

Test I

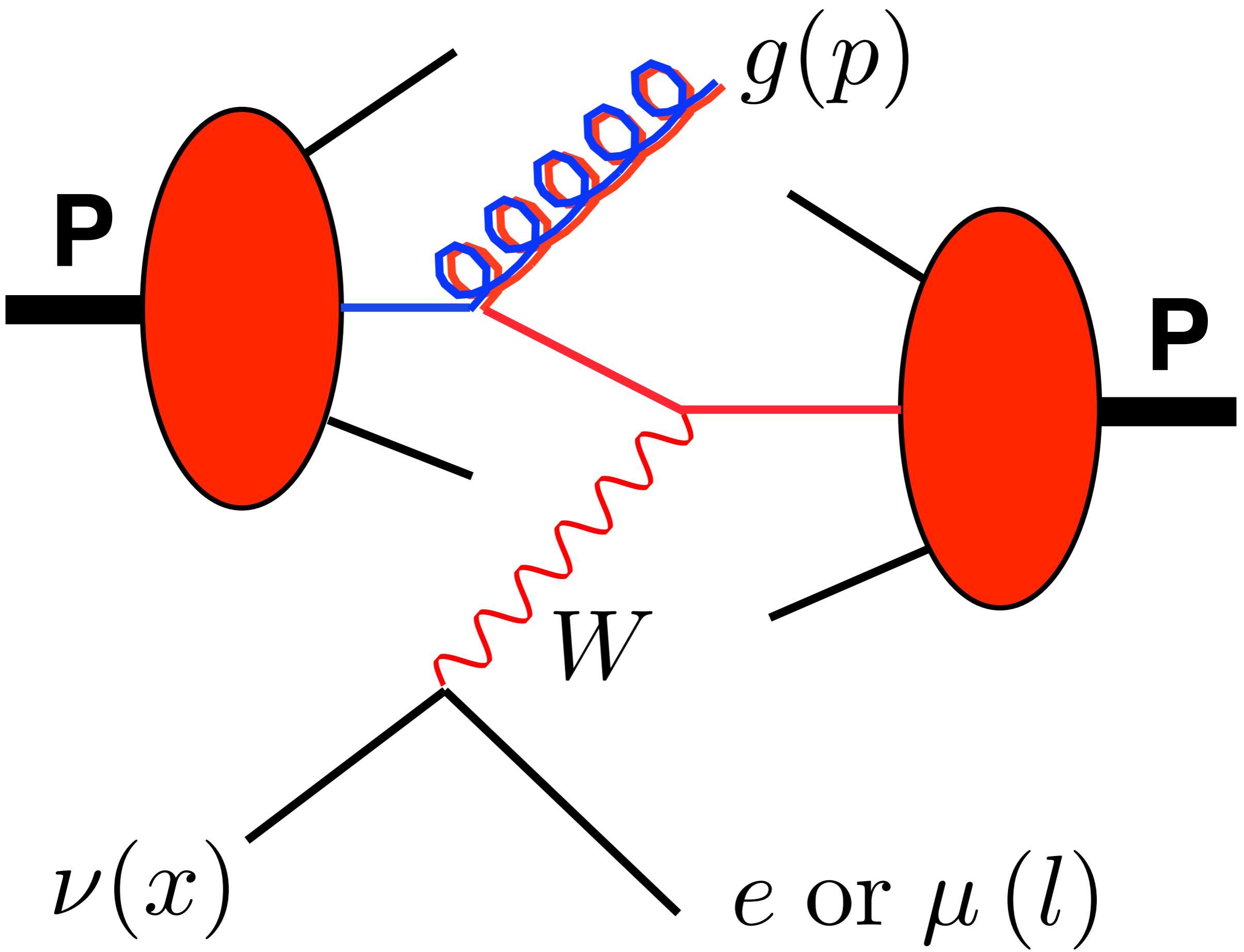
- Test 2

Test III

...

...

...



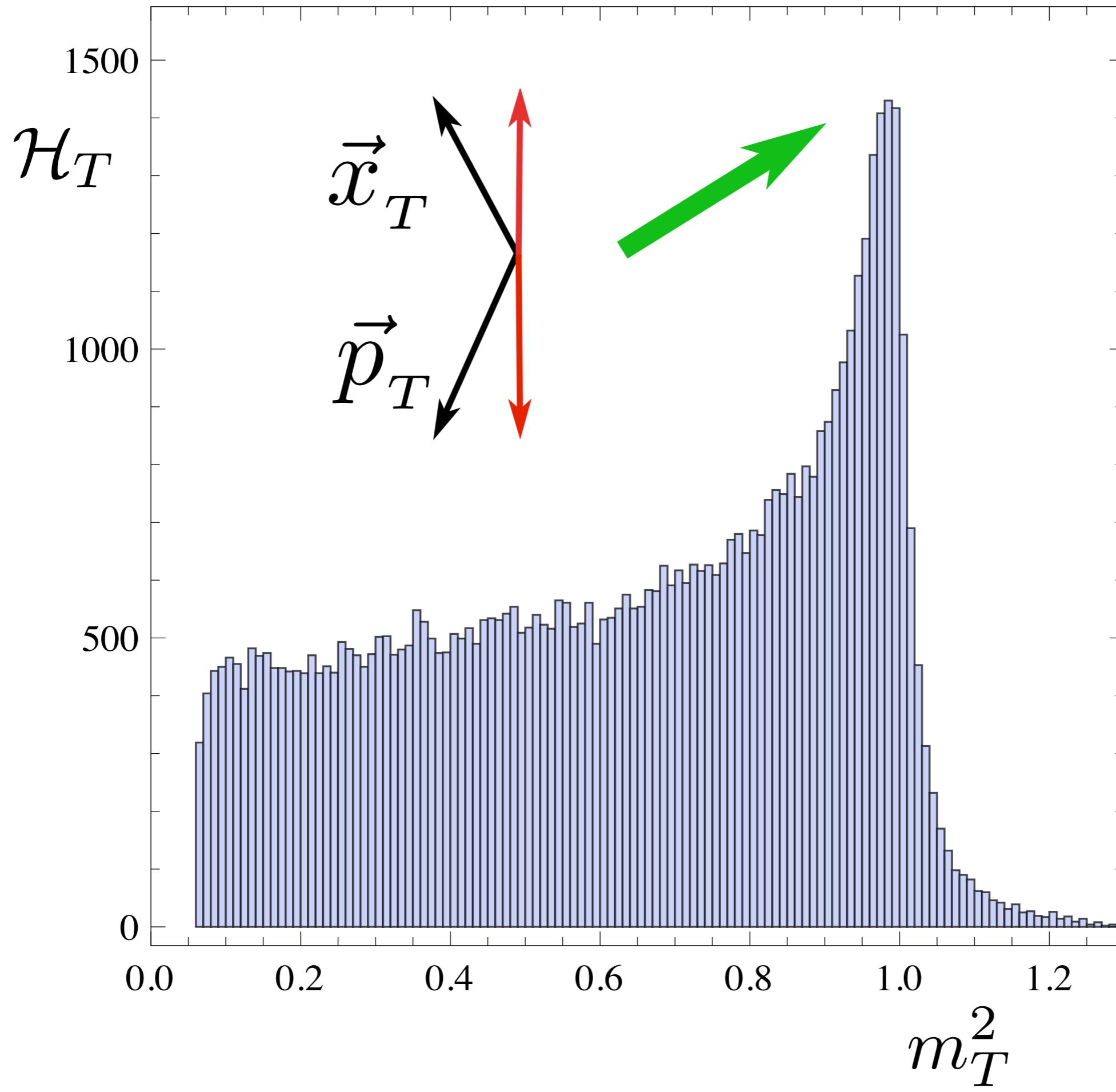
$$M^2(l, x) = 2(|\vec{l}| |\vec{x}| - \vec{l} \cdot \vec{x})$$

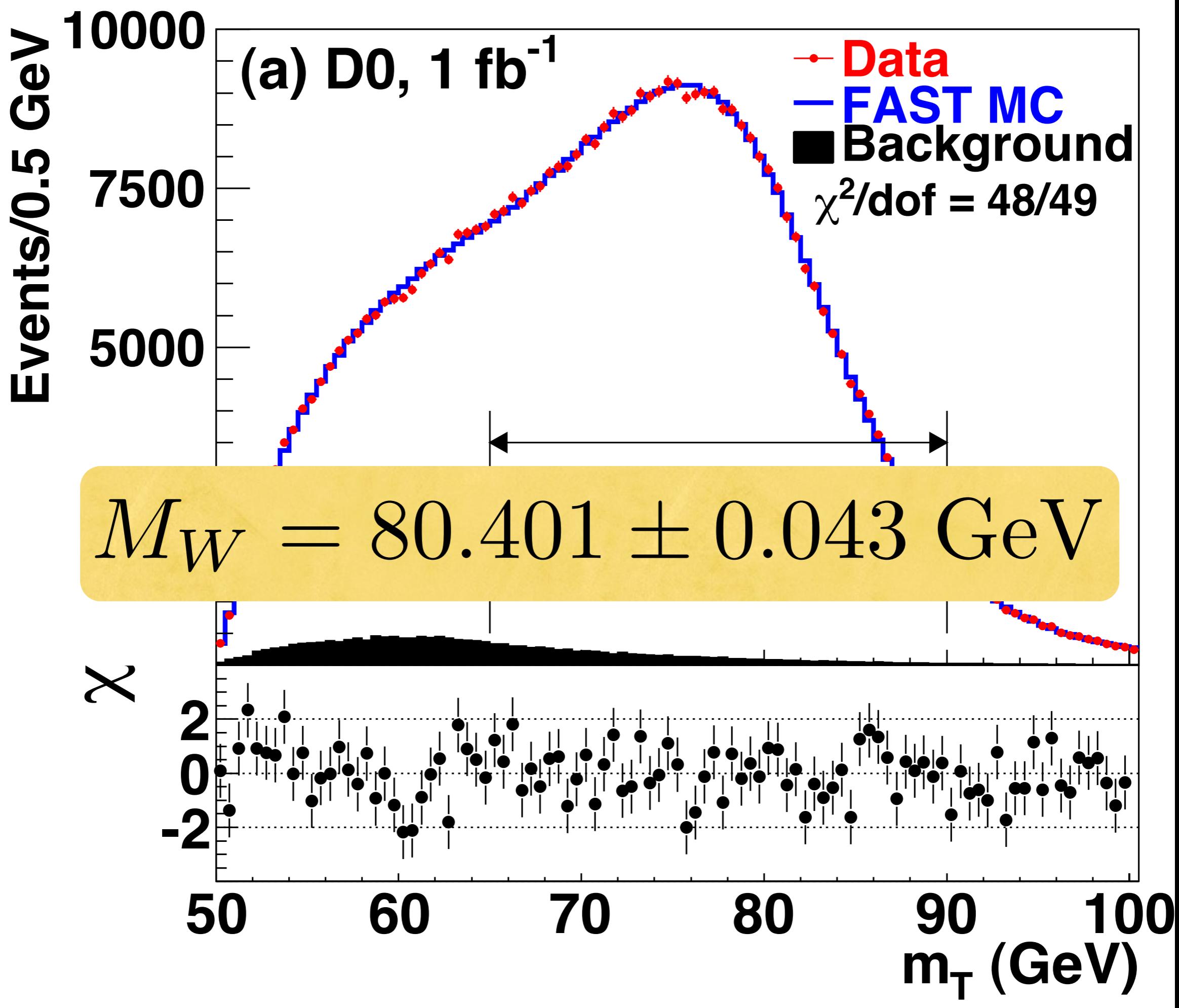
$$\vec{l}_T + \vec{x}_T + \vec{p}_T = 0 \Rightarrow \vec{x}_T$$

$$l_3 + x_3 + \cancel{p_3} = 0 \Rightarrow \cancel{x_3}$$

$$M_T^2(\vec{l}_T, \vec{p}_T) = 2(l_T x_T - \vec{l}_T \cdot \vec{x}_T) =$$

$$2 l_T x_T [1 - \cos \Delta\Phi(\vec{x}_T, \vec{l}_T)]$$





Limitations

Electron E calibration (Z)

>

PDF uncertainties

(pp , not $p\bar{p}$)

>

Statistics

$$E_1 \Rightarrow x^2 = 0$$

$$E_2\Rightarrow 2\,l\cdot x=M^2-m_l^2\approx M^2$$

$$E_3\Rightarrow l_1+x_1+p_1=0$$

$$E_4\Rightarrow l_2+x_2+p_2=0$$

$$\{x_0,x_1,x_2,x_3;\,M_W\}$$

With incomplete
information ...

How to optimize the
measurement of

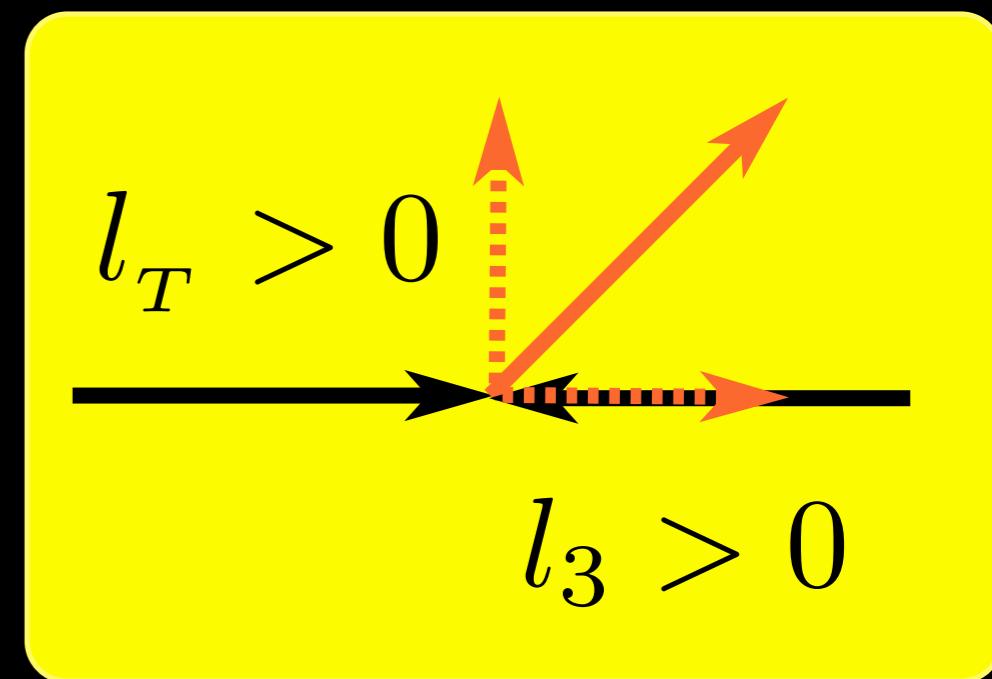
M_W ???

$$\vec{p}_T = 0$$

Eliminate x_0, x_1, x_2

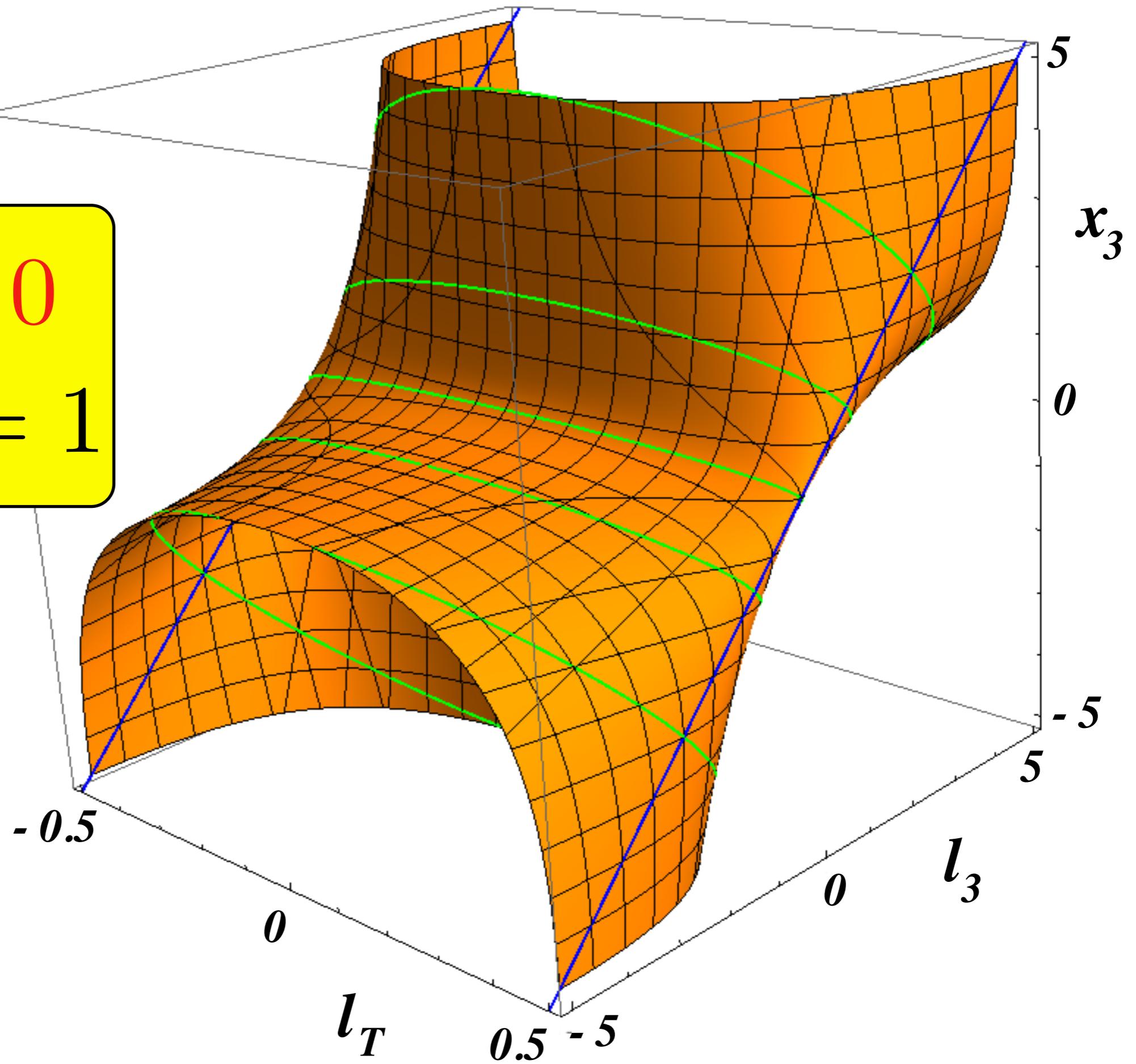
$$\begin{aligned}\Phi(l_T, l_3, x_3, M) \equiv & \\ (M^2 + 2l_3x_3 - 2l_T^2)^2 & \\ - 4l_0^2(l_T^2 + x_3^2) & = 0\end{aligned}$$

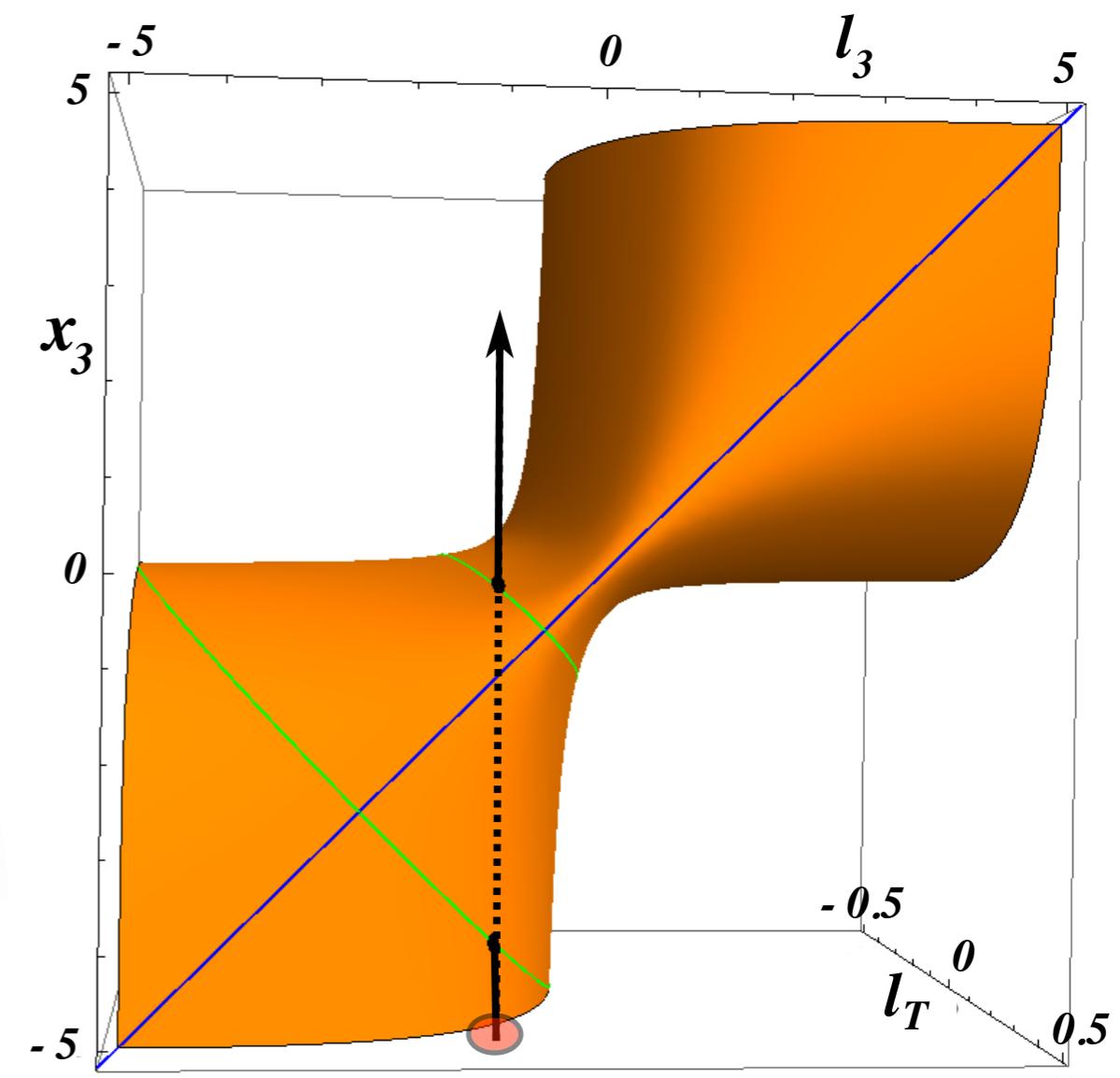
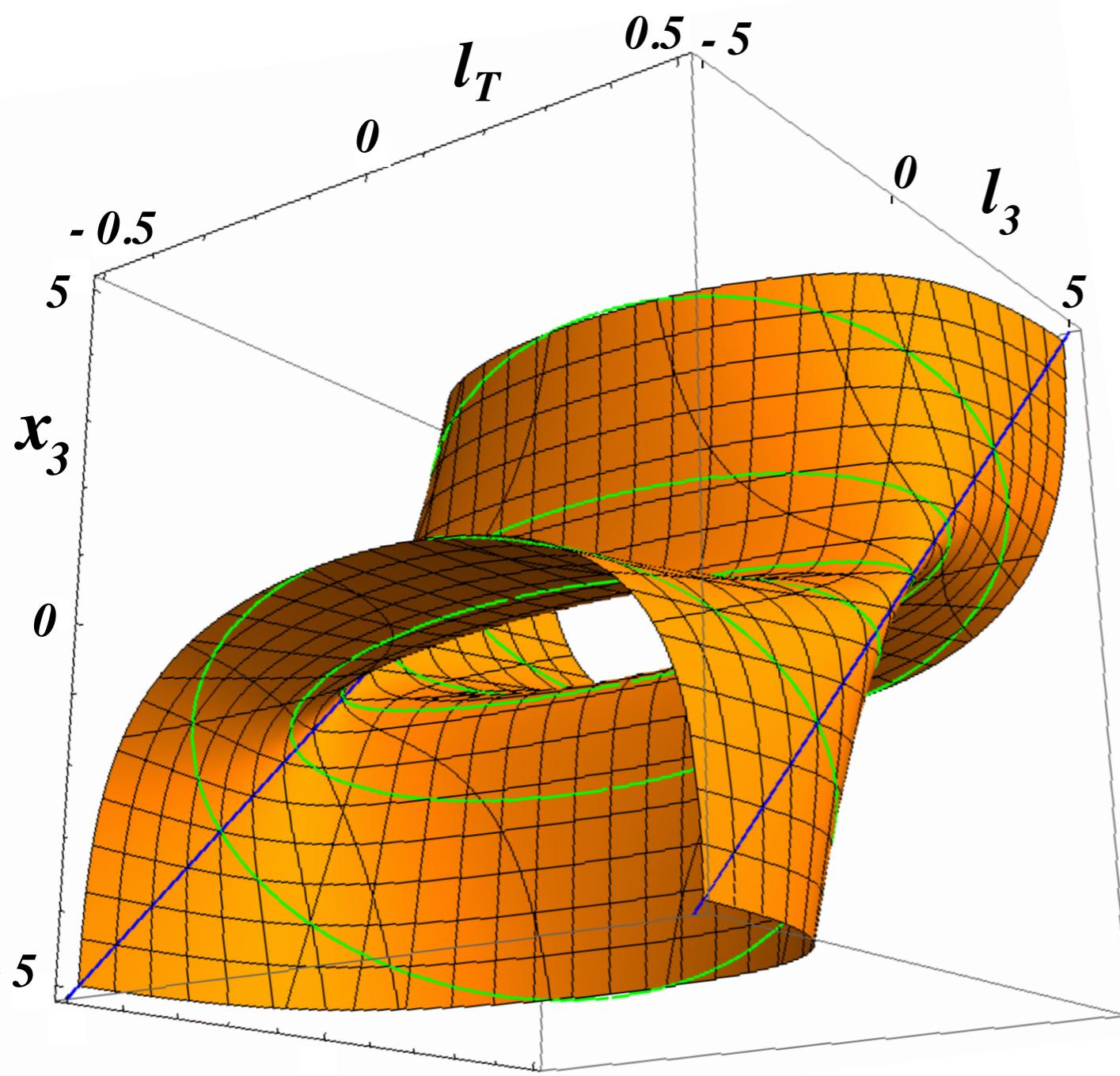
$$l_0 \equiv +\sqrt{l_T^2 + l_3^2}$$



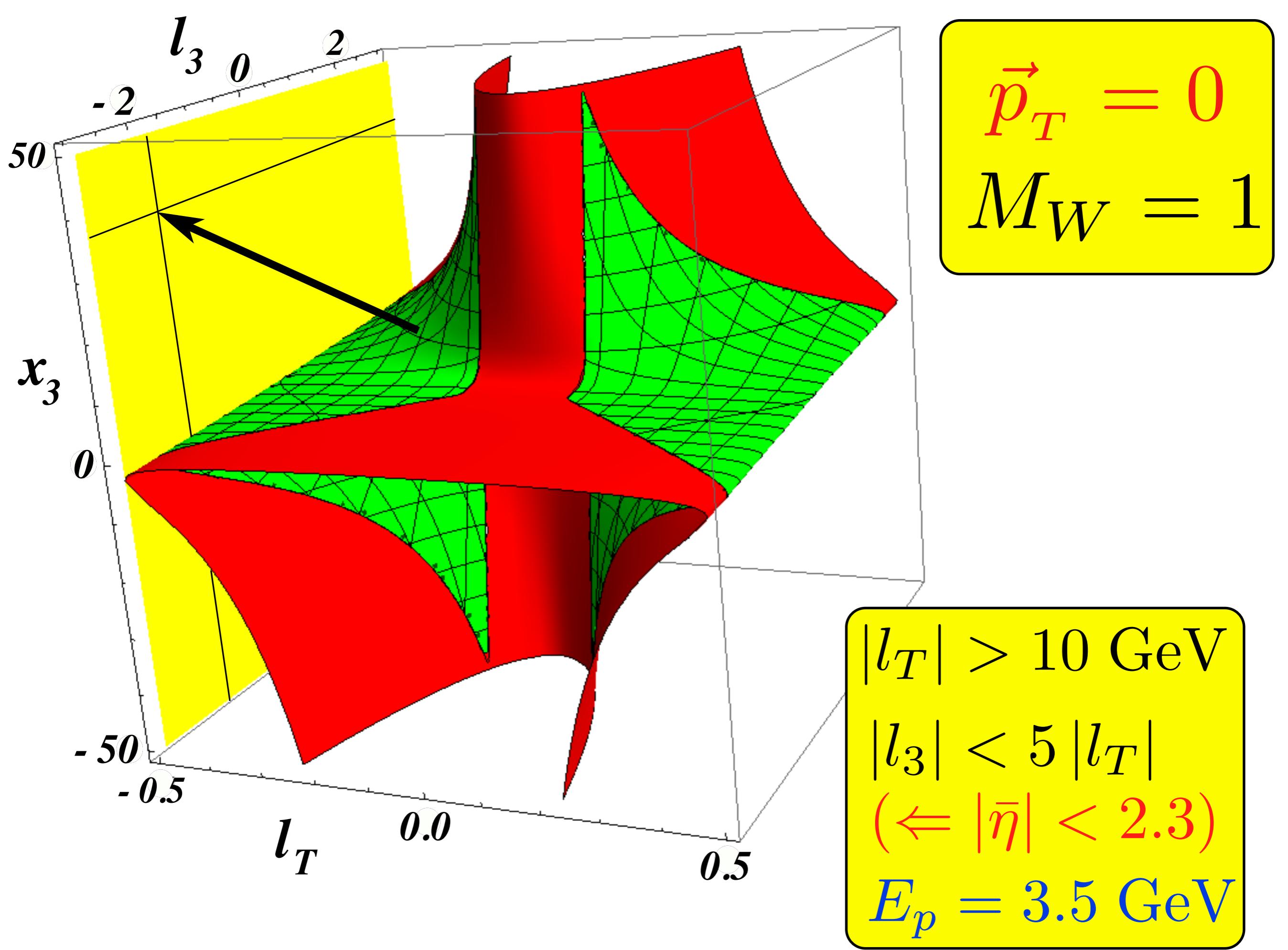
$$\vec{p}_T = 0$$

$$M_W = 1$$





l_T, l_3 measured
 \Rightarrow two values of x_3 ... at fixed M_W



$$0=\Phi(l_3,x_3,l_T,\cos\theta,p_T,M)\equiv$$

$$\left(-2\,l_T(\cos\theta\,p_T+l_T)+2\,l_3x_3+M^2\right)^2$$

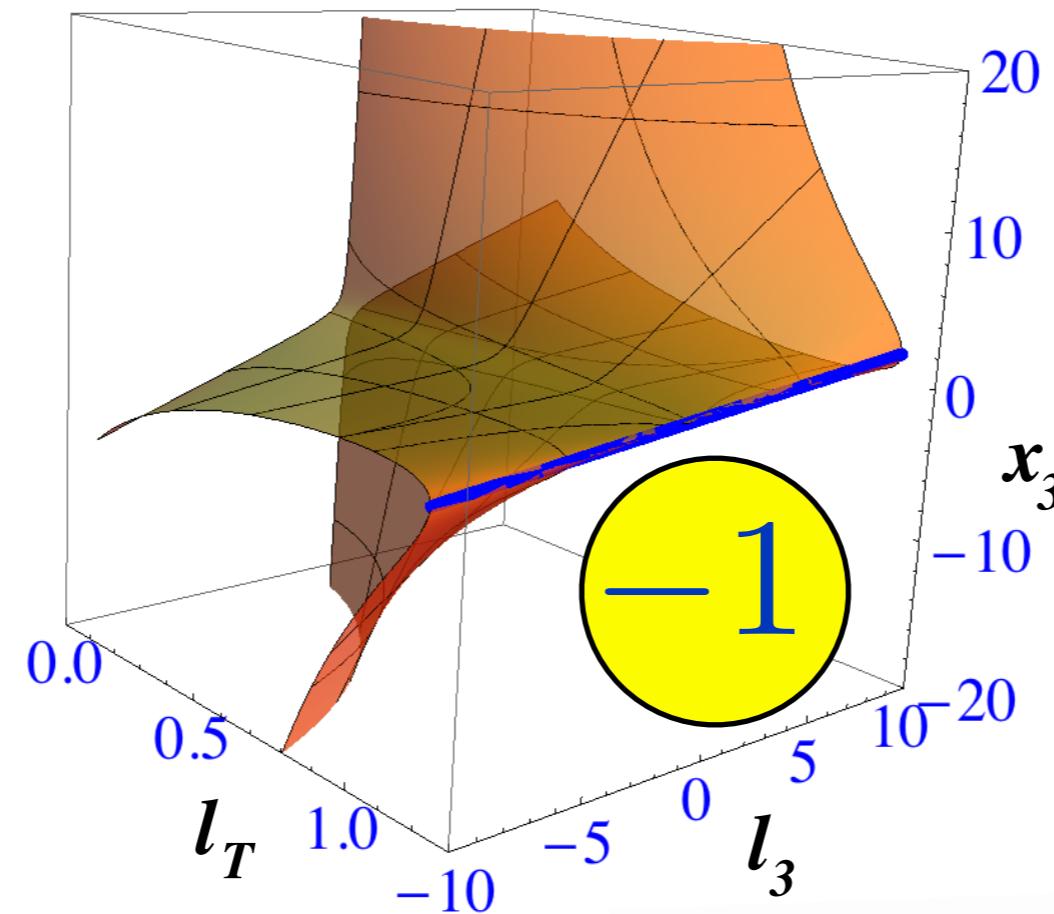
$$-4\left({l_3}^2+{l_T}^2\right)\times$$

$$\left(2\cos\theta\,l_T\,p_T+{l_T}^2+{p_T}^2+{x_3}^2\right)$$

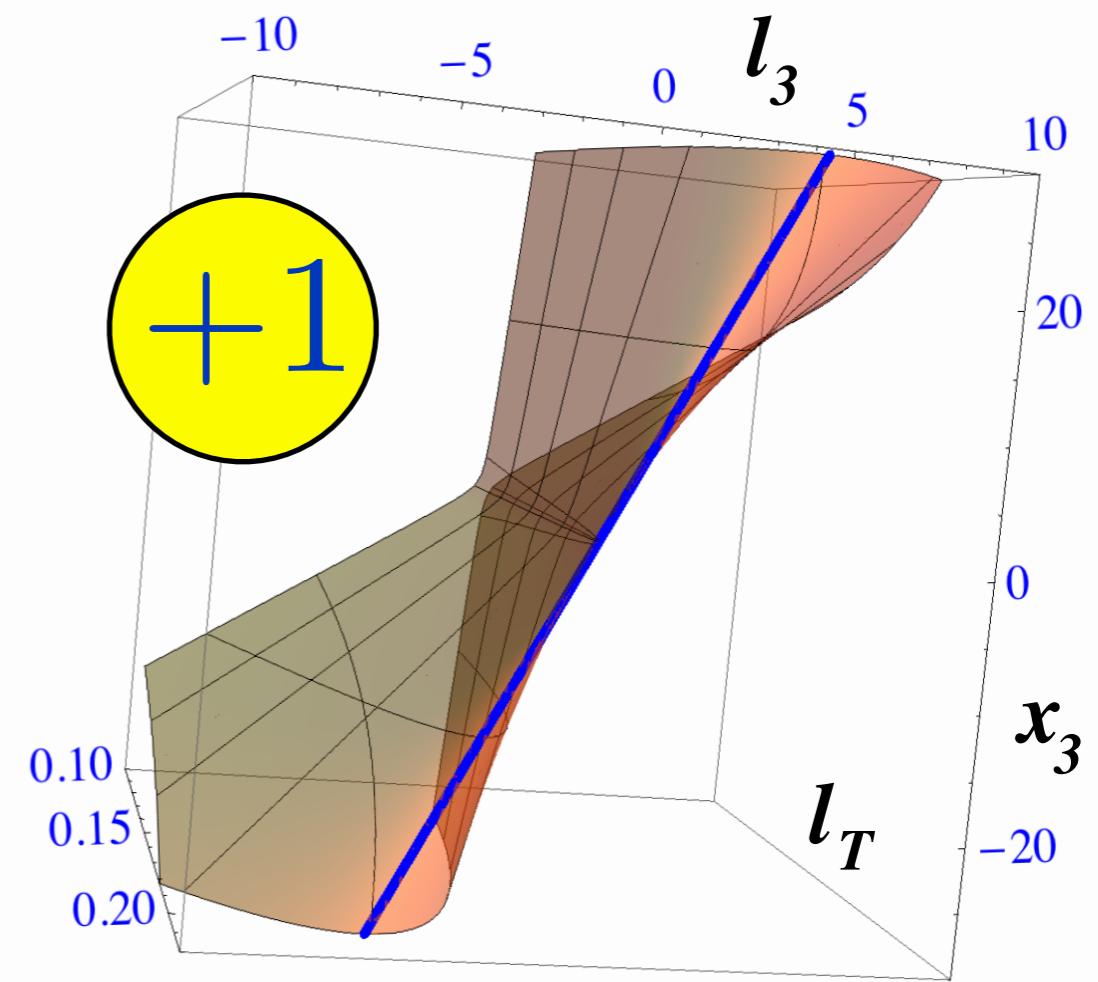
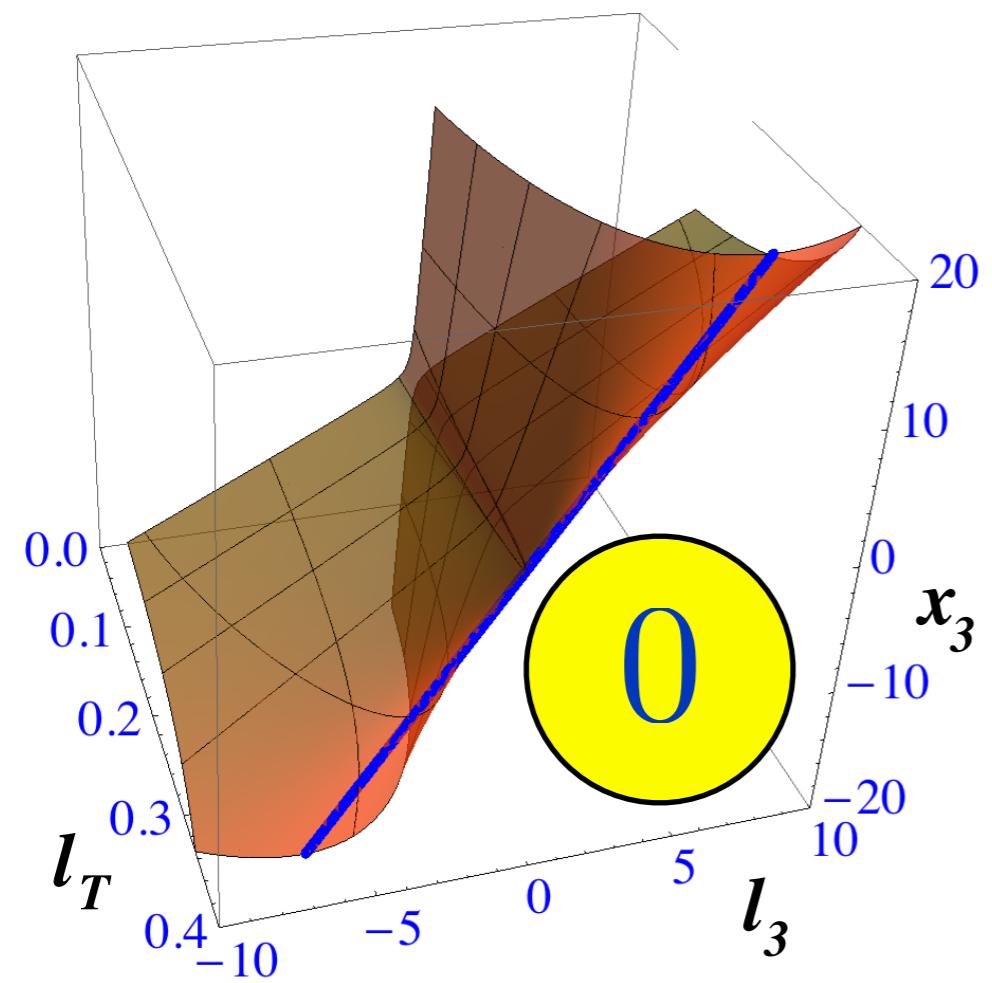
$$\theta \wedge \{\vec{p}_T, \vec{l}_T\}$$

$$|\vec{p}_T| = 1$$

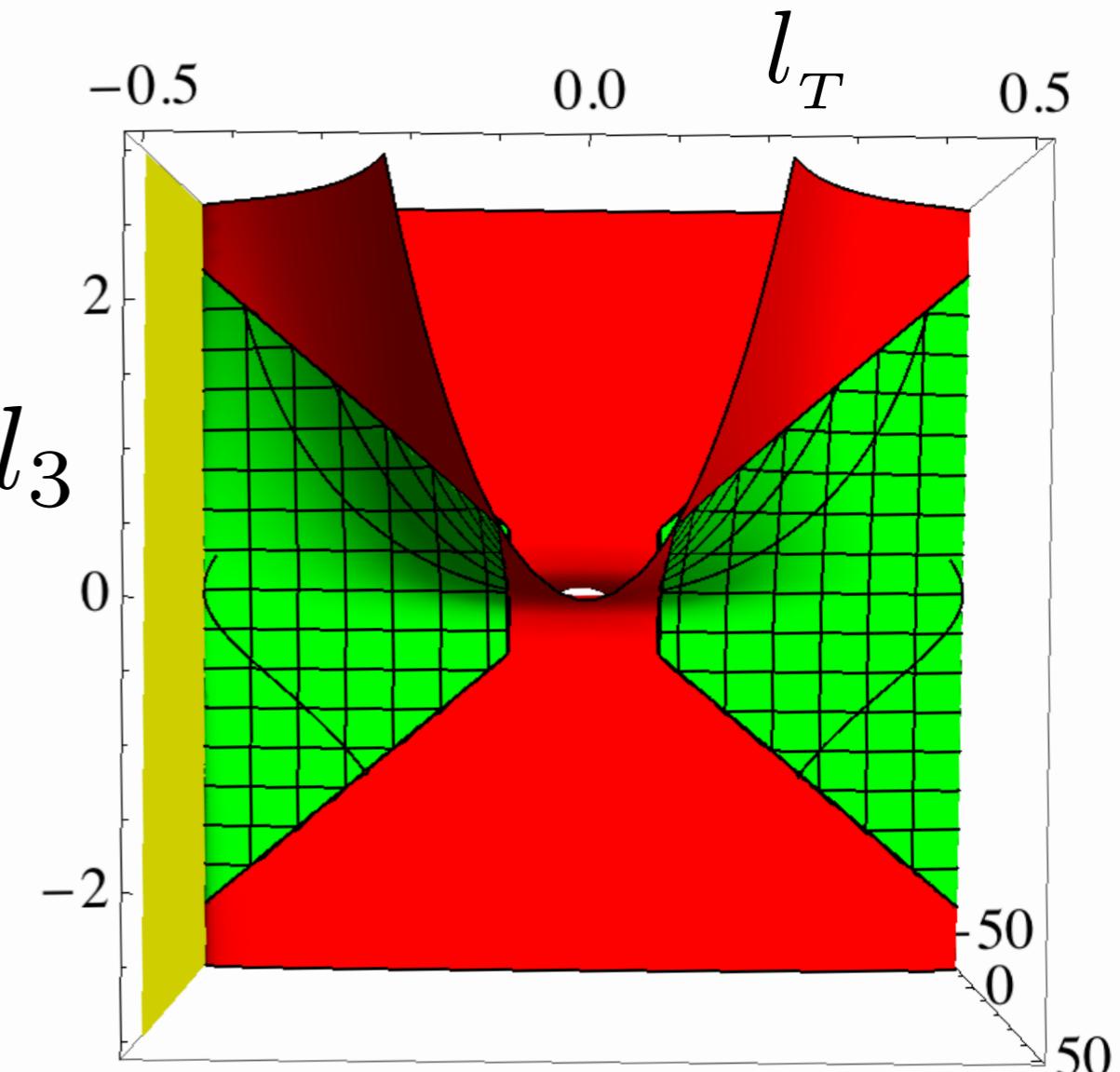
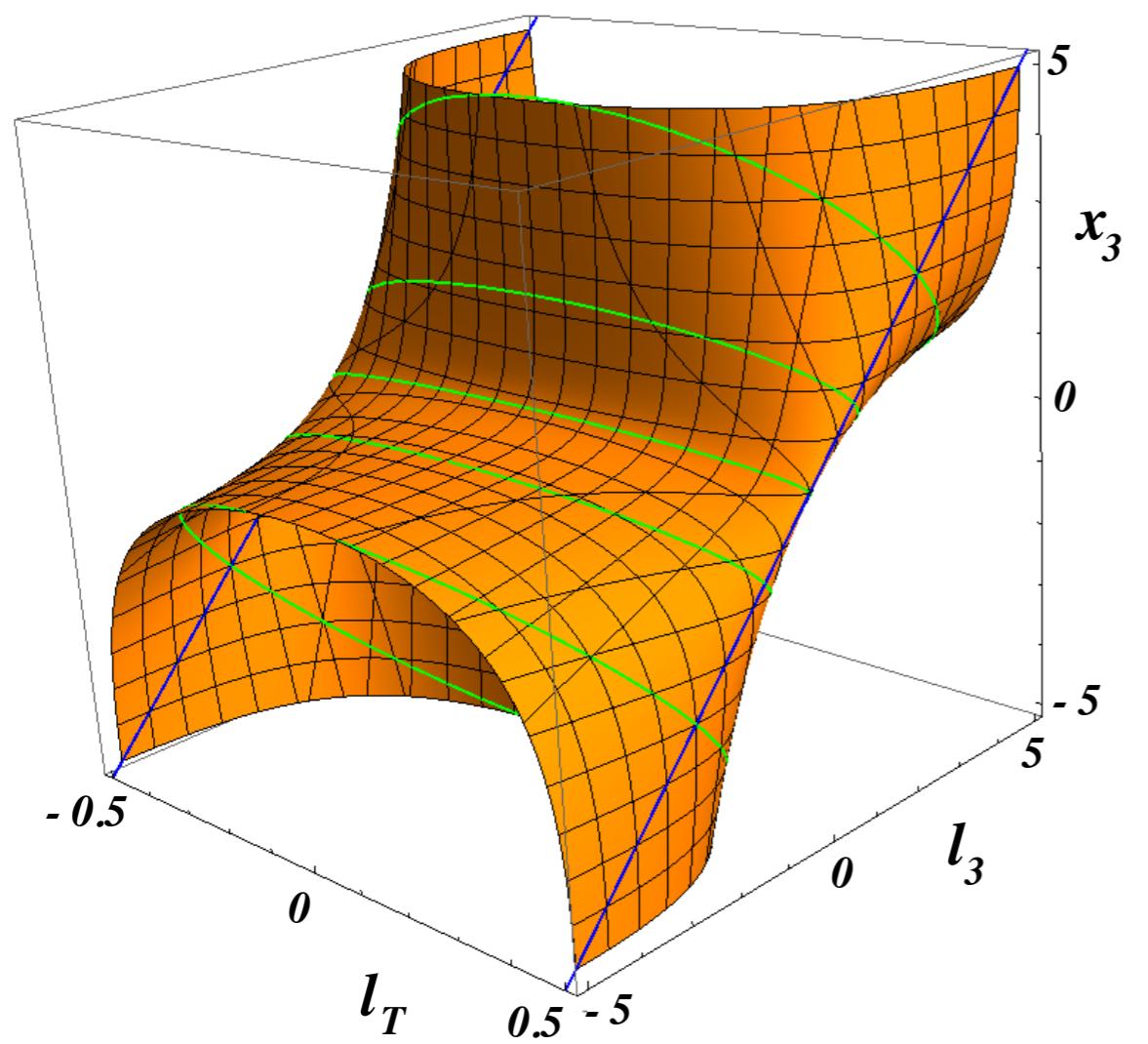
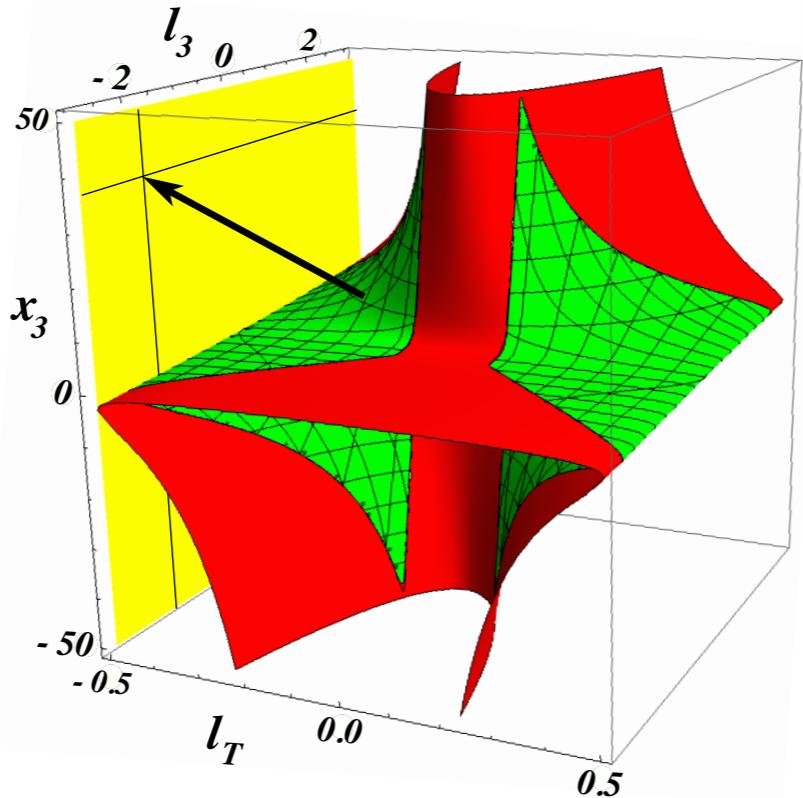
$$M_W = 1$$



$$\cos \theta =$$



$\vec{p}_T = 0$
 $M_W = 1$



$$\partial\Phi(l_T, l_3, x_3, M)/\partial x_3 = 0$$

$$\Phi(l_T, l_3, x_3, M) = 0$$

eliminate $M \Rightarrow x_3 = l_3$

eliminate $x_3 \Rightarrow 4l_T^2 = M^2$

Formal SINGULARITY CONDITION

I.W. Kim arXiv:0910.1149v1

$$D_{ij} \equiv \partial E_i / \partial x_j$$

SC: Rank(D) at Singularity < Rank(D) elsewhere

$$D = \frac{\partial(E_1, E_2, E_3, E_4)}{\partial(x_0, x_1, x_2, x_3)} = 2 \begin{pmatrix} x_0 & -x_1 & -x_2 & -x_3 \\ l_0 & -l_1 & -l_2 & -l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$E_{SC} \Rightarrow \text{Det } D \propto l_0 x_3 - l_3 x_0 = 0$$

$$E_1 \Rightarrow x^2 = 0$$

$$E_2 \Rightarrow 2 l \cdot x = M^2$$

$$E_3 \Rightarrow l_1 + x_1 + p_1 = 0$$

$$E_4 \Rightarrow l_2 + x_2 + p_2 = 0$$

$$E_{SC} \Rightarrow l_0 x_3 - l_3 x_0 = 0$$

$$\{x_0, x_1, x_2, x_3, M\}$$

Eliminate $x_0, x_1, x_2, M \Rightarrow x_3 = l_3$

Eliminate $x_0, x_1, x_2, x_3 \Rightarrow \text{SC}(l, p)$

$$0 = \Sigma_T(M, \vec{l}_T, \vec{p}_T) \equiv$$

$$M^4 - 4M^2 (\vec{l}_T \cdot \vec{p}_T + l_T^2)$$

$$+ 4 \left[(\vec{l}_T \cdot \vec{p}_T)^2 - l_T^2 p_T^2 \right]$$

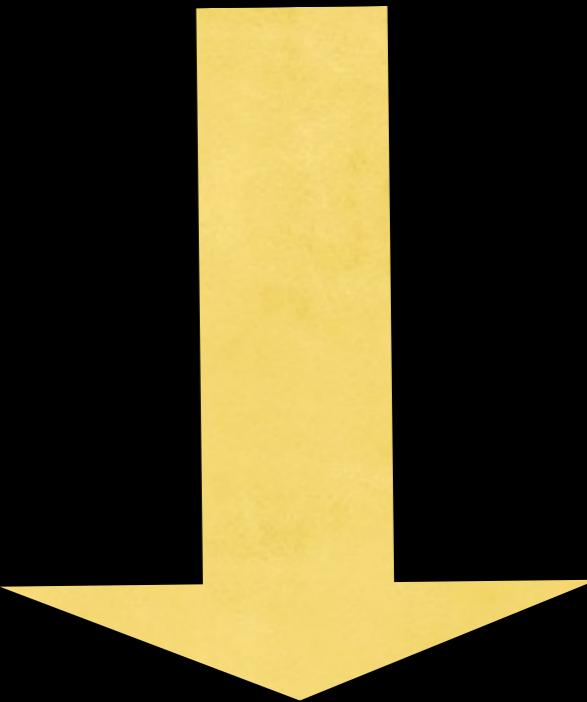
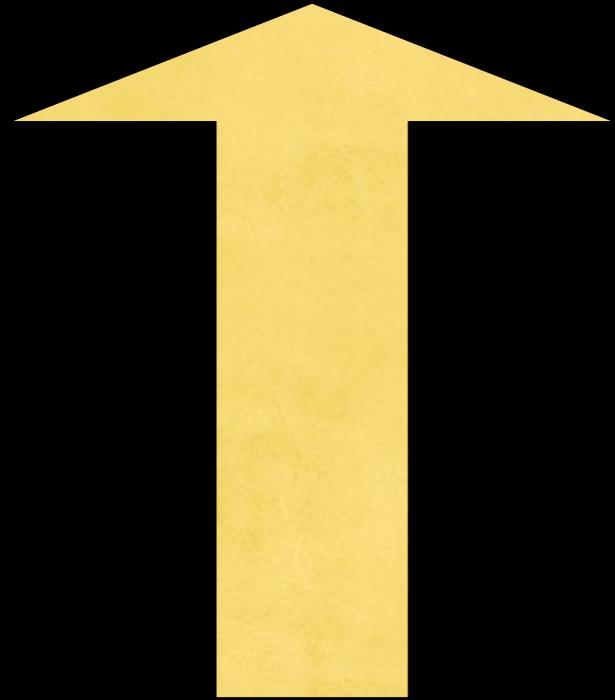
$$M_T^2(\vec{l}_T, \vec{p}_T) \equiv$$

$$2 \left[|l_T| |p + l|_T + \vec{l}_T \cdot (\vec{l}_T + \vec{p}_T) \right]$$

$$= 2 l_T x_T [1 - \cos \Delta\Phi(\vec{x}_T, \vec{l}_T)] !!!$$

Singularity Conditions

Singularity
VARIABLES



A Trivial “Euclidean” example; Gripaios, arXiv:1005.1229

$$\Phi := x^2 + l^2 - M^2 = 0$$

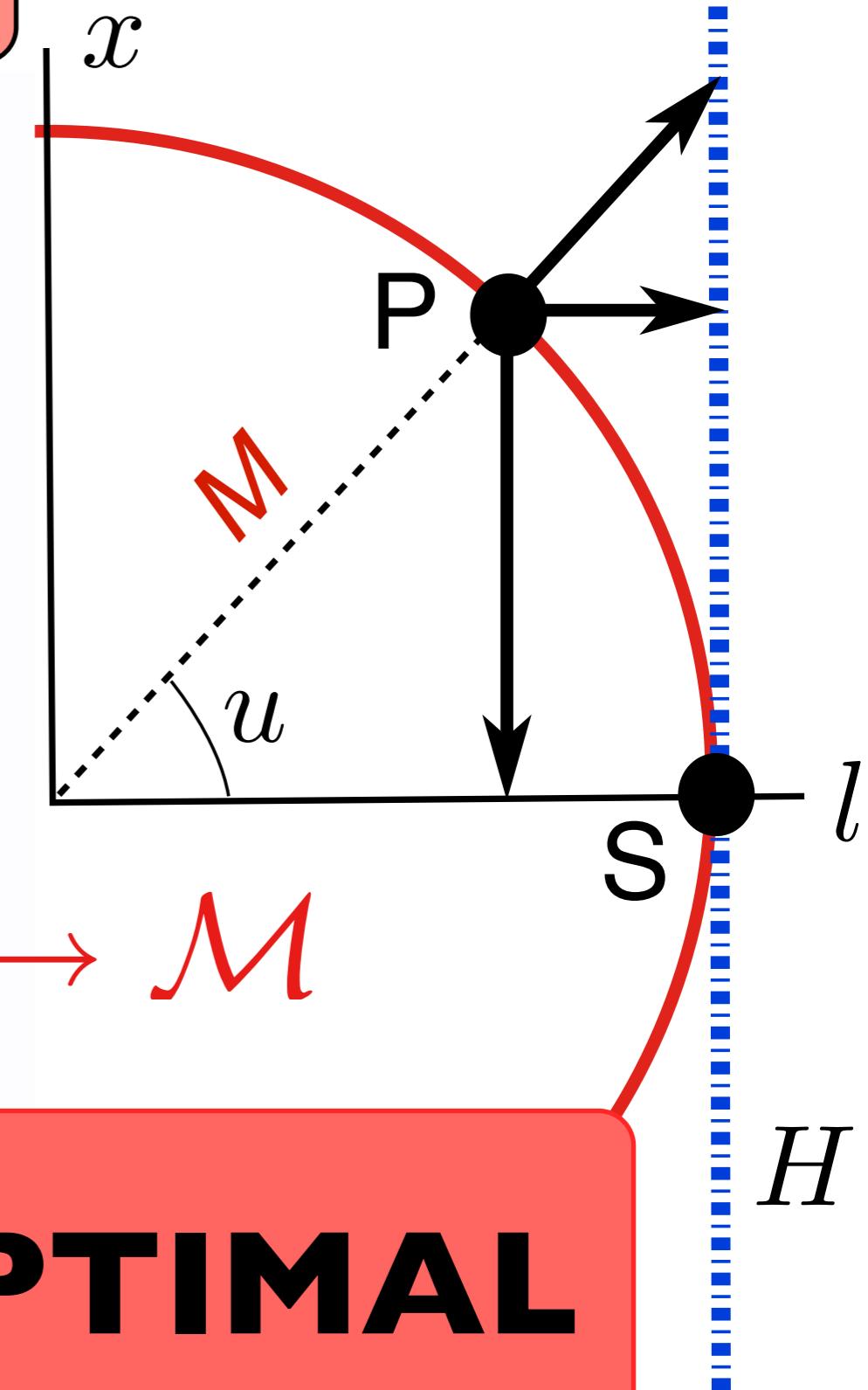
$$\text{SC} \Rightarrow D = \partial\Phi/\partial x = 2x$$

$$\text{Reduced Rank} \Rightarrow x = 0$$

$$\Sigma_K(\mathcal{M}, l) = u^2 \equiv$$

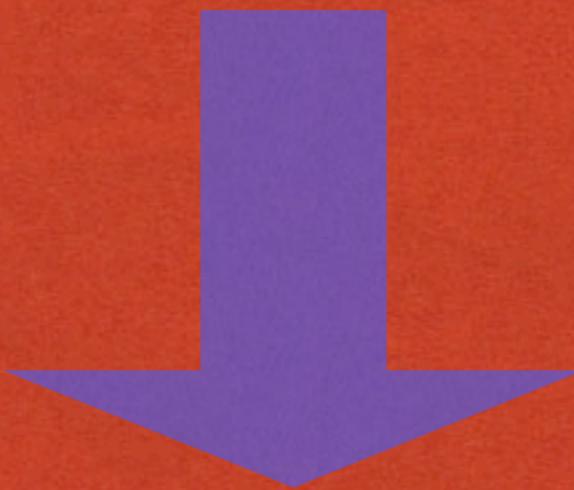
$$\left[\arccos \frac{|l|}{\mathcal{M}} \right]^2$$

$$M \rightarrow \mathcal{M}$$

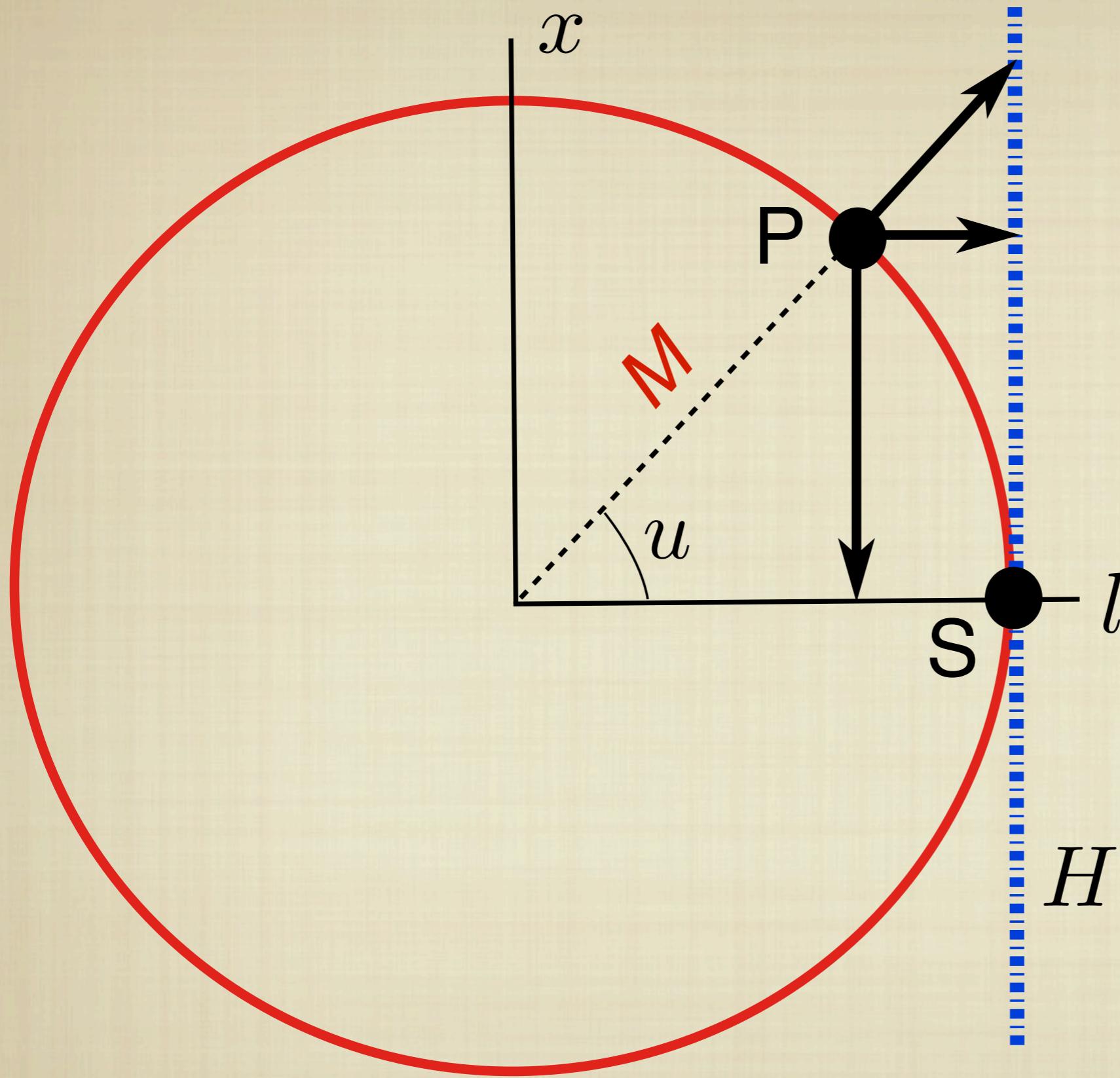


General but not OPTIMAL

M a letter



M everything
else



MANY
DIFFERENT
VARIABLES
MEASURE
THE DISTANCE
TO THE
SINGULARITY

$$\Sigma_K(\mathcal{M}, l) = u^2 \equiv \left[\arccos \frac{|l|}{\mathcal{M}} \right]^2$$

$$\Sigma_1(\mathcal{M}, l) = 2 [1 - \cos u]$$

$$\Sigma_2(\mathcal{M}, l) = 2 [1/\cos u - 1]$$

$$\Sigma_3(\mathcal{M}, l) = \sin^2 u$$

$$\int dx dl \delta(x^2 + l^2 - M^2) \\ \delta[\sigma - \Sigma_i(\mathcal{M}, l)]$$

$$= \mathcal{H}_i(\sigma, M, \mathcal{M}) \equiv \frac{dN}{d\sigma}$$

The Σ Contest

$$\hat{\chi}_i^2(\sigma) \equiv$$

$$\frac{1}{\mathcal{H}_i(\sigma, M, M)} \left[\frac{\partial \mathcal{H}_i(\sigma, M, \mathcal{M})}{\partial \mathcal{M}} \right]_{\mathcal{M}=M}^2$$

$$D_i = \int_{\sigma_{\min}}^{\sigma_{\max}} \hat{\chi}_i^2(\sigma) d\sigma$$

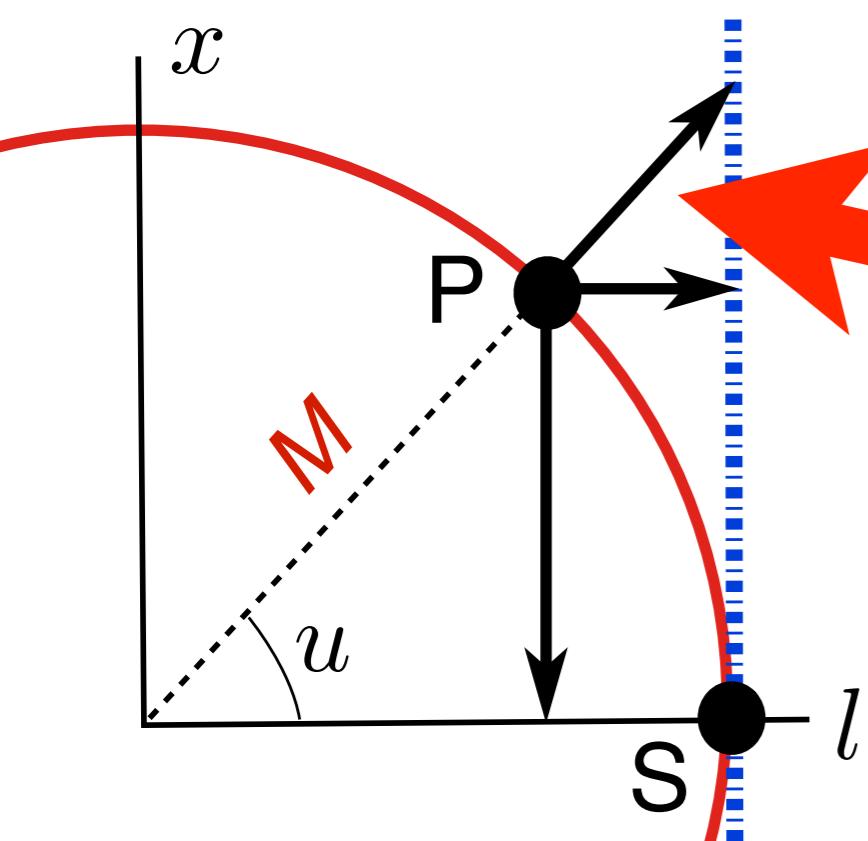
$$\Sigma_K(\mathcal{M}, l) = u^2 \equiv \left[\arccos \frac{|l|}{\mathcal{M}} \right]^2$$

$$\Sigma_1(\mathcal{M}, l) = 2 [1 - \cos u]$$

$$\Sigma_2(\mathcal{M}, l) = 2 [1/\cos u - 1]$$

$$\Sigma_3(\mathcal{M}, l) = \sin^2 u$$

$$\Sigma_2 \succ \Sigma_K \succ \Sigma_1 \succ \Sigma_3$$



$$\Sigma_2$$

$$x^2 + l^2 = M^2$$

OF COURSE

Dalitz plot

More Realistic Instances

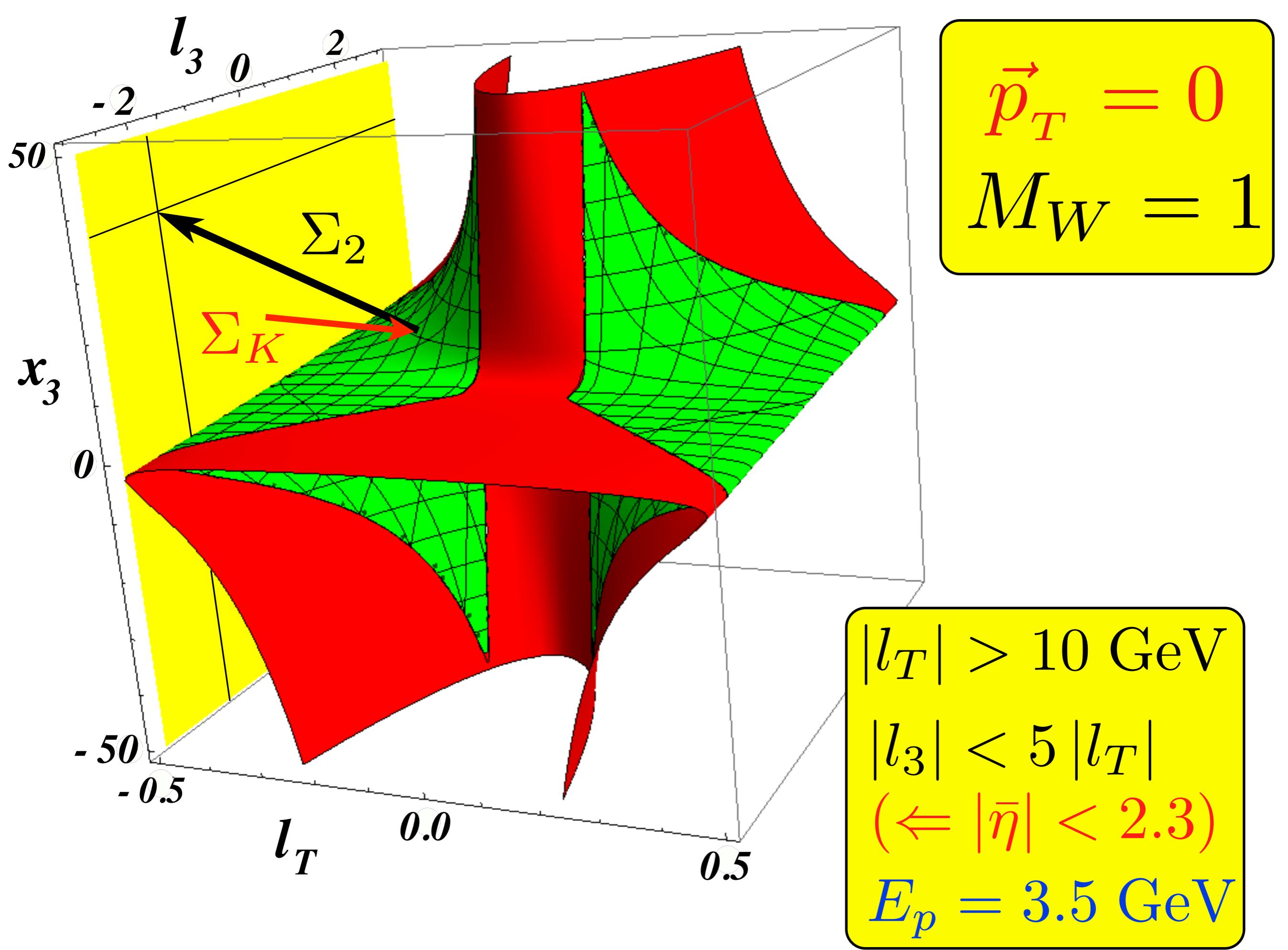
$$\delta(2l \cdot x - M^2) \times |\mathcal{ME}|^2$$

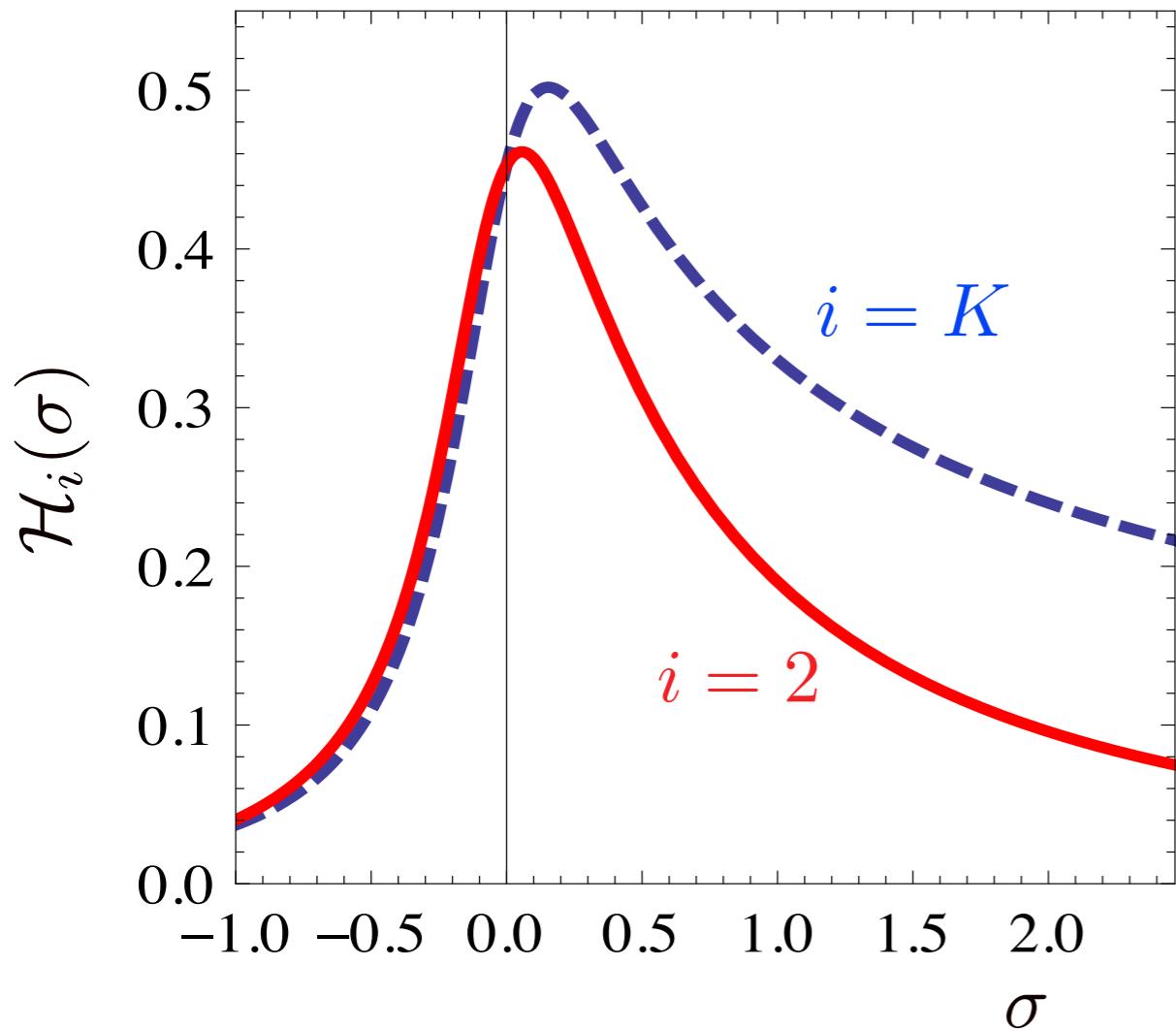
$$u \bar{d} \rightarrow W \rightarrow l \nu$$

PDFs: <http://durpdg.dur.ac.uk/HEPDATA/PDF>

\mathcal{ME} to LO in the SM

$\vec{p}_T = 0$ in graphs to follow

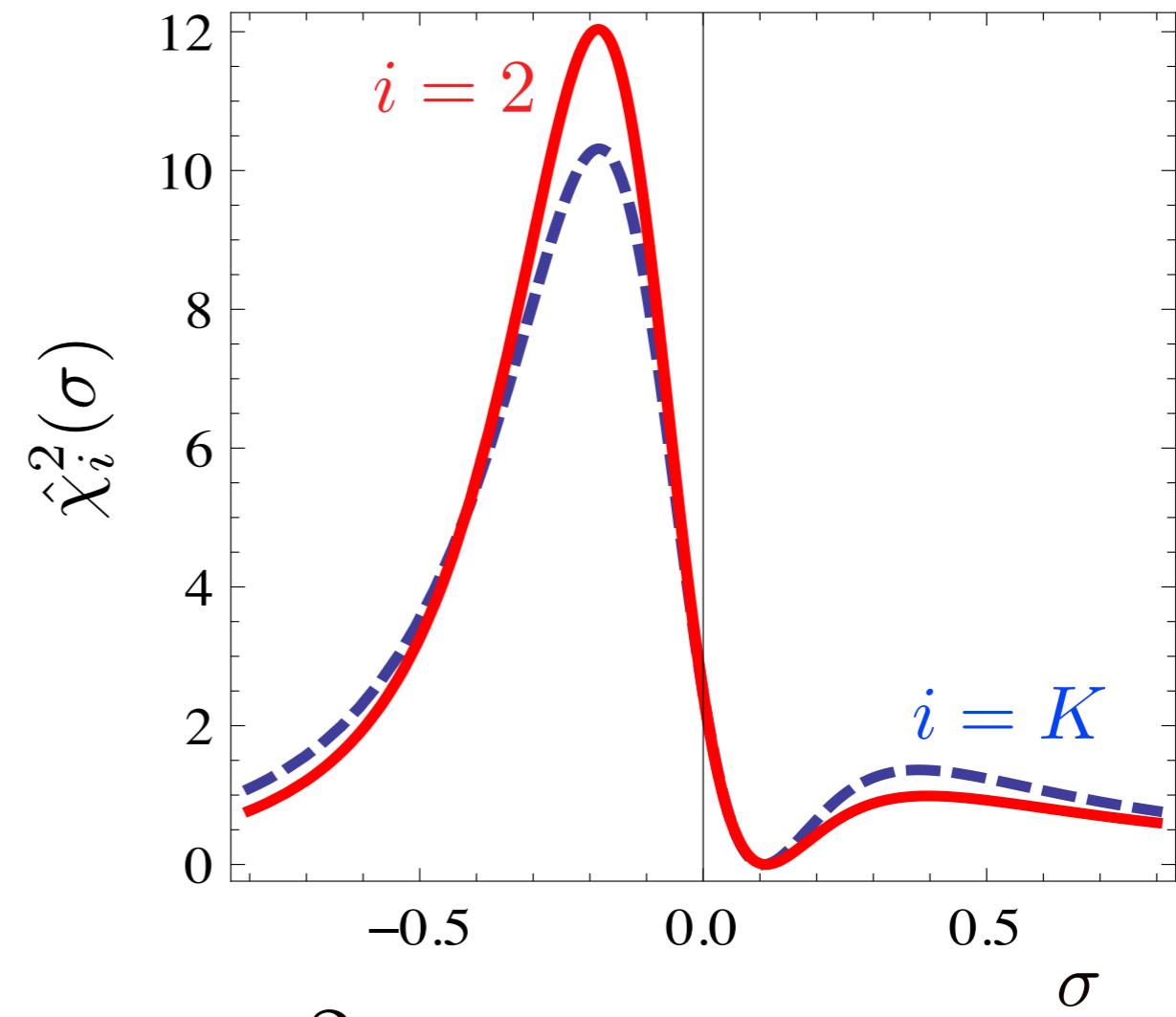




$d\mathcal{H}_i(\sigma, M, \Gamma, \mathcal{M})/d\sigma$

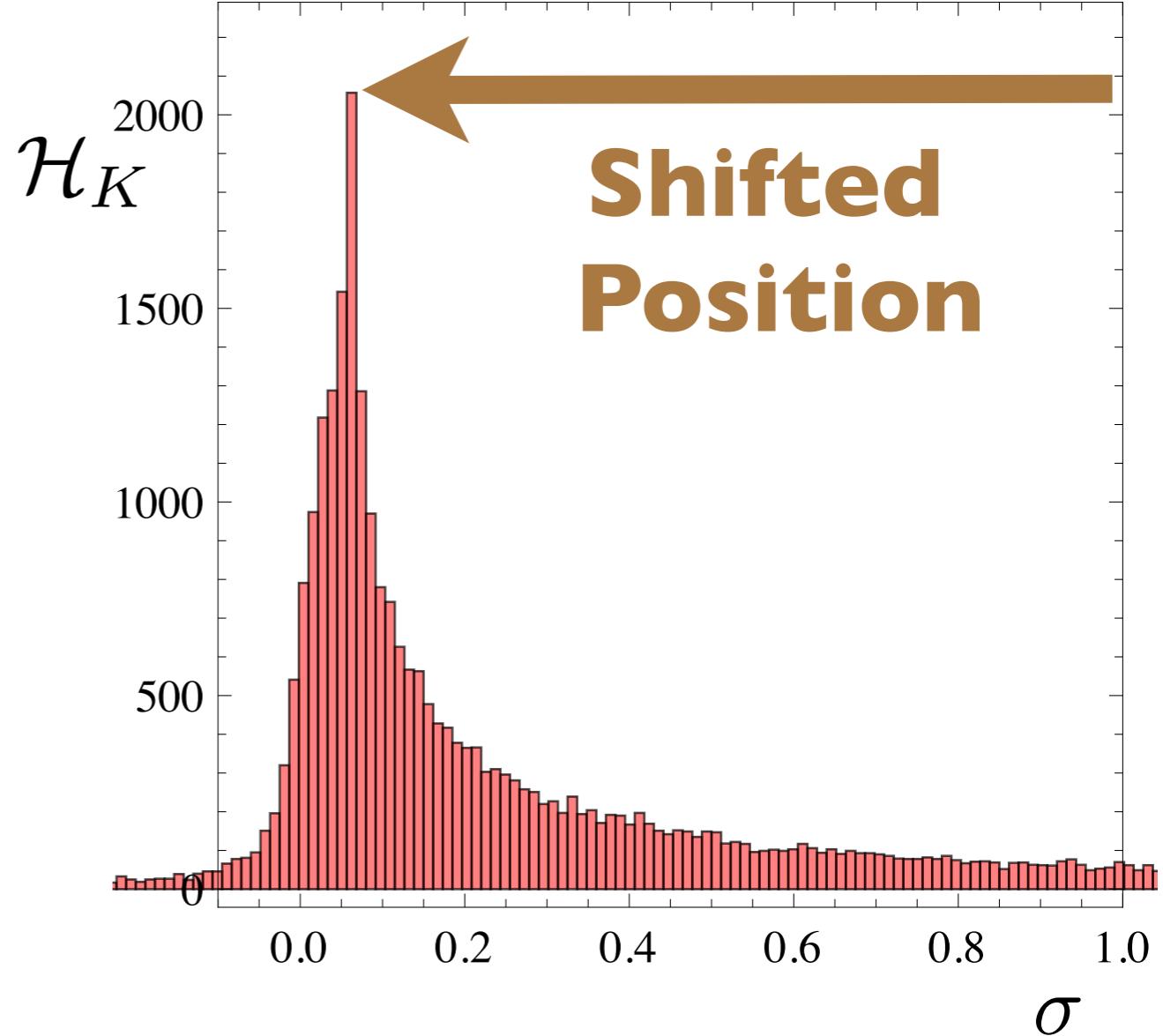
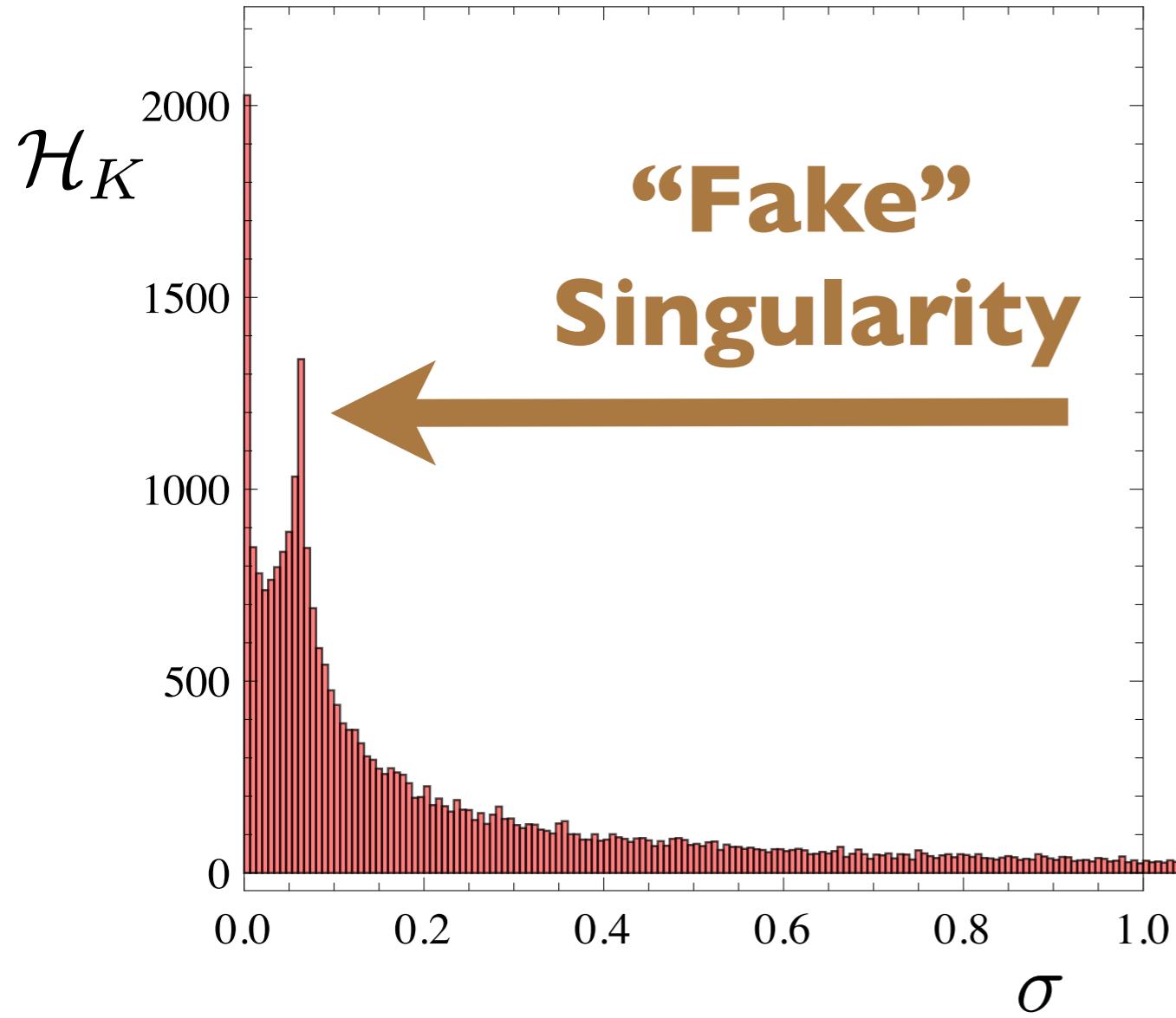
for

Σ_K and Σ_2



$\hat{\chi}_i^2(\sigma)$, their squared
statistical derivatives

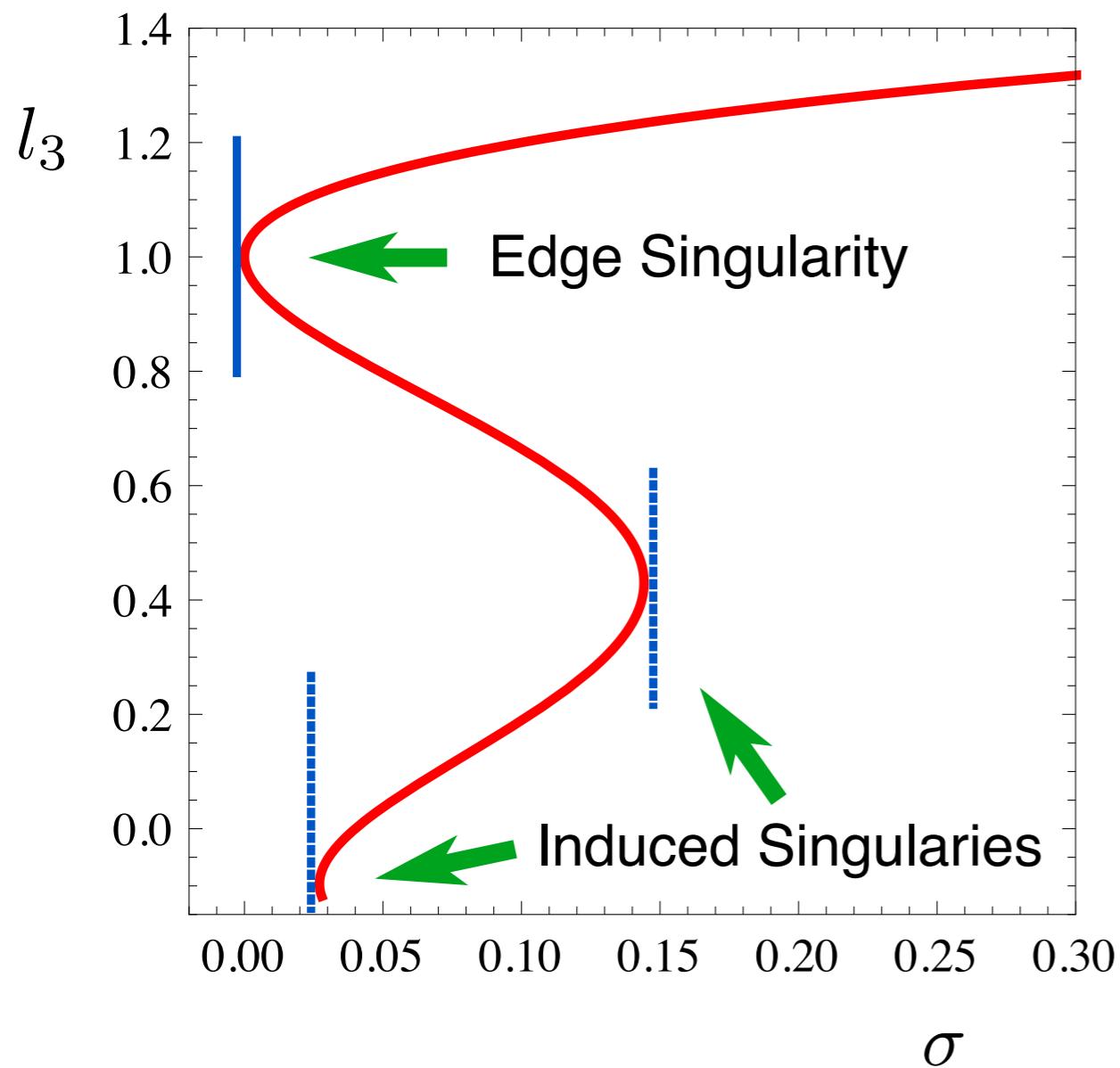
$M = \mathcal{M} = 1, \Gamma = 0.3$



$$\delta(x^2 + l^2 - M^2) \rightarrow \frac{1}{\pi} \frac{M \Gamma}{(l^2 + x^2 - M^2)^2 + M^2 \Gamma^2}$$

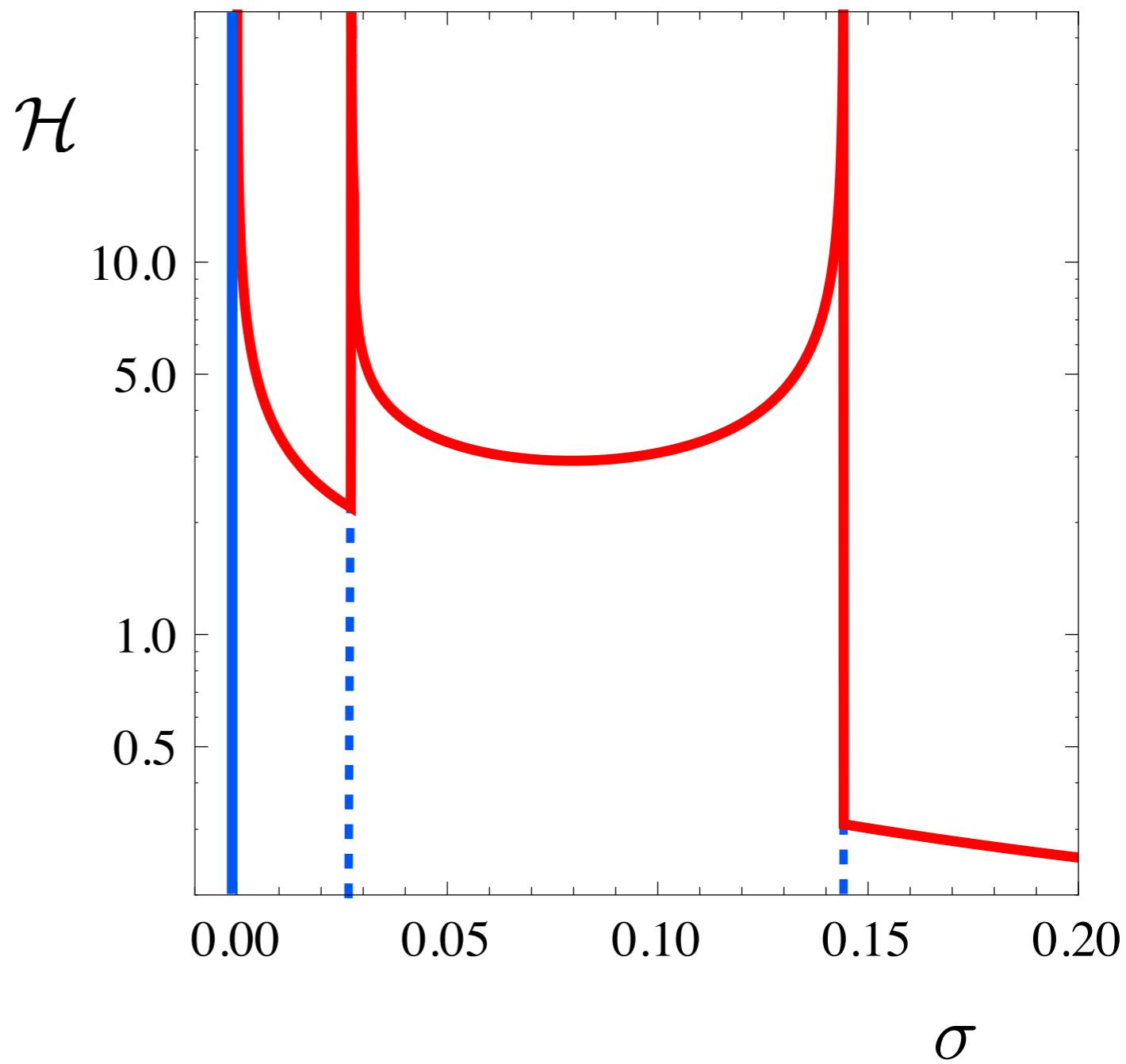
$$\mathcal{M} = M = 1; \quad \Gamma/M = \Gamma(W)/M(W)$$

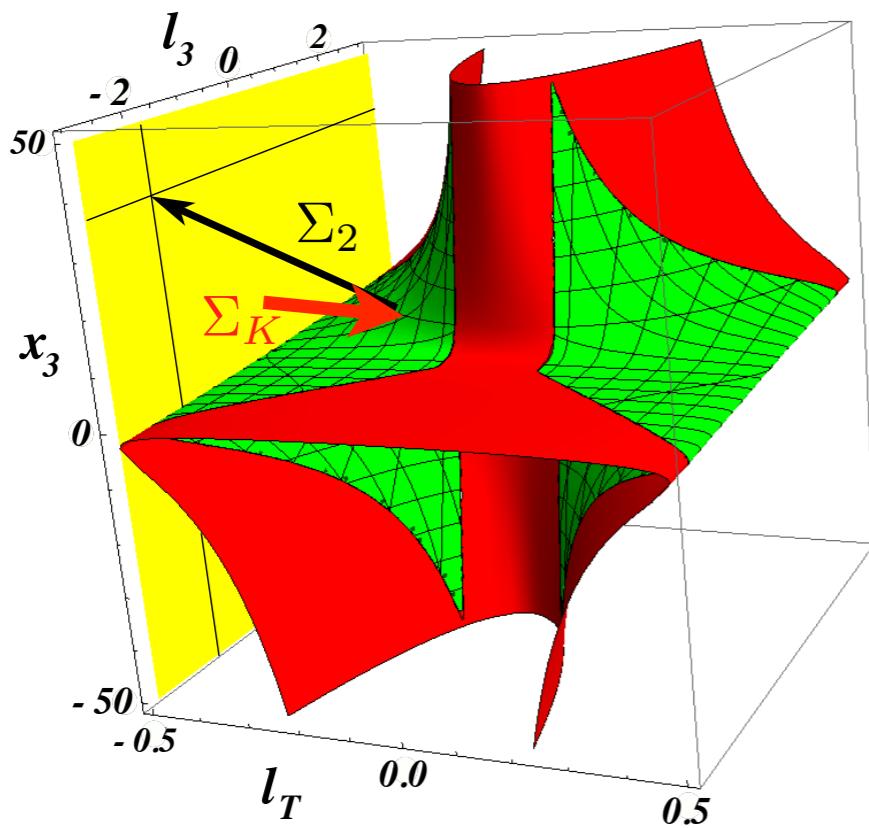
Kim's Variable



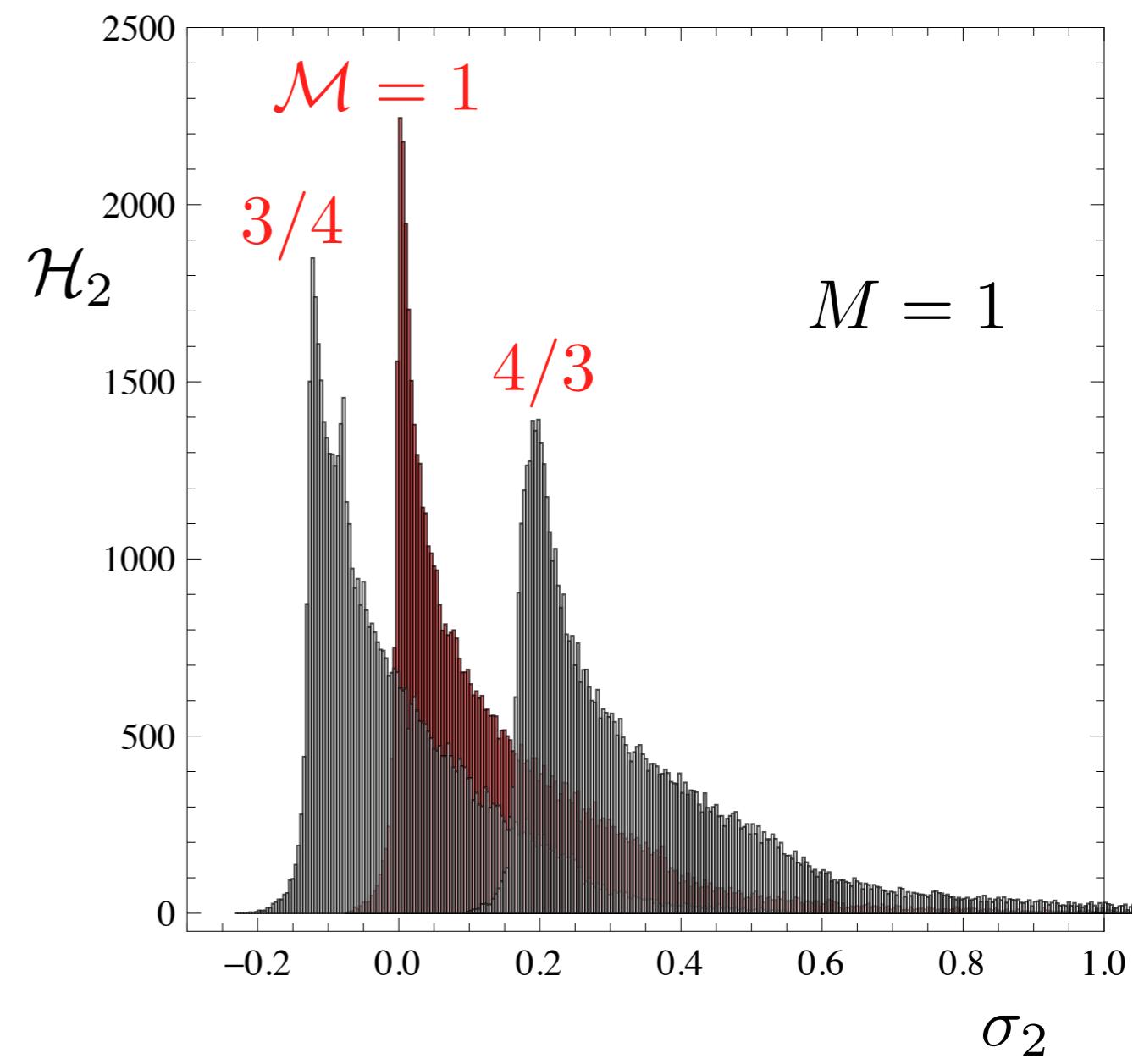
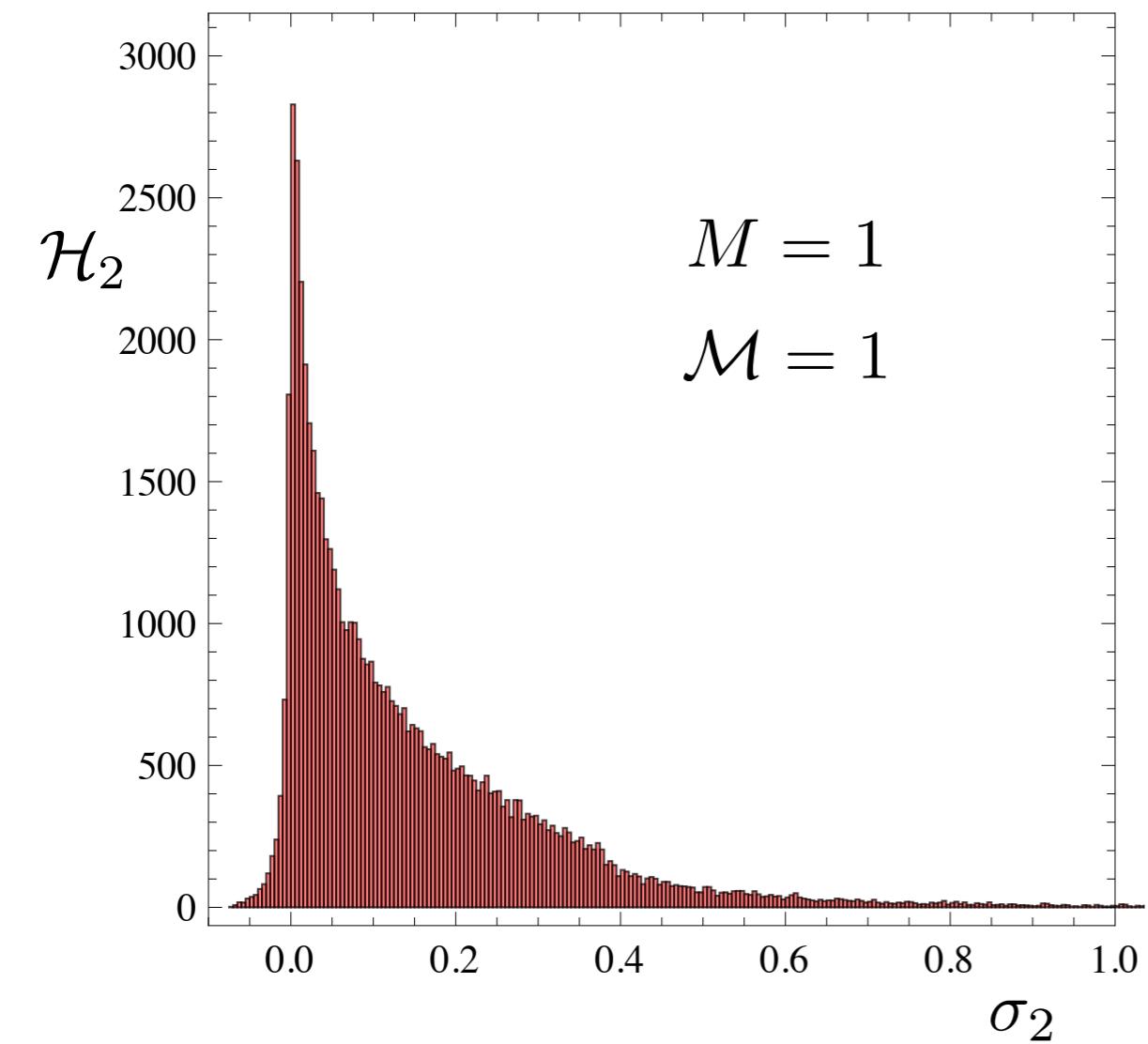
$$\mathcal{M} = M = 1; W_3 = 2$$

$$W_3 = l_3 + x_3$$





Orthogonal Variable



$$0=\Phi(l_3,x_3,l_T,\cos\theta,p_T,M)\equiv$$

$$\left(-2\,l_T(\cos\theta\,p_T+l_T)+2\,l_3x_3+M^2\right)^2$$

$$-4\left({l_3}^2+{l_T}^2\right)\times$$

$$\left(2\cos\theta\,l_T\,p_T+{l_T}^2+{p_T}^2+{x_3}^2\right)$$

$$\theta \wedge \{\vec{p}_T, \vec{l}_T\}$$

33

$$\vec{N} \equiv (N_1, N_2, N_3) = (\partial\Phi/\partial l_T, \partial\Phi/\partial l_3, \partial\Phi/\partial x_3)$$

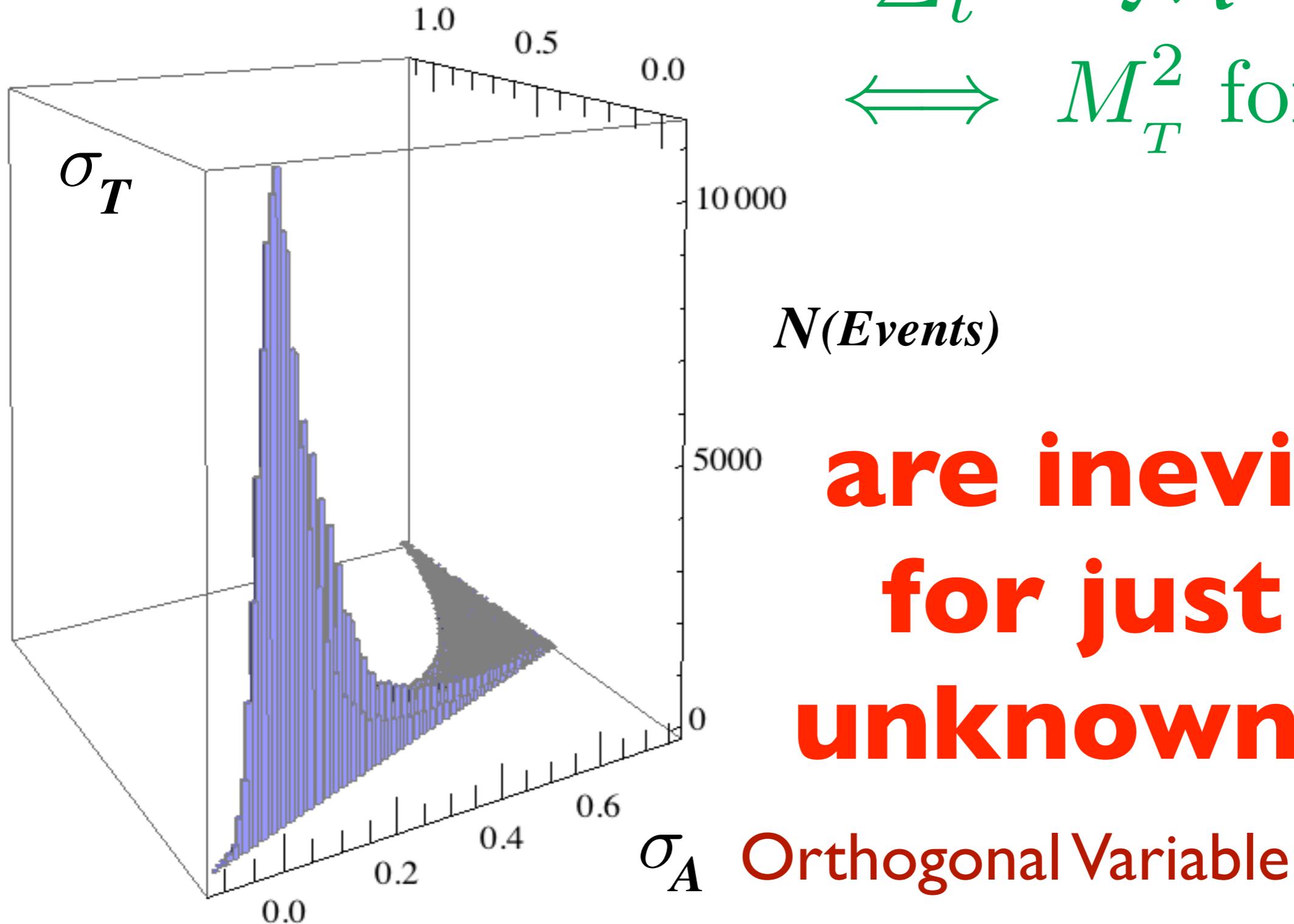
$$\vec{n} = \vec{N}/|N|$$

$$l_T^{\max}(M, \cos\theta, p_T) = \frac{M^2/2}{\sqrt{M^2 + p_T^2} + p_T \cos(\theta)}$$

Statistically Optimal Orthogonal Variable

$$\Sigma_A(l_3, x_3, l_T, \cos\theta, p_T, \mathcal{M}) = \frac{l_T - l_T^{\max}(\mathcal{M})}{n_1(\mathcal{M})}$$

Correlations



$$\Sigma_t = \mathcal{M} - 2 l_T$$
$$\iff M_T^2 \text{ for } \vec{p}_T = 0$$

$N(\text{Events})$

**are inevitable
for just one
unknown mass**

Orthogonal Variable

Conclusions

Φ -PROJECTIONS ONTO VISIBLE MOMENTA

**EDGES ARE SINGULARITY CONDITIONS
DISTANCES TO SINGULARITY ARE SVs**

**THE (STATISTICALLY) OPTIMAL SINGULARITY
VARIABLE MAXIMIZES THE FISHER INFORMATION,
AND IS EVERYWHERE ORTHOGONAL TO Φ**

**NON-OPTIMAL SVs DEVELOP FAKE
SINGULARITIES EVEN FOR $\mathcal{M} = M$**

The PDFs for pp are less constrained than for
 $p\bar{p} \rightarrow$ better M_W or better PDF-constraints ??

I.M.O. QUITE A BIT REMAINS TO BE DONE FOR
CASES WITH MORE THAN 1 INVISIBLE OBJECT