{3}\,(\rho_m + 3\,p)\,a}$$

$\lambda > 0 \rightarrow$ accelerates expansion. Repulsive force against $-G_N\,M/(a\,r_0)^2$, $M = (4/3)\pi\,r_0^3\,\rho_{eff}$.

◇ Similarly, 1st eq. can be understood as an energy balance → Connection of geometry with total energy density of Universe. Energy-conservation law in the presence of $\lambda \neq 0$:

$$\frac{1}{2}\dot{a}^2 - \left(\frac{4}{3}\pi a^3\right)\rho_m \frac{G_N}{a}\boxed{-\frac{1}{2}\left(\frac{1}{3}\lambda\right)a^2} = E$$

Redefine $a \to \sqrt{2|E|}\,a$ and set

$$k = -\frac{E}{|E|} \equiv \begin{cases} -1 & \text{if } E > 0 \text{ (open)} \\ 0 & \text{if } E = 0 \text{ (flat)} \\ +1 & \text{if } E < 0 \text{ (closed)} \end{cases}$$

and the original eq. is recovered.

◇ It is convenient to rephrase cosmological eqs. in terms of present day values ($p_0 = 0$!) of standard cosmological parameters:

$$\begin{cases} \Omega_0 = \rho_{m0}/\rho_{c0}\,; & \rho_{c0} \equiv 3H_0{}^2/8\pi G_N \text{ crit. density} \\ q_0 = -(\ddot{a}/H^2 a)_0 & \text{deceleration parameter} \end{cases}$$

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$$q_0 = \frac{\Omega_0}{2} - \frac{\lambda}{3\,H_0{}^2}$$

$$\frac{k}{a_0{}^2} = H_0{}^2(2q_0 - 1) + \lambda$$

◇ Substituting for λ:

$$\boxed{(k/{a_0}^2) = ({H_0}^2/2)\,(3\,\Omega_0 - 2\,q_0 - 2)}$$

$$\Downarrow \qquad (\text{for } \boxed{\lambda = 0} \Leftrightarrow \Omega_0 = 2\,q_0)$$

$$\begin{cases} k = -1 & \text{for } \Omega_0 < 1 \text{ (open and expand)} \\ k = 0 & \text{for } \Omega_0 = 1 \text{ (flat and expand)} \\ k = +1 & \text{for } \Omega_0 > 1 \text{ (closed and recollapse)} \end{cases}$$

◇ For $\boxed{\lambda \neq 0} \Rightarrow$ the 3 cosmological parameters (λ, Ω_0, q_0) are fully independent. The situation is more complicated, but more exciting!

$$\begin{cases} \Omega_0 < \Omega_q & \text{(open and expand, recollapse)} \\ \Omega_0 = \Omega_q & \text{(flat and expand, recollapse)} \\ \Omega_0 > \Omega_q & \text{(closed: recollapse, expand!!)} \end{cases}$$

with $\Omega_q \equiv \frac{2}{3}\,(q_0 + 1)$

◇ For $\boxed{\lambda \neq 0} \Rightarrow$ i) possibility of non-singular solutions; ii) closed Universes need not to recollapse!. "Unfortunately" non-singular solutions are excluded by incompatible observed high red-shifts and too small predicted Ω_0. And nowadays also because of too small measured $\lambda > 0$!!.

◇ For a practical determination of the present cosmological parameters, 1st cosmological eq. is usually rewritten

$$\boxed{\Omega_m + \Omega_\Lambda + \Omega_k = 1}$$

where

$$\Omega_m = \frac{\rho_m}{\rho_c}\,, \quad \Omega_\Lambda = \frac{\Lambda}{\rho_c} = \frac{\lambda}{3H^2}\,, \quad \Omega_k = -\frac{k}{a^2\,H^2}$$

It follows that ($p = \gamma\,\rho$ with $p_0 = 0$)

$$q_0 = \frac{1}{2}\Omega_m\,(3\,\gamma + 1) - \Omega_\Lambda \simeq \frac{1}{2}\Omega_m - \Omega_\Lambda$$

Notice that, if there is a de Sitter phase,

$$\Omega_k \propto\ e^{-H\,t} \to 0$$

and then

$$\boxed{\Omega \equiv \Omega_m + \Omega_\Lambda = 1}$$

$\Rightarrow$ popular scenario: Inflationary Universe. It automatically solves 4 of the 6 classical Cosmological Problems of the standard BB-model: 1) horizon problem, 2) flatness-curvature-entropy problem, 3) rotation problem, 4) monopole problem, 5) small-scale inhomogeneity problem and 6) the cosmological constant problem. ("66% C.L.")

Dark matter versus "**dark** λ"

◇ From primordial nucleosynthesis it is known that baryonic matter contributes $0.01 \lesssim \Omega_B\, h^2 \lesssim 0.02$ $(h = 0.5 - 0.8) \Rightarrow$

$$0.01 \lesssim \Omega_B \lesssim 0.08$$

◇ Luminous matter contribute at most 1/100:

$$\Omega_{vis} \lesssim 0.006\, h^{-1} \lesssim 0.01$$

◇ Non-relativistic matter density. Dynamical observations of clustered matter (galaxy distributions on scales of order 10 Mpc) yield

$$\Omega_{clust} = 0.3 \pm 0.1$$

Conclusions:

- There must be a lot of **dark matter**;
- Some of it is baryonic;

- A significant amount must be non-baryonic.

- But... dynamical methods cannot preclude a smooth (non-clustered) component of order $\Omega_{smooth} \simeq 0.8$! either in the form of $\Omega_\Lambda \neq 0$ or $\Omega_{DM} \neq 0$ or both!!.

$\diamond$ For inflationary models, the DM component is even more dramatic since the flat space condition $\Rightarrow$

$$\Omega \equiv \Omega_m + \Omega_\Lambda = 1$$

The question is then how much you put in Ω_m and how much you put in Ω_Λ. In many models (aimed to fit the power spectrum of the density fluctuations) one sets $\Omega_\Lambda = 0$ and just plays with

$$\Omega_m = \Omega_B + \Omega_{DM} = 1$$

with the DM component split into (MDM's)

$$\Omega_{DM} = \Omega_{CDM} + \Omega_{HDM}$$

$\diamond$ However, present day observations seem to suggest that the Ω_Λ component should be taken seriously

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$\boxed{\Lambda\, DM \text{ models!!}}$

Supernovae and the value of λ

◇ Recent analysis. Two teams: i) SCP: S. Perlmutter et. al. (Pilar Ruiz-Lapuente UB), Astrophys. J. 1999 and ii) HZT: A.G. Riess et. al. ⇒ Observation of high-redshift ($z \simeq 0.5$) Type Ia supernovae allow precise determination of cosmological parameters using the magnitude-redshift relation for objects with same absolute lumin.– "standard candles":

$$m(z) = M + 5 \log D_L(z, \Omega_m, \Omega_\Lambda) + const.$$

◇ For fixed $z = 0.4 - 0.8$ ⇒

$$\boxed{D_L(z, \Omega_m, \Omega_\Lambda) = \frac{1}{H} \left(\frac{\mathcal{L}}{4\pi \mathcal{F}} \right)^{1/2} = const.}$$

→ isolines in the $(\Omega_m, \Omega_\Lambda)$-plane ∼ slope.
Striking results!!:

- Simplest inflationary model in the market,
  $\Omega_m = 1$, $\Omega_\Lambda = 0$ ⇒ strongly excluded!!.

- $\boxed{\Omega_\Lambda > 0 \text{ at } 3\sigma \Leftrightarrow \; P(\Omega_\Lambda > 0) > 99.7\%}$!!.

- The Universe will expand forever. Inconclusive curvature.

- The rate of expansion is accelerating: $\boxed{q_0 < 0}$.

- General constraint from standard candle luminosity distance (magnitude-redshift relation):

$$\boxed{0.8\,\Omega_m - 0.6\,\Omega_\Lambda = -0.2 \pm 0.1}$$

- $\Lambda \neq 0$ Inflationary Universe it's **OK**.
No longer age problem!!.($\simeq 15\,Gyr$)

$$(q_0 \simeq -0.6)$$

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$$\boxed{\Omega_m = 0.28 \pm 0.10\,; \quad \Omega_\Lambda = 0.72 \pm 0.20}$$

Roughly

$$\boxed{\Omega_m \lesssim 1/3 \gg \Omega_B;\ \ \Omega_\Lambda \gtrsim 2/3}$$

.

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$$\boxed{\Omega_\Lambda \gtrsim 2\,\Omega_m \ \ \Rightarrow \ \ \ \Lambda\, DM\ \ !!}$$

Despite Einstein's exorcisms to get rid of his creature, λ wants to resurrect from ashes!!

Λ: Alfa and Omega

tempo	**passage**	**age**	$\bar{\rho}$	$\bar{p}$	$\bar{\rho}+3\,\bar{p}$
prestissimo	string?	$< 10^{-43}\,s$	?	?	?
presto	vacuum	$< 10^{-38}\,s$	Λ_e	$-\Lambda_e$	$-$
allegro	matter	$< 10^{-36}\,s$	ρ_ϕ	0	$+$
andante	radiation	$< 10^{4}\,yr$	T^4	T^4	$+$
largo	matter	$\lesssim 10^{10}\,yr$	$\rho+\Lambda$	$-\Lambda$	$\pm$
largissimo	vacuum	∞ !!	Λ	$-\Lambda$	$-$

$\diamond$ Immediately after the birth of the Universe, the cosmic evolution could have been driven by a huge effective Cosmological Constant $\Lambda_e > 0$ (Inflationary Epoch). Whether or not the Planckian epoch was also dominated by a Planckian $\Lambda_P \sim M_P^4 \sim 10^{76}\,GeV^4$ cannot be stated.

$\diamond$ What we do know for certain is that the residual Cosmological Constant (if it is $\Lambda > 0$, as suggested by Supernovae data) will drive away the final fate of the Universe into a diluting inflationary Ω-phase:

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THE END?...

Saying: "A mischievous genie, once let out of a bottle, is not easily reconfined!".

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$\diamond$ For... what is the natural value of λ in QFT...and in particular in the SM?

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Here starts the very nature of the old and the NEW Cosmological Constant Problem!!

Λ in QFT: the Vacuum Energy

◇ Action integral for a scalar QFT:

$$S[\phi] = \int d^4x \sqrt{-g}\, \mathcal{L}(\phi, \partial_\mu \phi)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi - V_{eff}(\phi)$$

◇ Matter field energy-momentum tensor:

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta\, S}{\delta\, g^{\mu\nu}} = 2\, \frac{\partial \mathcal{L}}{\partial\, g^{\mu\nu}} - g_{\mu\nu}\, \mathcal{L}$$

$$= \left[\partial_\mu \phi\, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu}\ \partial_\alpha \phi\, \partial^\alpha \phi \right] + g_{\mu\nu}\, V_{eff}$$

◇ For static equilibrium configurations $\Rightarrow$

$$\boxed{< T_{\mu\nu} > = g_{\mu\nu}\ < V_{eff} >}$$

◇ Comparing with Einstein's eqs $\Rightarrow$

$$\lambda \to \lambda + \lambda_{ind} = \lambda + 8\, \pi\, G_N\ < V_{eff} >$$

$$\Downarrow$$

$$\Lambda \to \Lambda + < V_{eff} > \Leftrightarrow \quad \boxed{\Lambda_{ph} = \Lambda_v + \Lambda_{ind}}$$

◇ The Cosmological Constant Problem in modern QFT is the realization that the Physical Cosmological Constant is an Effective Cosmological Constant!!
(Zeldovich 1967; Linde 1974, Veltman 1975):

$$\boxed{\Lambda_{ph} = \Lambda_v + \Lambda_{ind}}$$

where

- Λ_v is the vacuum cosmological term ("bare cosmological constant")

- Λ_{ind} is the induced cosmological term by quantum fluctuations in QFT.

Λ in the SM and beyond

Source	**Effect** (GeV^4)	Λ/Λ_{exp}
electron 0-point	10^{-16}	10^{31}
QCD chiral	10^{-4}	10^{43}
QCD gluon	10^{-2}	10^{45}
Electroweak SM	10^{+9}	10^{56}
typical GUT	10^{+64}	10^{111}
Quantum Gravity	10^{+76}	10^{123} !!

◇ From the observed Ω_Λ and

$$\rho_{c0} = 8.1\, h^2 \times 10^{-47}\, GeV^4$$
$$\simeq (2-5) \times 10^{-47}\, GeV^4 \quad (h \simeq 0.5 - 0.8)$$

we have

$$\boxed{\Lambda_{exp} \gtrsim +2 \times 10^{-47}\, GeV^4}$$

◇ Consider the zero-point energy of the electron:

$$< V >_0 = \sum_\omega \frac{1}{2}\omega = \int_0^{E_{max}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m_e^2}$$
$$= E_{max}^4/16\pi^2 = m_e^4/16\pi^2 \sim 1/\lambda_C^4$$

corresponding to $r_{min} = 1/m_e \equiv \lambda_C$.

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Radius of the corresponding Einstein's Universe

$$a_E = \frac{1}{\sqrt{\lambda}} = \frac{1}{\sqrt{8\,\pi\,G_N\,< V >_0}} \simeq 10^7\ Km$$

instead of 10^{23} Km!!.

◇ Pauli did a "slightly" different estimate. He took the classical electron radius $r_e = \alpha/m_e$. He was quite disappointed to learn that the radius of Einstein's Universe would not even reach out the moon!!.

◇ The usual way we get rid of the 0-point energy contributions in QFT is through "normal ordering" of the ladder operators acting on Fock-space states. After all "only energy differences matter in (flat!) QFT"...

◇ However, NOT so in GRAVITY !!:

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GEOMETRY $\Leftrightarrow$ TOTAL ENERGY!!

◇ The contribution from QCD comes from the VEV of the anomalous trace

$$< T^{\mu}_{\mu} >=< \frac{\beta_3(g_s)}{2\,g_s} F^{\mu\nu}_a\, F^a_{\mu\nu} > +(1+\gamma_q) < m_q\,\bar{q}\,q >$$

with

$$< m_q\,\bar{q}\,q >= -f_\pi^2\, m_\pi^2 \sim 10^{-4}\, GeV^4$$
$$< \frac{\beta_3(g_s)}{2\,g_s} F^{\mu\nu}_a\, F^a_{\mu\nu} > \sim 10^{-2}\, GeV^4$$

Even the fairly modest QCD contribution would produce a tiny Einstein's Universe of only

$$\boxed{a_{QCD} = 1\, Km \;\; !!}$$

◇ What about the electroweak contribution?. Later on. But by now just notice that, if it there at all, it would distort space curvature (and redshift !!) already at the level of

$$\boxed{a_{EW} = 1\, cm \;\; !!}$$

◇ Extreme case: Planckian $\Lambda \Rightarrow$ Universe of Planckian size!! ($10^{-33}\, cm$!!!!).

Trying to solve the PROBLEM

◇ Which problem ???. In the past we were "happy" with the bound

$$|\Lambda| < \rho_c \simeq 10^{-47}\, GeV^4 \;\Leftrightarrow |\lambda| < 10^{-84}\, GeV^2$$

because we thought that this was compelling "evidence" that $\Lambda = 0$. Similarly as we eagerly conclude $m_\gamma = 0$ from the bound

$$m_\gamma^2 < 10^{-50}\, GeV^2$$

from terrestrial measurements of the magnetic field. $\Rightarrow$ Most mechanisms in the past addressed a strategy for proving that $\Lambda = 0$.

◇ BUT...

The NEW Cosmological Constant Problem is now quite another!!. We must prove that, amid the jungle of multifarious phase transitions, somewhere in the theory of the world there is a small $+$ number that remains:

$$\Lambda \simeq +2 \times 10^{-47}\, GeV^4 \text{ !!!}$$

General (pessimistic) message: NO satisfactory solution to any of the two problems up to now.

List of some mechanisms

◇ Anthropic Principle

◇ A marvelous ("deep") SYMMETRY?

◇ Antisymmetric tensor field

◇ Baby Universes

◇ Undynamical scalar field in Weyl gravity

◇ Quintessence

◇ Topological string mechanisms

◇ Dynamical adjustment

◇ Brane World, Neutrinos and Cosmons
⇒ new submm. macroscopic forces!!

◇ M-Theory (somewhere inside the hexagon...)
◇ ...

Sketching some avenues:

◇ Anthropic Principle (Weinberg,1989,1996)

- "We know in our bones...that $\Lambda_{ph} = 0$!! (too naïve?...)

- Upper bound on Λ comes from requirement that formation of sufficiently large gravitational condensation should be possible. Galaxies formed already at $z \simeq 4 \rightarrow \rho_m(z) = (1+z)^3 \rho_{m0} \rightarrow \Lambda < 125\, \rho_{m0}$ (Anthropic bound). Poor...

- Anthropic mean value follows from condition that spherically symmetric fluctuations recondense at recombination. From the No. of indep. observers arising from a fluctuation ($\propto M_{coll}$)$\Rightarrow$

$$< \Lambda > < (7-9)\, \rho_{m0}$$ Not bad!.

◇ SYMMETRY?

NO. Not even SUSY !!!

- It implies $E_{vac} = 0$ only if SUSY is exact, which is NOT !!. $\Rightarrow$ Imperfect cancellation between boson and fermion loops.

- Formal proof from Graded Lie algebra:

$$\left\{Q_\alpha, Q_\beta^\dagger\right\} = (\sigma_\mu)_{\alpha\beta}\ P^\mu$$

SUSY $\Leftrightarrow$ $Q_\alpha|0>= Q_\alpha^\dagger|0>= 0$

$$\Downarrow$$

$$\boxed{<0|P^\mu|0>= 0}$$

- In general $<V>= \sum_i \left|\frac{\partial W(\phi)}{\partial \phi_i}\right|^2 > 0$

- In broken local SUSY (Supergravity) fine-tuning $V = 0$ (Käller) technically possible. But it is not the vacuum state: $< V >< 0$!!.

◇ Baby Universes (Coleman, 1988)

- Quantum Cosmology $\rightarrow$ Wormholes: little baby universes that branch off from our region of Universe. Topological fluctuations of Euclidean spacetime. For macroscopic sizes $\rightarrow$ **Blackholes**!!

- In Lorentzian (hyperbolic) metrics baby universes cannot branch off (Geroch) because Cauchy's problem is well-defined. However, in Euclidean QG (positive-definite metric) $\Rightarrow$ no restrictions on the possible topologies and metrics of spacetime. All of them must be summed over in the Euclidean path integral!!

$$Z_E = \int Dg\, D\Phi \, \exp\left[- \int d^4x_E \, \mathcal{L}(g_{\mu\nu}, \Phi, \lambda_i) \right]$$

- Wormholes exist only in "imaginary time" !!. But they can produce "real effects" $\rightarrow$ introduce apparent interactions between particles in a given region of Universe.

- WH's renormalize the parameters and the partition function is that of an ensemble of worlds with a statistical distribution of coupling constants. In particular, Λ.

- Most important, they could perhaps help to extinguish Λ or make it very small. Sketch:

$\boxed{\mathcal{L}(x) = \sum_i \lambda_i \, O_i(x)}$

$$S_E \rightarrow S_E - \frac{1}{2} \sum_{ij} C'_{ij} \int d^4x \, d^4x' \, O_i(x) \, O_j(x')$$

$$C'_{ij} = C_{ij} \, \exp\left(-S_{WH}\right)$$

Partition function?

$$Z_E \rightarrow \int \prod_i d\alpha_i \, e^{-\frac{1}{2}\alpha_i \, D'_{ij} \alpha_j} \int Dg D\Phi$$

$$\times \exp\left[- \int d^4x \sum_i \lambda_i(\alpha) \, O_i(x) \right]$$

$$\boxed{\lambda_i \to \lambda_i(\alpha) = \lambda_i + \alpha_i\, e^{S_i}}$$

$$\Downarrow$$

$$\rho(\alpha) \;\sim\; \exp\left[-\frac{1}{2}\alpha_i\, D'_{ij}\alpha_j\right] \exp\left[\frac{+3\,\pi}{G_N(\alpha)\,\lambda(\alpha)}\right]$$
$$\times \exp\left\{\exp\left[\frac{+3\,\pi}{G_N(\alpha)\lambda(\alpha)}\right]\right\}$$

Big deal? $\Rightarrow$ big fix !!

◇ Nice result, but life it's not so easy

Some problems:

-Euclidean QG is ill-defined (action unbounded from below)
-All metrics contribute: wild fluctuations
-Dilute wormhole approx. enough?
-Did we use the correct saddle point?
-Unclear analytical continuation to Lorentzian manifolds.

◇ Dinamical Adjustment
(Dolgov,Wilczek, Abbot, PSW,...)

◇ Assume that the trace of the energy-momentum tensor is the source for a dynamical scalar field ϕ.

$$\Box\phi = c\,T^{\mu}_{\mu}$$

In general

$$\begin{aligned} T_{\mu\nu} \;=\; & g_{\mu\nu} < V > \quad \text{vacuum condensates} \\ & + T^{m}_{\mu\nu} \quad \text{incoherent matter} \\ & + T_{\mu\nu}(\phi) \quad \text{coherent matter fields} \end{aligned}$$

Recall that by Einstein's eqs. $R \propto T^{\mu}_{\mu} \Rightarrow$ if there is a component $T^{\mu}_{\mu} \supset T(\phi) \rightarrow$ one might be able to find an automatic dynamical solution with zero curvature:

$$\boxed{\Box\phi = c\,T(\phi_0) = R = 0}$$

◇ Natural idea, similar to PQ-symmetry for solving strong CP-problem. The $\bar{\theta}$-parameter becomes a scalar field!!. Axion oscillations around the true vacuum, driven by the $U(1)$-anomaly, adjust $\bar{\theta} \rightarrow 0$.

◇ Why is not that easy? Tried by PSW (1987)!!. All dynamical mechanisms follow sort of a similar scheme and end up with similar illness:

- Look for equilibrium solutions of the field eqs. in flat space:

$$\frac{\partial \mathcal{L}}{\partial \phi} = 0\,, \qquad \frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = 0$$

- For a natural solution of these eqs. we expect that for the stationary solutions:

$$g_{\mu\nu}\,\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = \frac{\partial \mathcal{L}}{\partial \phi}\, f(\phi)$$

- But in flat space $T^{\mu}_{\mu} = 2\, g_{\mu\nu} \boxed{\frac{\partial \mathcal{L}}{\partial\, g_{\mu\nu}}} - 4\, \mathcal{L}$.

$< T^{\mu}_{\mu} >= -4 \; < \mathcal{L} >= +4 \; < V_{eff}(\phi) >= 0$?

Trouble! $\rightarrow$ "NO-GO THEOREM"

Scape? $\Rightarrow$ Quintessence?...Fine-tuning !!

Neutrinos, Cosmons, new Macroscopic forces?

(Shapiro, J.S)

◇ We should not discard the possibility that $\Lambda \sim \rho_{m0}$ is a consequence of the existence of very light degrees of freedom S $\Rightarrow$ new, sub-millimeter, macroscopic forces !!.. Typically, we expect

$$\boxed{m_S = \Lambda^{1/4} \simeq 10^{-3}\, eV} \rightarrow \boxed{\lambda_C \simeq 0.1\, mm}$$

◇ Take SM Higgs potential

$$V_{cl} = -\frac{1}{2} m^2 \phi^2 + \frac{f}{8}\phi^4.$$

Minimization yields the SSB relation:

$$\phi = \sqrt{\frac{2m^2}{f}} \equiv v = 246\, GeV \quad \text{and} \quad f = \frac{M_H^2}{v^2}$$

$$\Lambda_{ind} = < V_{cl} > = -\frac{m^4}{2\, f}$$

For $M \gtrsim 100\, GeV \Rightarrow$

$$\boxed{\Lambda_{ind} \simeq 10^8\, GeV^4 \sim 10^{55}\, \Lambda_{ph}}$$

◇ Renormalization group equations for the Higgs potential parameters. Follow from RGE-invariance of the one-loop V_{eff}:

$$\mu \frac{d\, V_{eff}}{d\, \mu} = 0$$

$$\left(\beta_f \frac{\partial}{\partial f} + \beta_{m^2}\, m^2 \frac{\partial}{\partial m^2} - \gamma\, \phi \frac{\partial}{\partial \phi} \right) V_0 = - \frac{\partial V_1}{\partial \ln \mu}$$

$$\Downarrow$$

$$(4\pi)^2 \frac{dm^2}{dt} = m^2 \left(6\, f - \frac{9}{2} g^2 - \frac{3}{2} g'^2 + 2 \sum_i N_i\, h_i^2\right)$$

$$\begin{aligned} (4\pi)^2 \frac{df}{dt} \;=\; & 12\, f^2 - 9\, f g^2 - 3\, f g'^2 + \frac{9}{4} g^4 + \frac{3}{2} g^2 g'^2 \\ & + \frac{3}{4} g'^4 + 4 \sum_i N_i\, h_i^2 (f - h_i^2) \end{aligned}$$

$$\begin{aligned} \boxed{\frac{d\Lambda_{ind}}{dt}} \;&=\; \frac{m^4}{2\, f^2} \frac{d\, f}{dt} - \frac{m^2}{f} \frac{dm^2}{dt} \\ &=\; - \frac{1}{(4\pi)^2} \cdot \frac{2 m^4}{f^2} \cdot \sum_j h_j^4 = \boxed{- \frac{2}{(4\pi)^2} \sum_j m_j^4} \end{aligned}$$

◇ Important cancellation of the leading terms (for light fermions) $\boxed{m^4\, h_j^2/f}$!!

◇ One-loop contribution from fermions to vacuum cosmological term. Exactly the same as the induced term !! (checked!!)

$$\Downarrow$$

$$\boxed{(4\pi)^2\,\frac{d\Lambda_{ph}}{dt} = -\,4\,\sum_j m_j^4} \qquad \Lambda_{ph}(0) = 0)$$

◇ Assume that Λ is fine-tuned at the far IR and ask for neutrino contribution to Λ_{ph}. Take the "double-hierarchy" neutrino scenario

$$m_{\nu_e} \simeq m_{\nu_s} \sim 10^{-3}\, eV \ll m_{\nu_\mu} \simeq m_{\nu_\tau} \sim 1\, eV$$

It gives Λ_{ph} in the right ballpark but $\boxed{\Lambda_{ph} < 0}$!!.

◇ Assume the existence of a very light scalar $m_S \gtrsim 4\, m_\nu$. The new RGE for the physical Λ is

$$\boxed{(4\pi)^2 \frac{d\Lambda_{ph}}{dt} = \beta_\Lambda \equiv \tfrac{1}{2}\, m_S^4 \,-\, 4\,\sum_i\, m_{\nu_i}^4}$$

A light scalar?...Very easy to postulate , but very difficult to accomodate in Particle Physics!!.

◇ A pseudo-dilaton can do it!!. Construct a (non-linear) dilatation-invariant formulation of the SM following the rule

$$\boxed{m \to m\left(\frac{\chi}{M}\right) = m\, e^{S/M}}$$

◇ How can we expect to get any low-energy effect from a Goldstone boson? (only derivative couplings to matter!!).

◇ Due to the trace anomaly $\Rightarrow$ S becomes a pseudo-GH!! with mass

$$\boxed{m_S^2 = \frac{\Lambda_{QCD}^4}{M^2}}$$

where $\Lambda_{QCD} \sim 100\, MeV$. Taking $M \gtrsim 10^{10} GeV$

$$\boxed{m_S \lesssim \mathcal{O}(10^{-3})\, eV!!.}$$

◇ For a nucleon N, the "Cosmon charge" is given by the nucleonic matrix element

$$\begin{aligned} Q &= \kappa \left\langle N \mid \Theta^{\mu}_{\mu} \mid N \right\rangle \\ &= \kappa \left\langle N \mid \frac{\beta(g_3)}{2g_3} F^{\mu}_{a} F^{a}_{\mu} + m_q \gamma(g_s) \overline{q} q \mid N \right\rangle \end{aligned}$$

where

$$\kappa = \sqrt{\frac{16\pi h}{1+12h}}\,.$$

and

$$h \equiv \frac{1}{16\pi} \frac{M_P^2}{M^2}\,,$$

For $M \ll M_P$

$$\boxed{\frac{\kappa^2}{4\pi} \;\rightarrow\; \frac{1}{3}}$$

$$\Downarrow$$

Cosmon exchange between bulks of matter should be testable in the next generation of experiments: it's a composition-dependent macroscopic force of the order of 1/3 of gravity and with a range of order $0.1\,mm$.

Conclusions

- Λ is a parameter allowed by general covariance and there is no reason whatsoever to reject it in Einstein's equations.

- All attempts to "sweep Λ under the rug" have turned into an ever increasing theoretical and experimental fiasco.

- Present day experimental observations point towards $\Lambda \neq 0$ at the 99.7% C.L.

- Its present value and positive sign helps to understand kinematics of cosmic evolution (e.g. solves the age problem). It predicts that the Universe will expand forever.

- The dynamical origin of Λ is completely unknown.

- The Inflationary Paradigm is compatible with the observed Λ. A great deal of dark matter could just be "dark Λ".

- The fact that $\Lambda \neq 0$ severely aggravates the Cosmological Constant Problem from the point of view of the blatant inconsistency of our most cherished QFT's with astronomic observations.

- Maybe the origin of Λ is connected with the existence of new, very light, degrees of freedom, like the Cosmon field $\Rightarrow$ Testable hypothesis: existence of new (sub-millimeter), composition-dependent, macroscopic forces of $\boxed{\sim 1/3(\alpha_G)}$.

$\Downarrow$

$\diamond$ The Cosmological Constant is the "deepest mystery" - the "smallest and biggest" parameter– in our Universe...