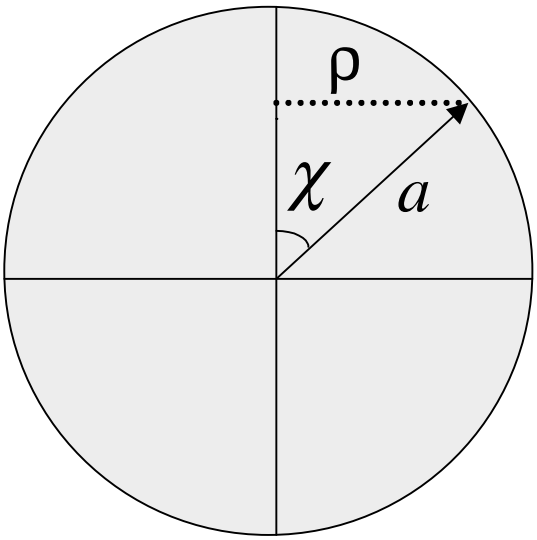
Einstein's World (2D)



$$a = \frac{1}{\sqrt{\lambda_E}}$$

$$\lambda_E \equiv 4\pi G_N \rho_{m0}$$

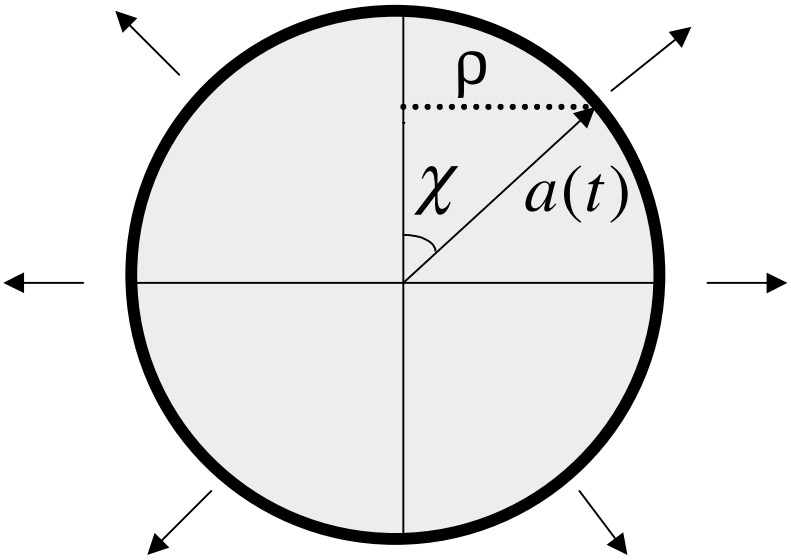
$$ds^2 = dt^2 - a^2 \left[\frac{dr^2}{1 - r^2} + r^2 d\phi^2 \right]$$

$$\rho = ar = a \sin\chi$$

$$ds^2 = dt^2 - (a^2 d\chi^2 + \rho^2 d\phi^2)$$

$$ds^2 = dt^2 - a^2 (d\chi^2 + \sin^2 \chi d\phi^2)$$

Friedmann-L-R-W World

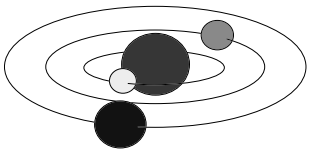


$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + \sin^2\theta d\phi^2\right]$$

$$\rho = ar = a\sin\chi$$

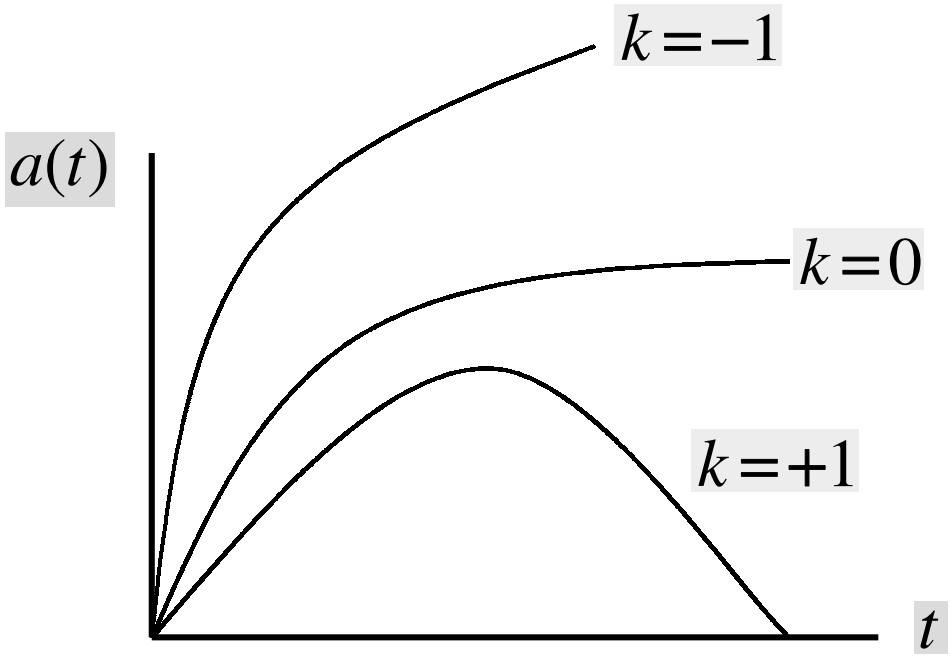
$$k = \begin{cases} -1 & (hyperbolic \rightarrow open) \\ 0 & (flat \rightarrow Euclidean) \\ +1 & (elliptic \rightarrow closed) \end{cases}$$

$$Curvature \not\rightarrow (in)finiteness\ of\ space$$

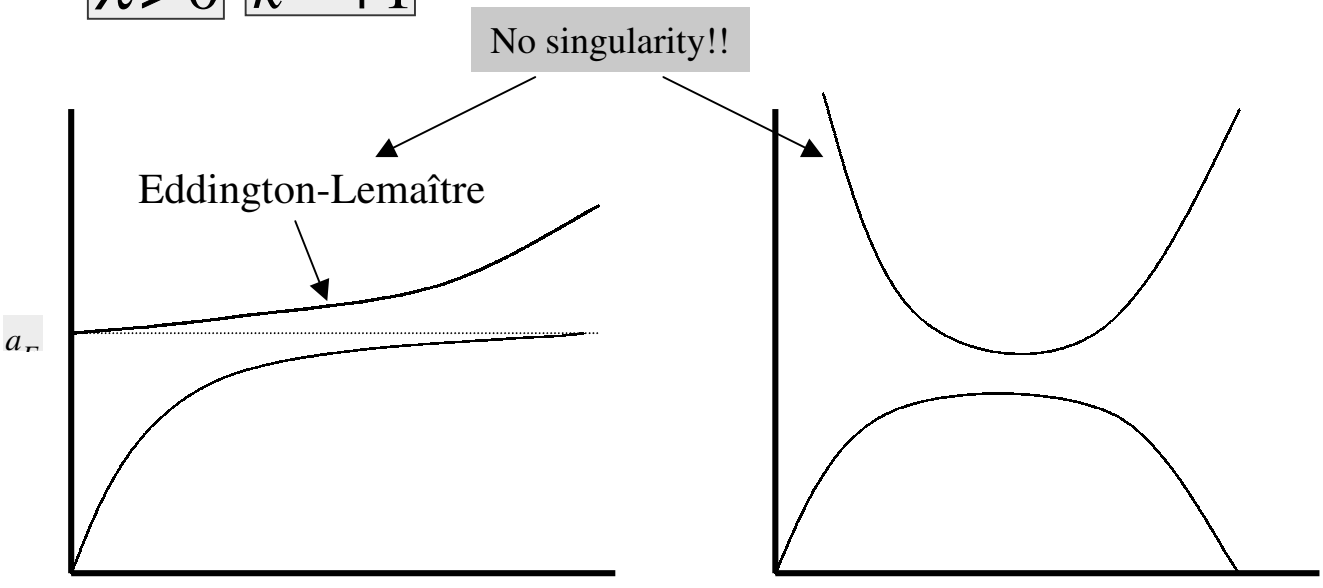


Cosmologies with $\lambda=0$, $\lambda\neq 0$

$\lambda=0$



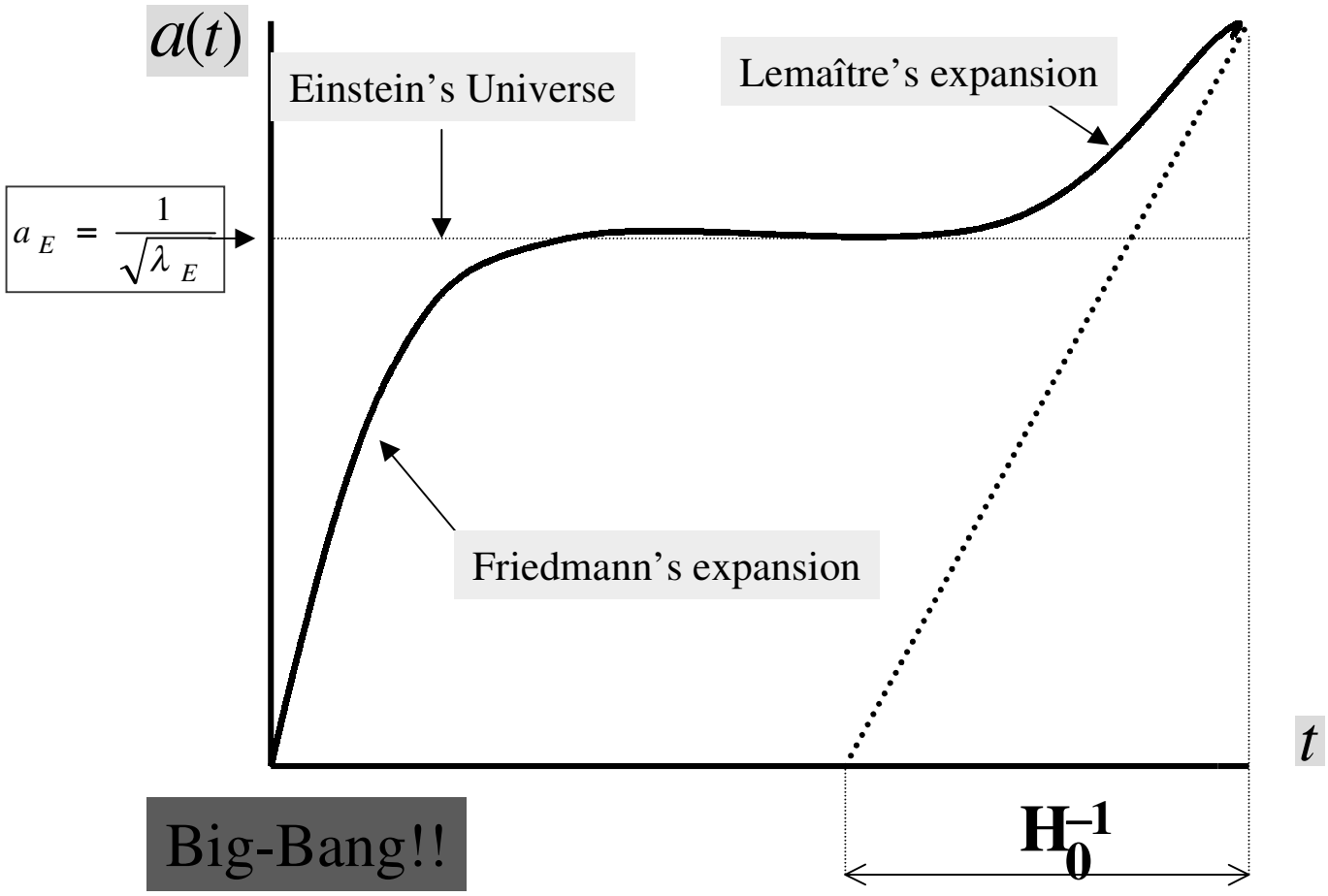
$\lambda>0$ $k=+1$



$\lambda>0$ $k=0,-1$ → expand recollapse

$\lambda<0$ → recollapse

Lemaître's hesitation Universe

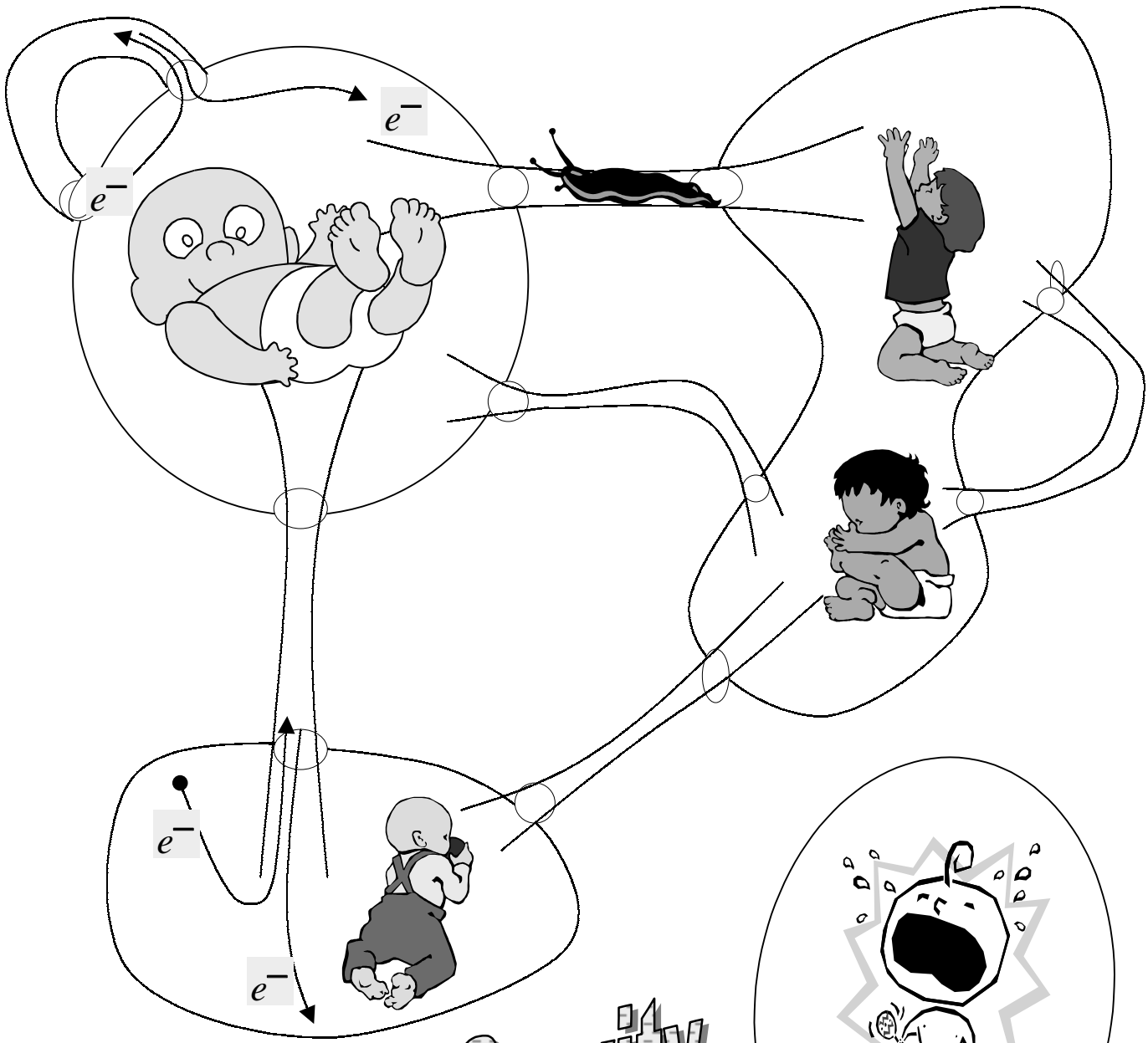


$\lambda > 0$ $k = +1$

Critical value $\longrightarrow$ $\lambda_c = \lambda_E \equiv 4\pi G_N \rho_{m0}$ $\longrightarrow$ $\lambda = \lambda_c + \varepsilon$

Used in the past to explain the age puzzle

Baby Universes and wormholes



Euclidean Gravity

$$Z_E = \int Dg\, D\phi\, \exp\left[-S_E(g_{\mu\nu},\phi,\alpha_i)\right]$$