Cosmology (I)

Winter Meeting 2005
(XXXIII I.M.F.P.)

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Overview

I. The accelerating Universe

Dark Energy

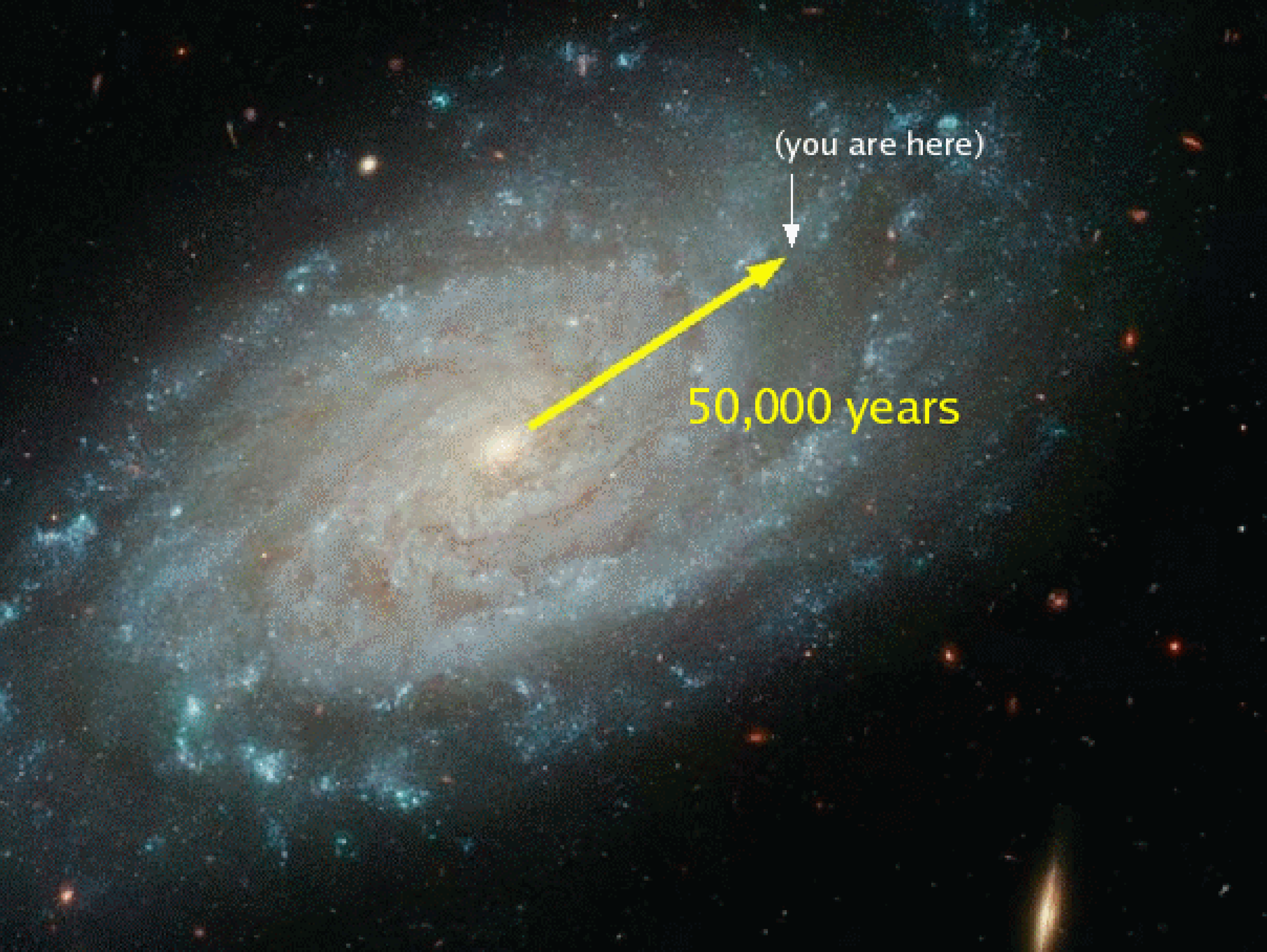
II. Structure Formation

Dark Matter

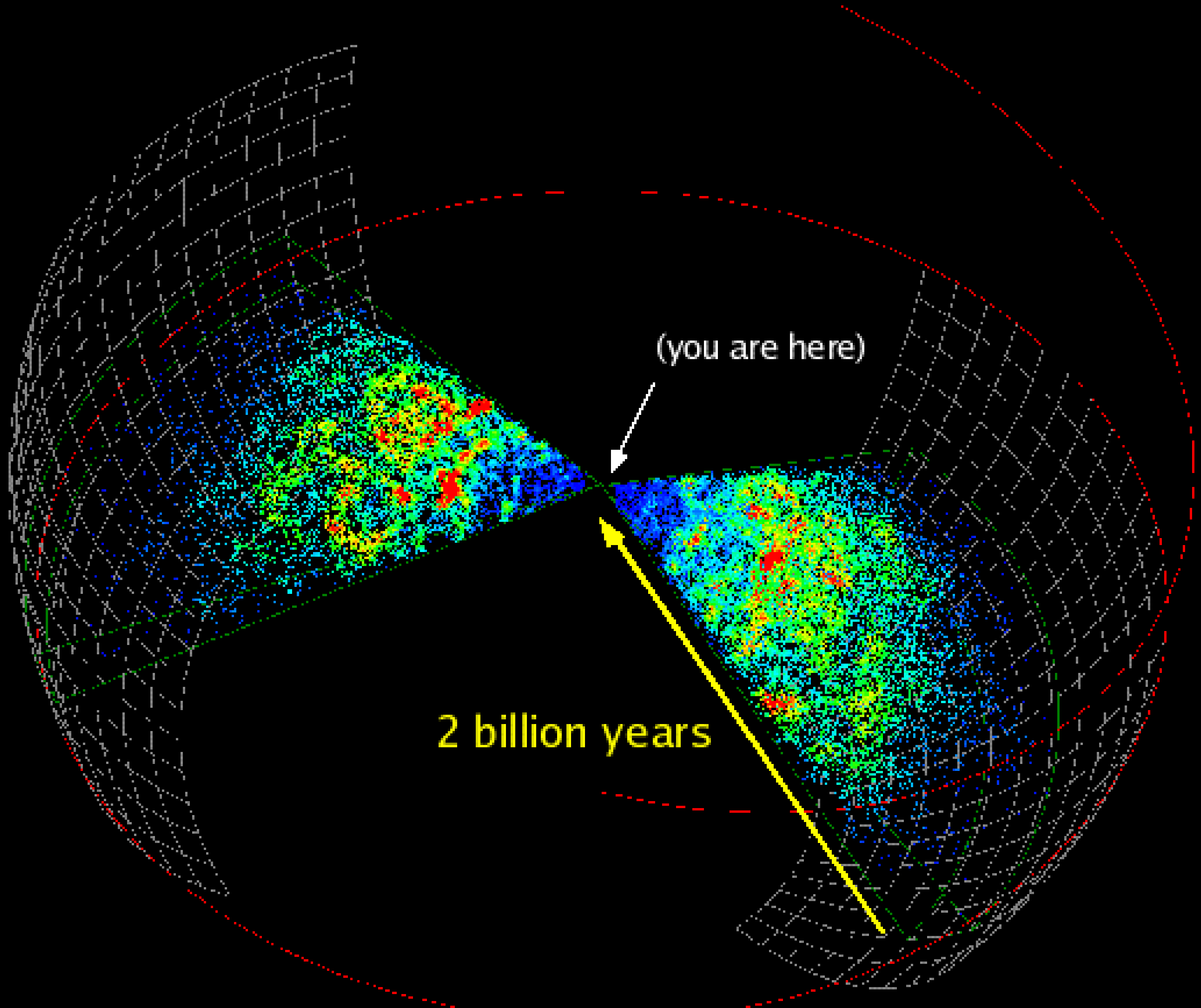
III. CMB Anisotropies

Inflation


8 minutes


(you are here)
50,000 years


100 million years
(you are here)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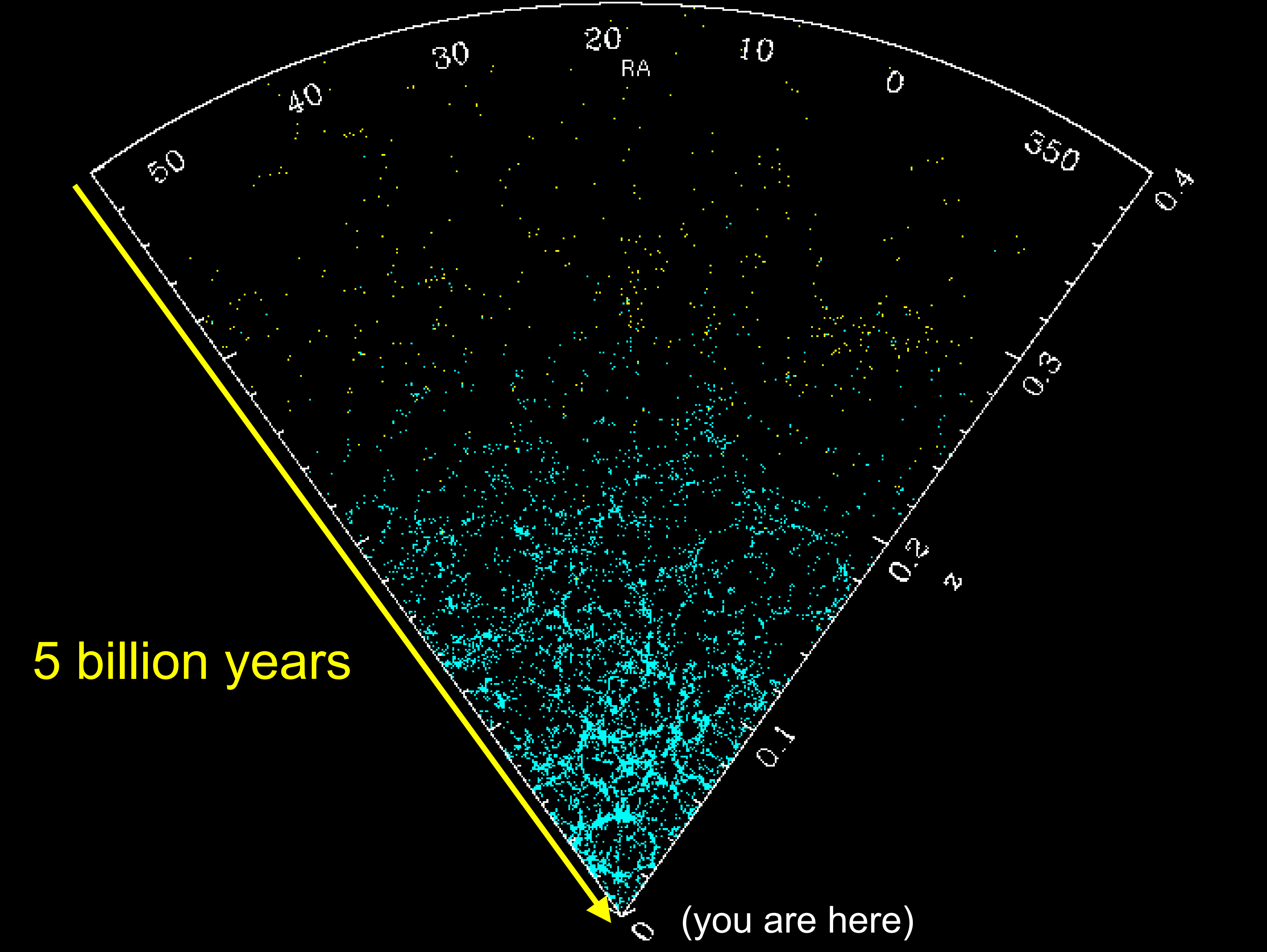
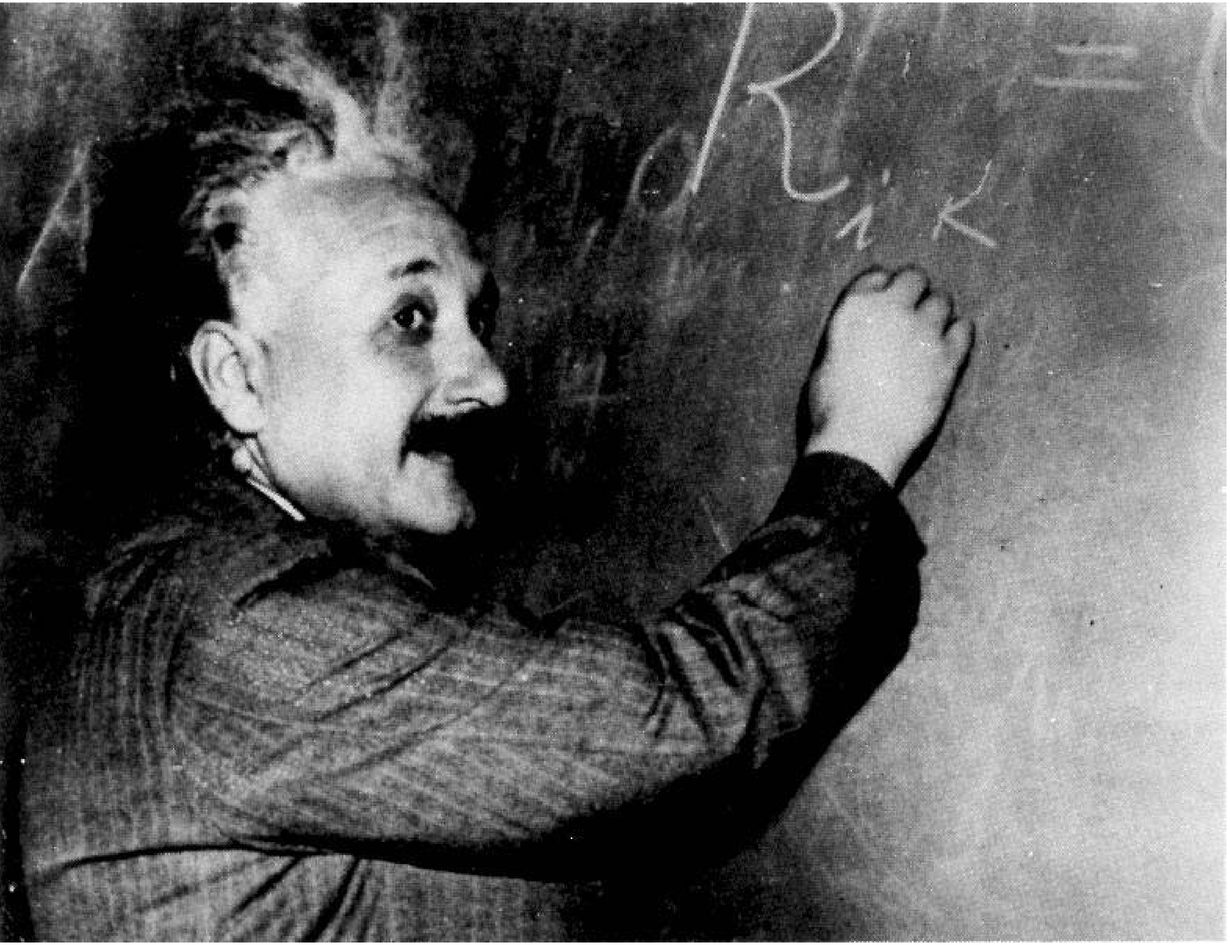
2 billion years



50
40
30
20
RA
10
0
350
0.4
0.3
0.2
z
0.1
0
5 billion years
(you are here)



World Year of
PHYSICS
2005

www.physics2005.org

Hot Big Bang

General Relativity

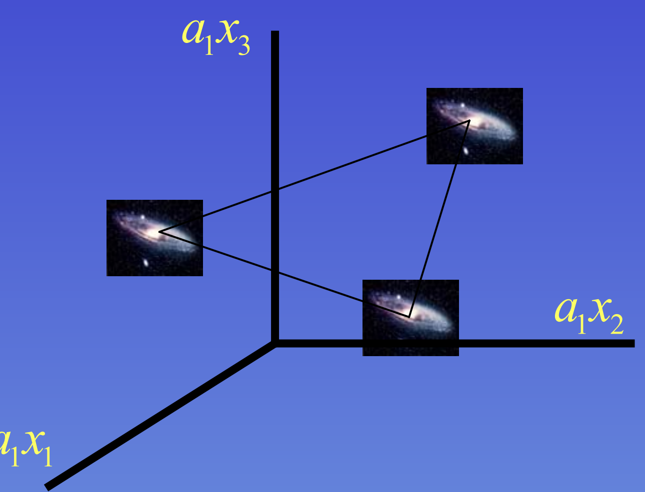
$$G_{\mu\nu} = R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Homogeneity and Isotropy

$$ds^2 = -dt^2 + \underline{a^2(t)}\left[\frac{dr^2}{1-\underline{K}r^2} + r^2 d\Omega\right] \qquad FRW$$

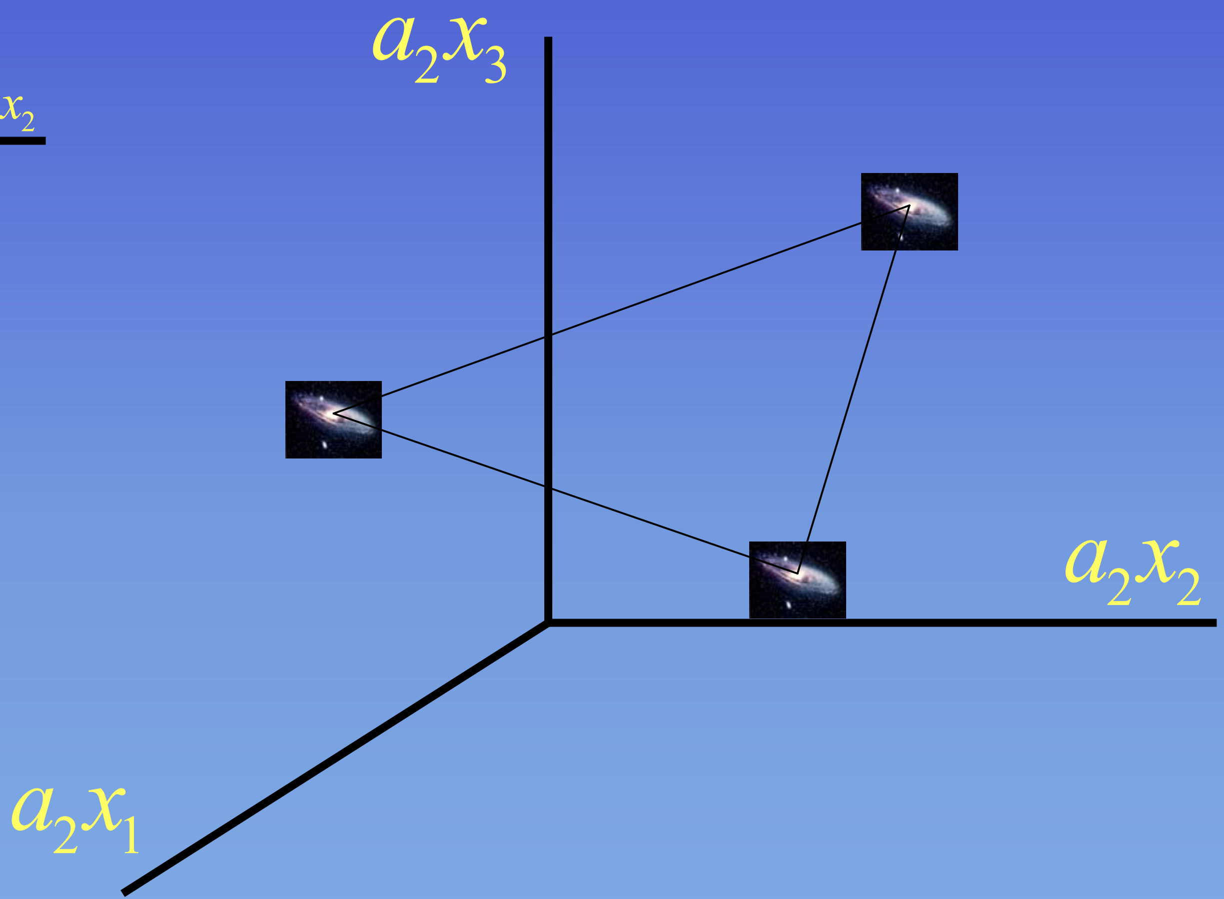
$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2)$$

flat space



scale factor

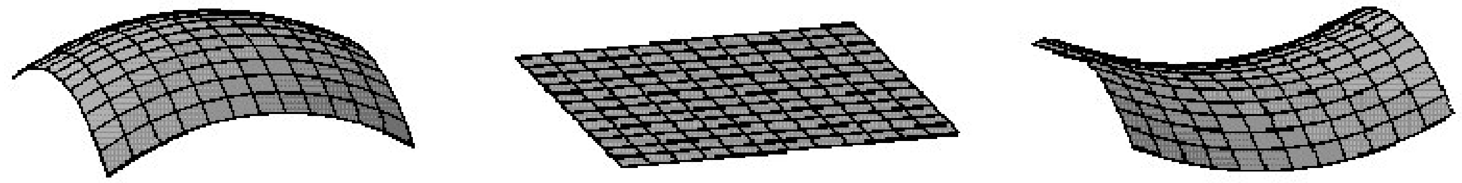
$$a(t_2) > a(t_1)$$



Spatial Curvature

$$^{(3)}R = \frac{6K}{a^2(t)}$$

Closed	Flat	Open
$K = +1$	$K = 0$	$K = -1$



Matter Content: Perfect Fluid

$$T_{\mu\nu} = p\, g_{\mu\nu} + (\rho + p) U_\mu U_\nu$$

Isotropic in its rest frame:

$$T^{\mu}{}_{\nu} = diag(-\rho(t), p(t), p(t), p(t))$$

Energy density conservation:

$$D_\mu T^{\mu}{}_{\nu} = 0 \Rightarrow \dot{\rho}(t) + 3\frac{\dot{a}}{a}(\rho(t) + p(t)) = 0$$

Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} \qquad ij + 00$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad 00$$

Equation of state

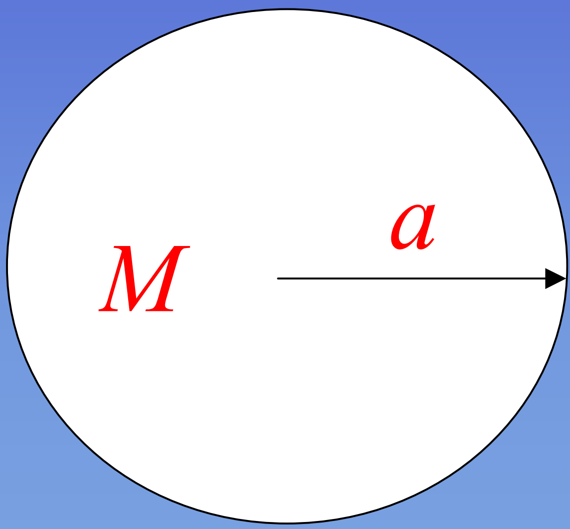
$$p(t) = w\rho(t)$$

Friedmann equation

$$\frac{1}{2}\dot{a}^2 - \frac{GM}{a} = -\frac{K}{2}$$

$$T + V = E$$

$$M = \frac{4\pi}{3}\rho\, a^3$$



$K = 0$ escape velocity

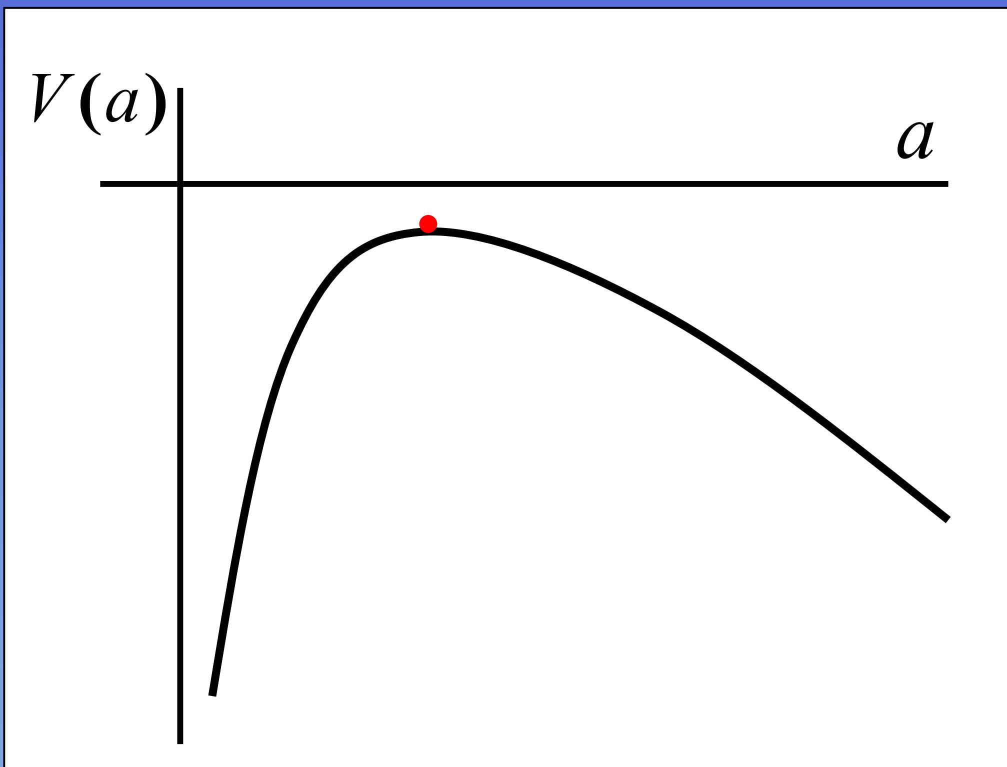
$K > 0$ recollapse

$K < 0$ expand forever

Einstein-de Sitter model

$$\frac{1}{2}\dot{a}^2 - \frac{GM}{a} - \frac{\Lambda}{6}a^2 = -\frac{K}{2}$$

$$T + V = E$$



$$a = \left(\frac{3GM}{\Lambda}\right)^{1/3}$$

coasting point (unstable)

Universe dynamics (K=0)

Radiation: $p = \rho / 3$

$\rho_R \propto a^{-4}$ $a_R \propto t^{1/2}$

Matter: $p << \rho$

$\rho_M \propto a^{-3}$ $a_M \propto t^{2/3}$

Vacuum: $p = -\rho$

$\rho_V \propto a^{0}$ $a_V \propto e^{Ht}$

Cosmological Parameters

H_0	Rate of expansion
t_0	Age of the Universe
q_0	Acceleration Parameter
Ω_K	Spatial Curvature
Ω_M	Dark Matter
Ω_Λ	Cosmological Constant
Ω_B	Baryon Density
Ω_ν	Neutrino Density

Cosmological Parameters

Rate of Expansion (Hubble)

$$H_0 = \frac{\dot{a}}{a}(t_0) = 100\, h \text{ km/s/Mpc}$$

$$H_0^{-1} = 9.773\, h^{-1} \text{ Gyr}$$

$$cH_0^{-1} = 3000\, h^{-1} \text{ Mpc}$$

$$1 \text{ pc} = 3.262 \text{ ly} = 3.086 \times 10^{16} \text{ m}$$

Critical density (K=0)

$$\rho_c(t_0) = \frac{3H_0^2}{8\pi G}$$

$$= 1.88\ h^2\ 10^{-29}\ \text{g/cm}^3$$

$$= 2.77\ h^{-1}\ 10^{11}\ \text{M}_\Theta/(h^{-1}\text{Mpc})^3$$

$$= 11.26\ h^2\ \text{protons/m}^3$$

Density parameter

$$\Omega_0 = \frac{8\pi G}{3H^2}\rho(t_0) = \frac{\rho}{\rho_c}(t_0)$$

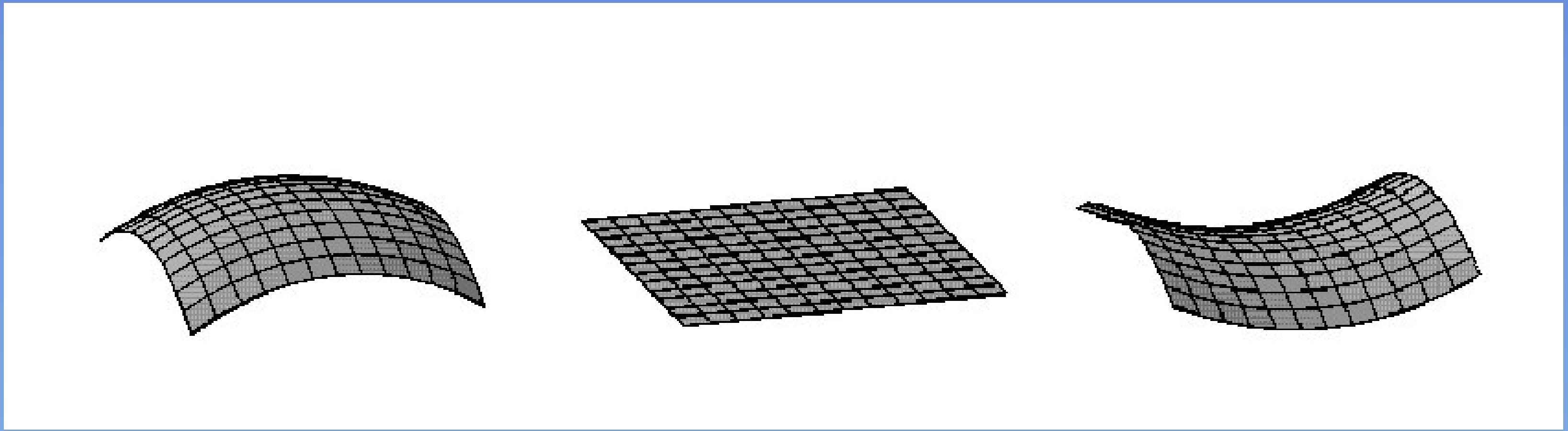
$$\Omega_0 = \Omega_R + \Omega_M + \Omega_\Lambda$$

$$\Omega_R = \frac{\rho_R}{\rho_c}(t_0) \qquad \Omega_M = \frac{\rho_M}{\rho_c}(t_0)$$

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} \qquad \Omega_K = \frac{-K}{a_0^2 H_0^2}$$

Spatial Curvature

$K = +1$	$K = 0$	$K = -1$
Closed	Flat	Open
$\Omega_0 > 1$	$\Omega_0 = 1$	$\Omega_0 < 1$



Cosmic Sum Rule

Friedmann equation

$$H^2 = \frac{8\pi G}{3}(\rho_R + \rho_M) + \frac{\Lambda}{3} - \frac{K}{a^2}$$

Today: $$1 = \Omega_R + \Omega_M + \Omega_\Lambda + \Omega_K$$

No vacuum: $\Omega_\Lambda = 0 \quad \Rightarrow \quad \Omega_K = 1 - \Omega_M$

Flat space: $\Omega_K = 0 \quad \Rightarrow \quad \Omega_\Lambda = 1 - \Omega_M$

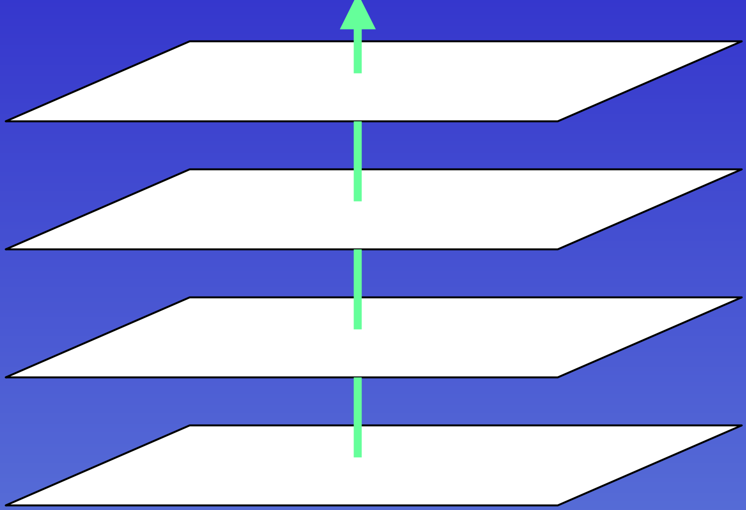
Deceleration parameter

$$q_0 = -\frac{a\ddot{a}}{\dot{a}^2}(t_0) = \frac{4\pi G}{3H_0^2}(\rho + 3p)$$

$$q_0 = \Omega_R + \frac{1}{2}\Omega_M - \Omega_\Lambda + \frac{1}{2}\sum_x (1+3w_x)\Omega_x$$

Matter domination: $q_0 > 0$

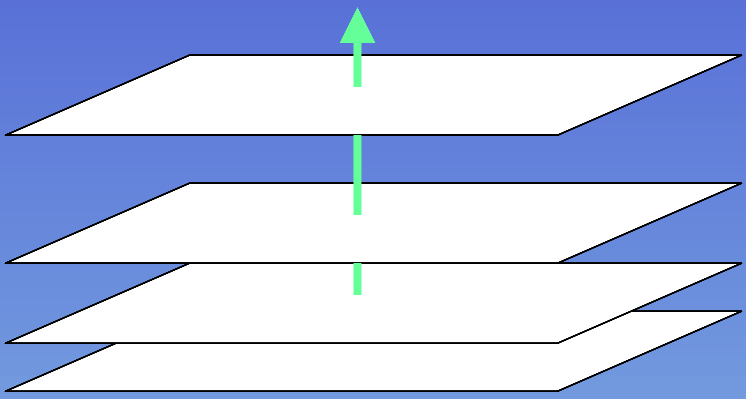
Vacuum domination: $q_0 < 0$



Uniform expansion

$$q_0 = 0$$

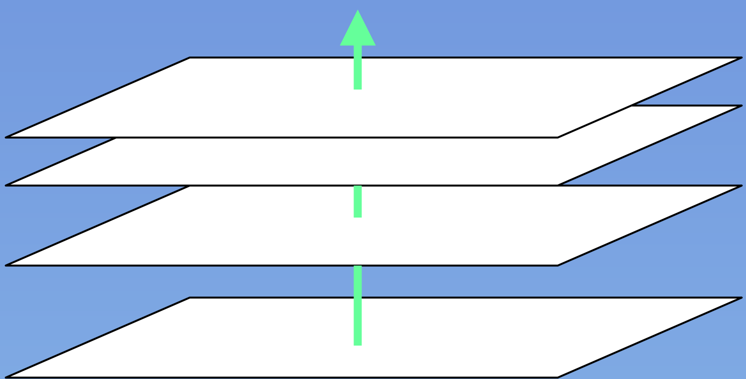
$$\Omega_M = 2\Omega_\Lambda$$



Accelerated expansion

$$q_0 < 0$$

$$\Omega_M < 2\Omega_\Lambda$$



Decelerated expansion

$$q_0 > 0$$

$$\Omega_M > 2\Omega_\Lambda$$

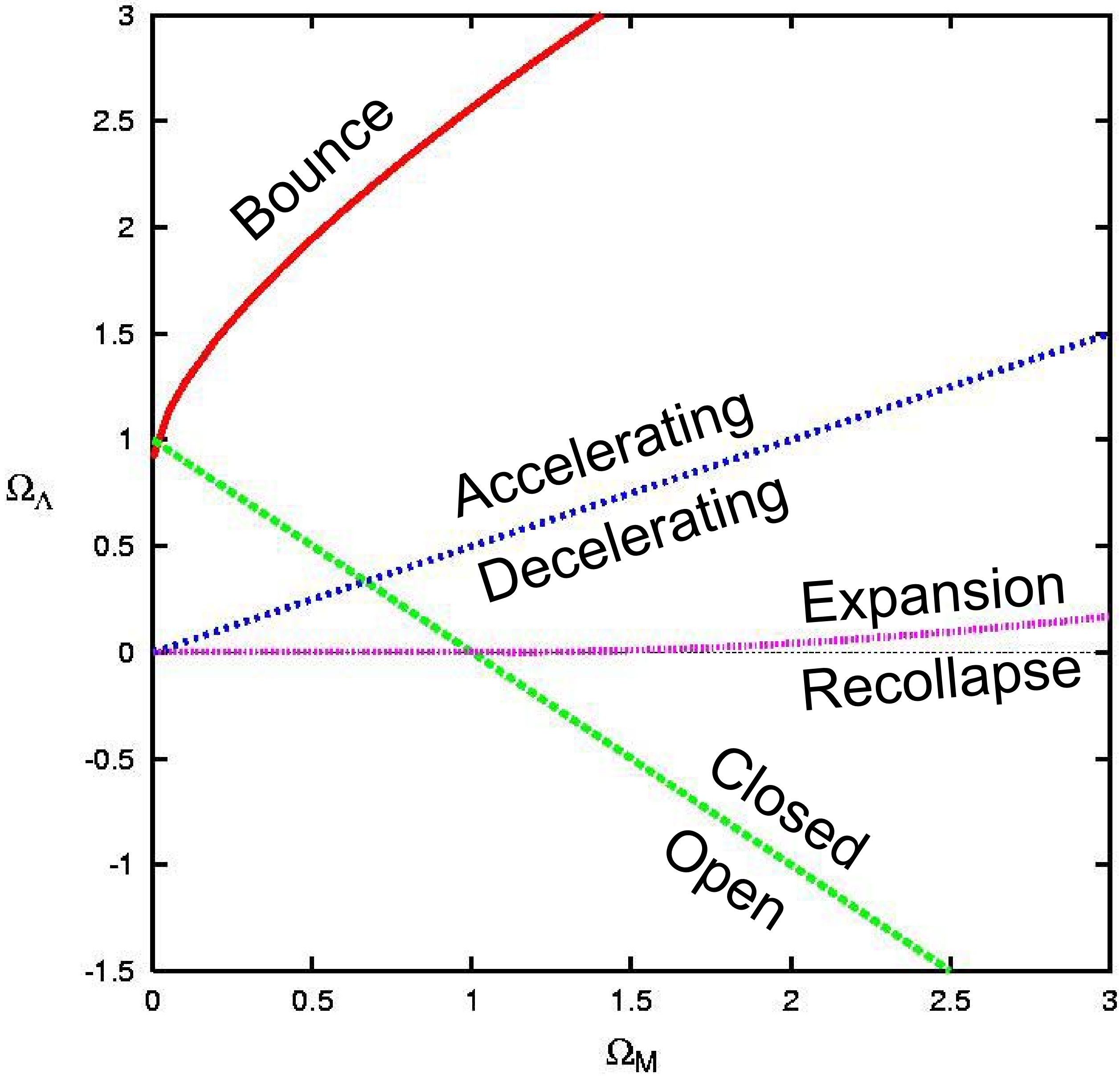
Bounce $$H_0 t_0 = \int_0^1 \frac{da}{\sqrt{1+\Omega_M(\frac{1}{a}-1)+\Omega_\Lambda(a^2-1)}} = \infty$$

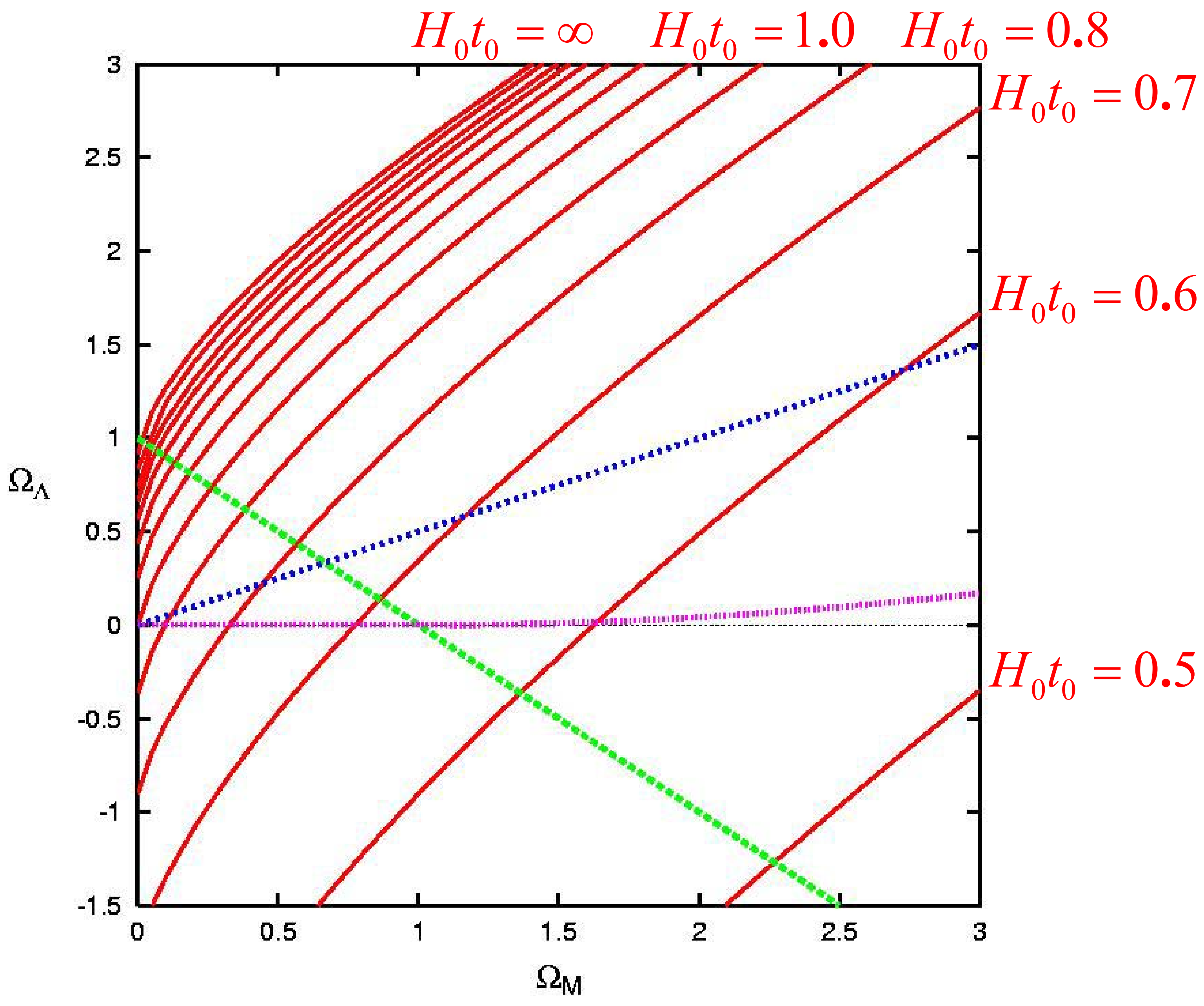
Uniform exp. $$q_0 = 0 \quad \Rightarrow \quad \Omega_\Lambda = \tfrac{1}{2}\Omega_M$$

Critical univ.

$$\Omega_\Lambda = \begin{cases} 0 & \Omega_M \le 1 \\ 4\Omega_M \sin^3[\frac{1}{3}\arcsin(\frac{\Omega_M - 1}{\Omega_M})] & \Omega_M > 1 \end{cases}$$

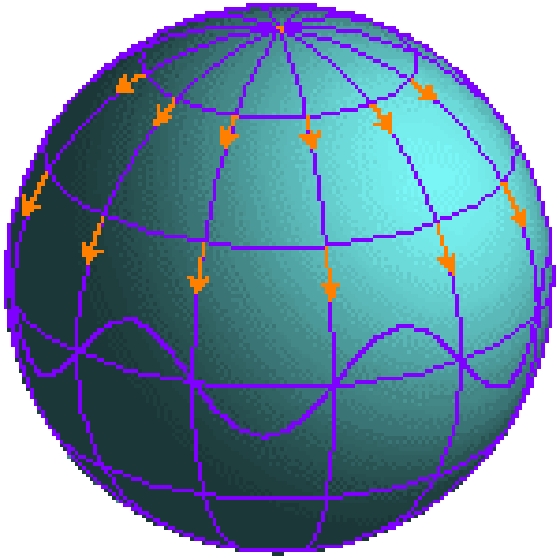
Flat space $$\Omega_K = 0 \quad \Rightarrow \quad \Omega_\Lambda = 1 - \Omega_M$$


Bounce
Accelerating
Decelerating
Expansion
Recollapse
Closed
Open
Ω_Λ
Ω_M
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5
3



$H_0 t_0 = \infty$
$H_0 t_0 = 1.0$
$H_0 t_0 = 0.8$
$H_0 t_0 = 0.7$
$H_0 t_0 = 0.6$
$H_0 t_0 = 0.5$
Ω_Λ
Ω_M
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.5
1
1.5
2
2.5
3

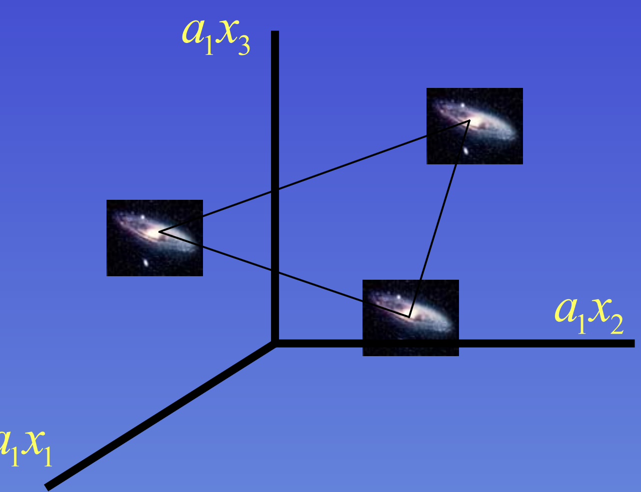
The Expanding Universe

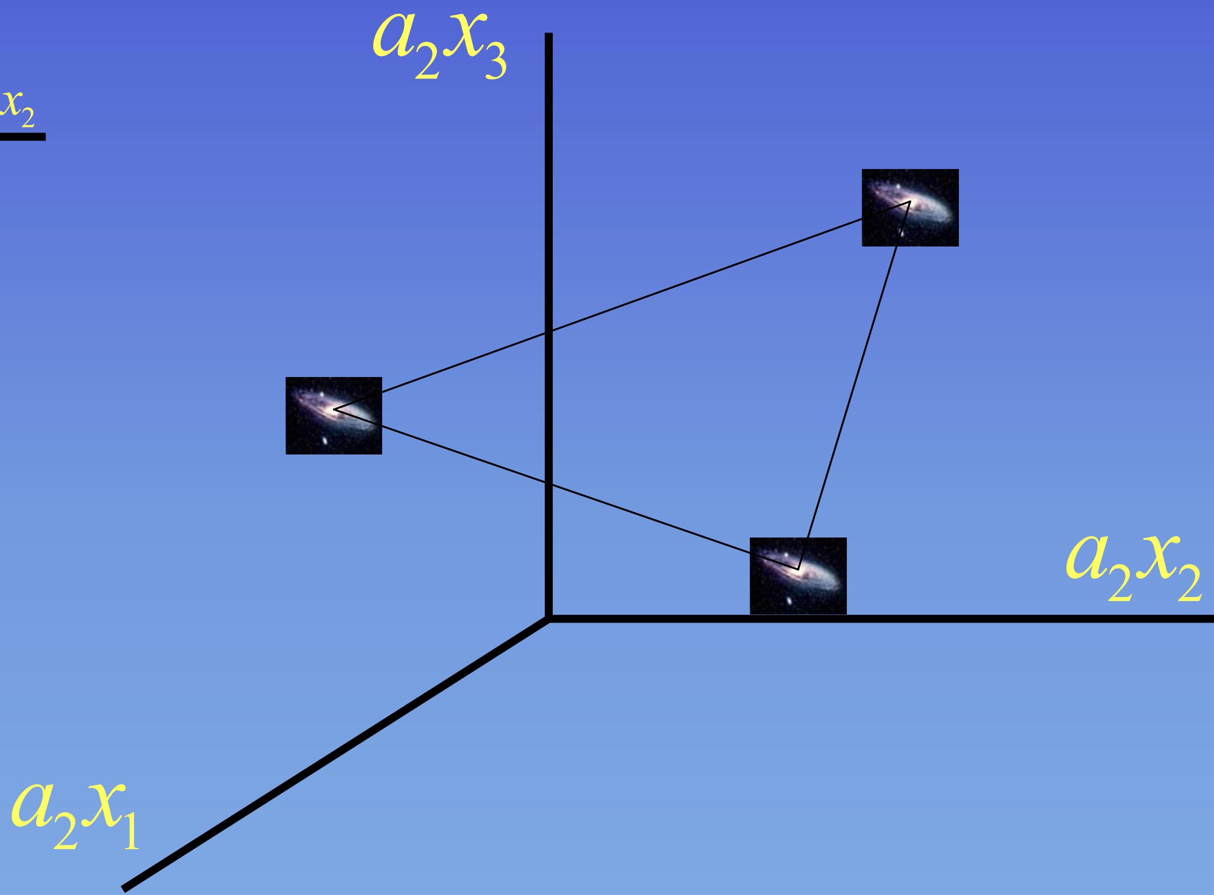


$$\frac{\lambda_{obs}}{\lambda_{em}} = \frac{a_0}{a_1} = 1 + z$$

$$ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2)$$

flat space





scale factor

$$\frac{a(t_2)}{a(t_1)} = \frac{1+z_1}{1+z_2}$$

Geodesic motion

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0\,; \qquad u^\mu = (\gamma, \gamma v^i)$$

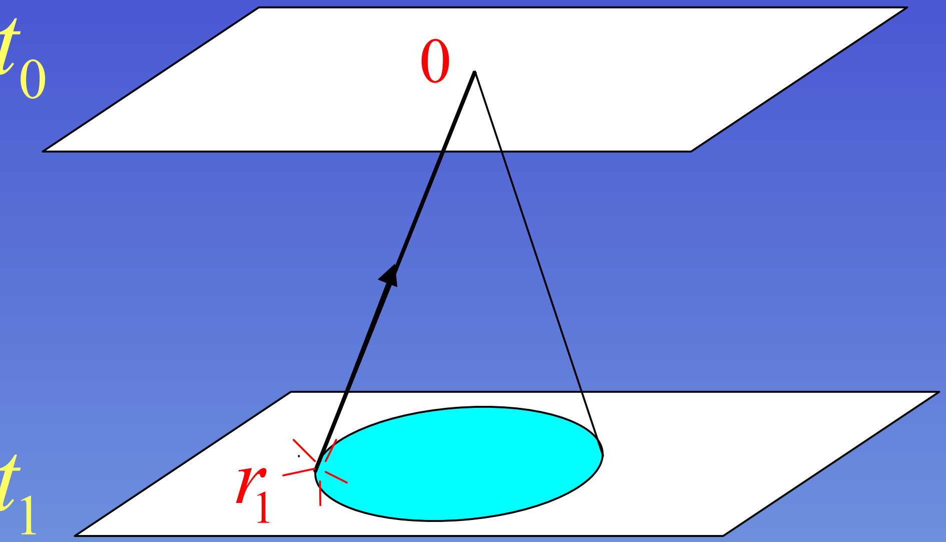
$$\Gamma^0_{ij} = \frac{\dot{a}}{a} g_{ij} \quad \Rightarrow \quad |\vec{u}| \propto \frac{1}{a} \quad \Rightarrow \quad |\vec{p}| \propto \frac{1}{a}$$

Photon redshift

$$p = \frac{h}{\lambda}$$

$$\frac{\lambda_1}{\lambda_0} = \frac{a(t_1)}{a(t_0)} \quad \Rightarrow \quad z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{a_0}{a_1} - 1$$

FRW kinematics



Physical distance

$$d = a_0 r_1 \quad (1^{st}\ order)$$

Light cone:

$$0 = -dt^2 + a^2(t)\frac{dr^2}{1-Kr^2}$$

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{\sqrt{1-Kr^2}} = f(r_1) = \begin{cases} \mathbf{arcsin}\, r_1 & K=1 \\ r_1 & K=0 \\ \mathbf{arcsinh}\, r_1 & K=-1 \end{cases}$$

Taylor expansion

$$\frac{1}{1+z} = \frac{a(t)}{a_0} = 1 + H_0(t - t_0) + O(t - t_0)^2$$

To first approximation

$$r_1 \approx f(r_1) = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0}(t_1 - t_0) + \ldots = \frac{z}{a_0 H_0} + \ldots$$

Hubble law

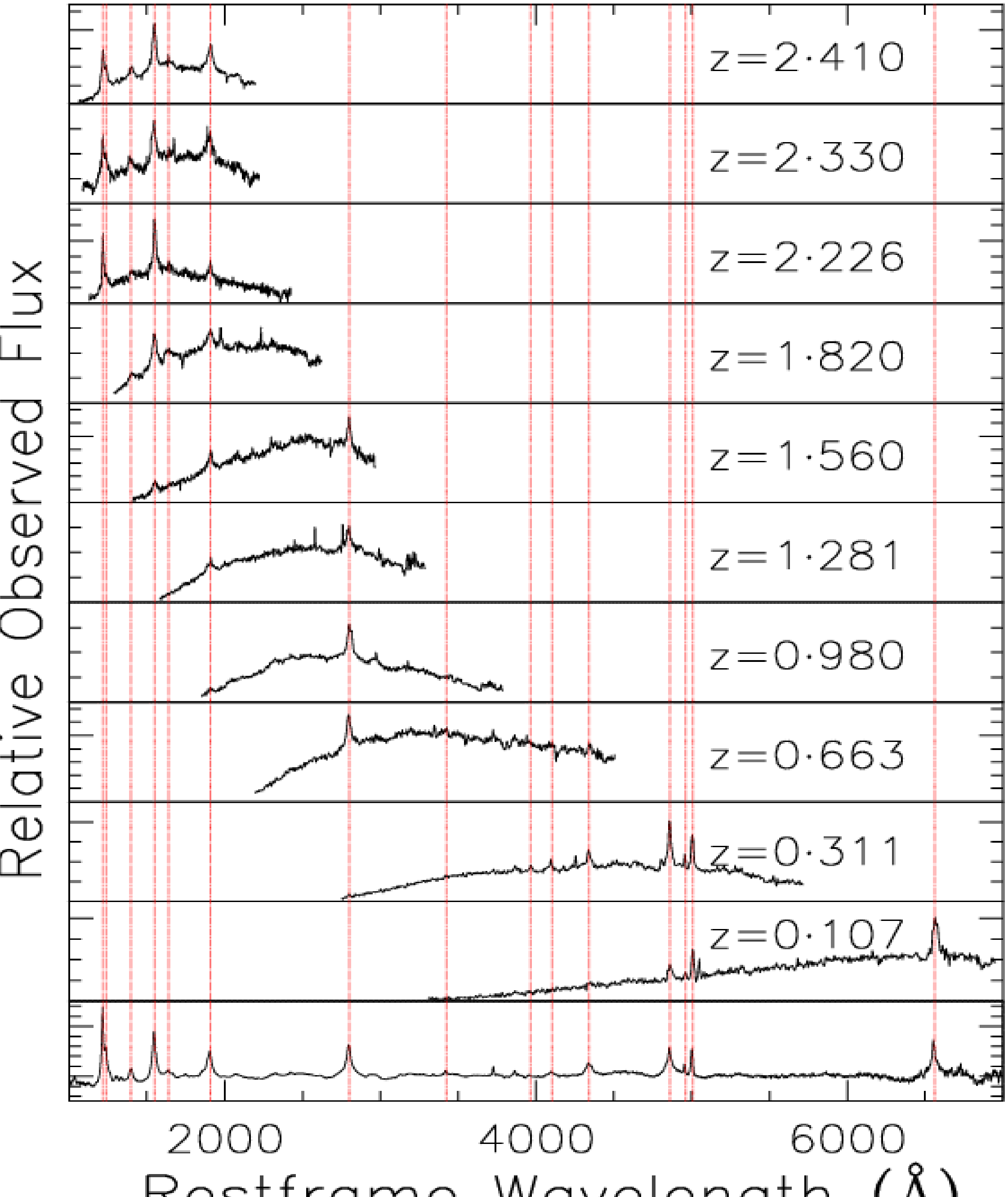
$$H_0 d = H_0 a_0 r_1 = z \approx vc$$

Edwin P. Hubble

(1920s)

Mount Wilson

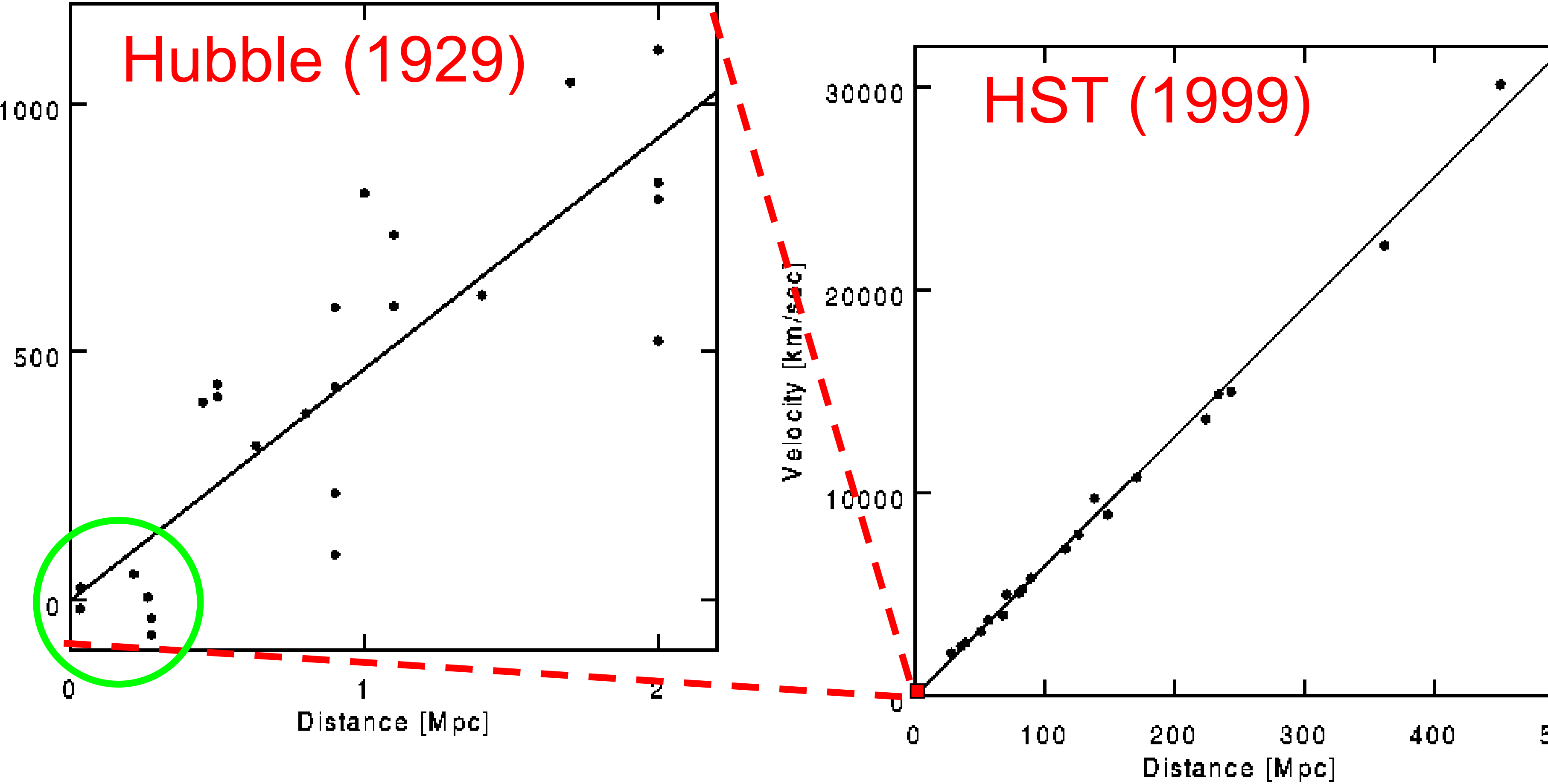
Mount Palomar



Redshifts to galaxies

Hubble law

$$H_0 d = z \approx vc$$

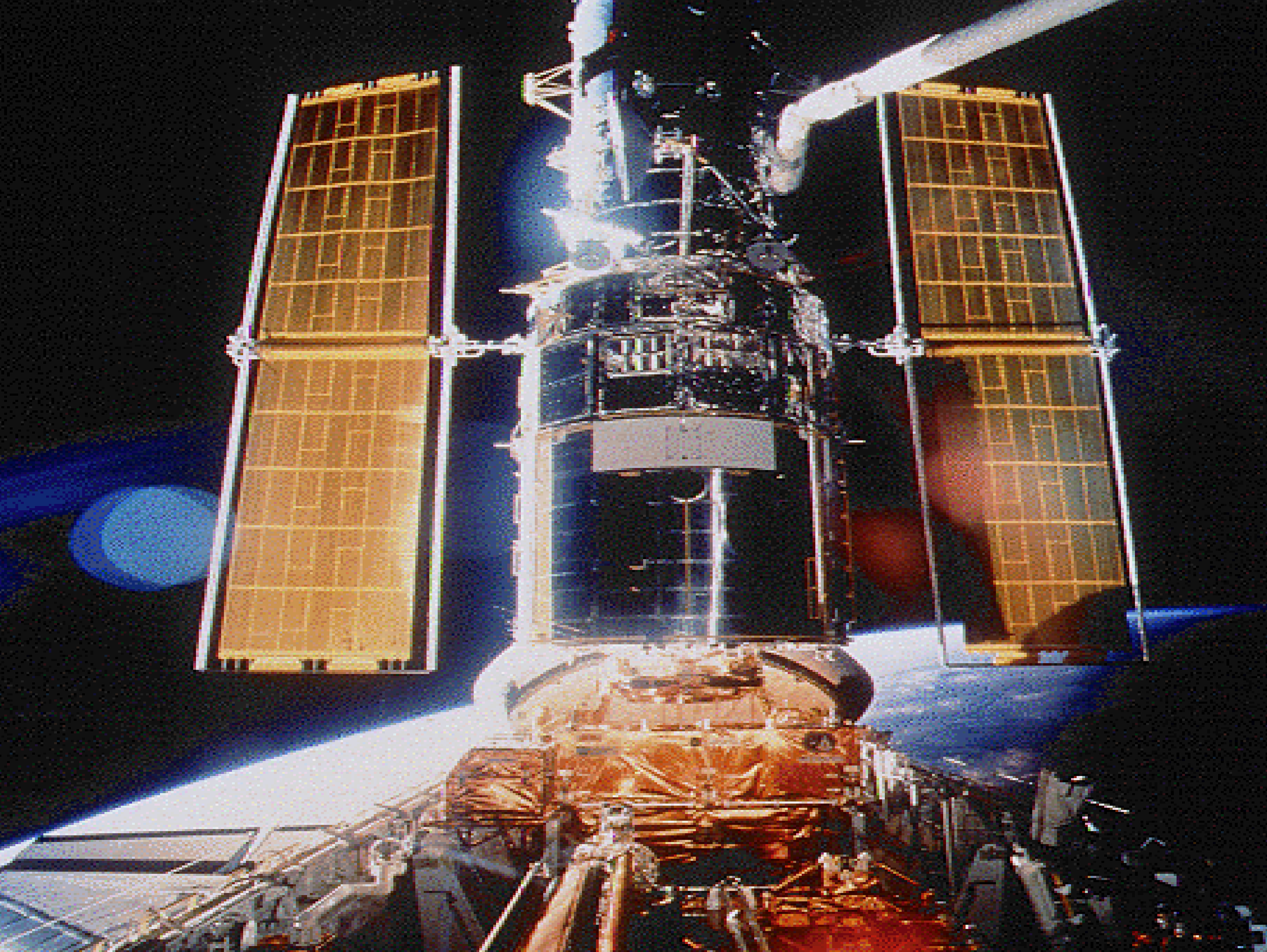


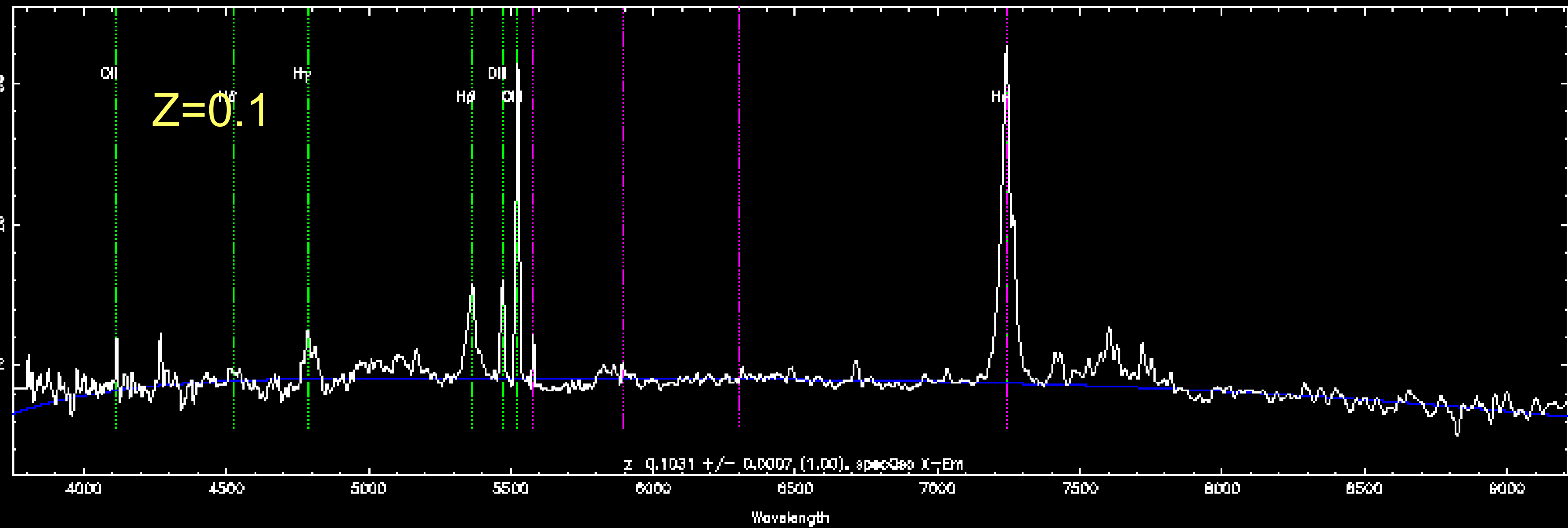
$H_0 = 500$ km/s/Mpc

Dominated by systematic errors!

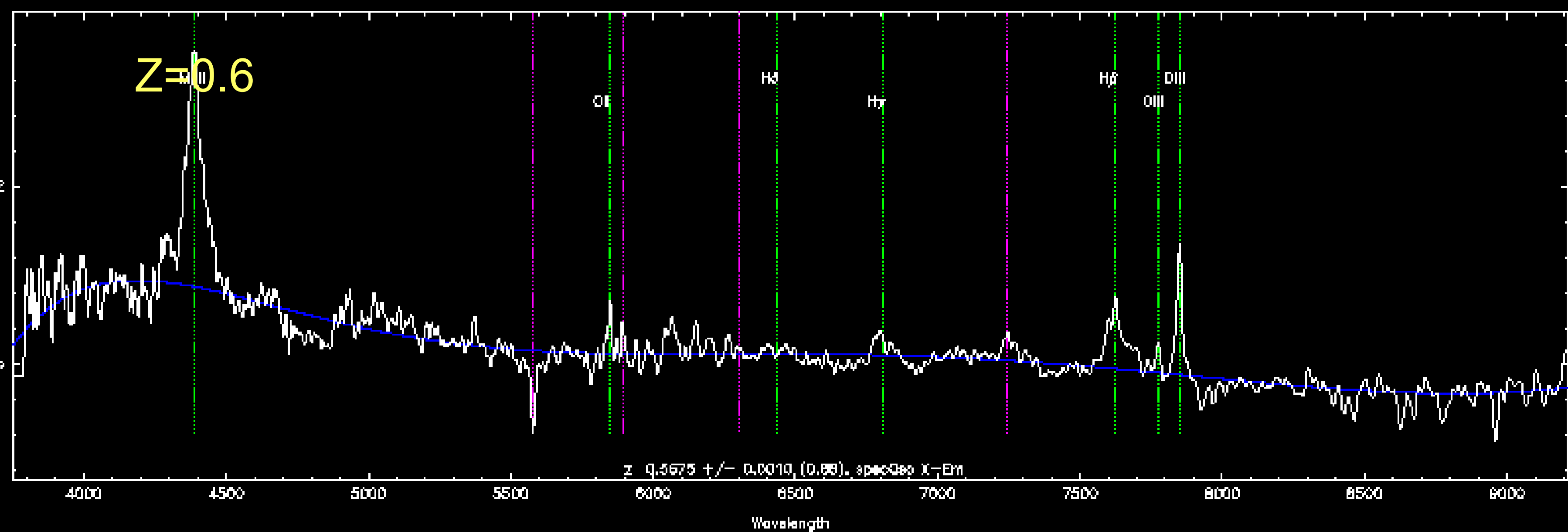
$H_0 = 70$ km/s/Mpc

$z \leq 0.1$

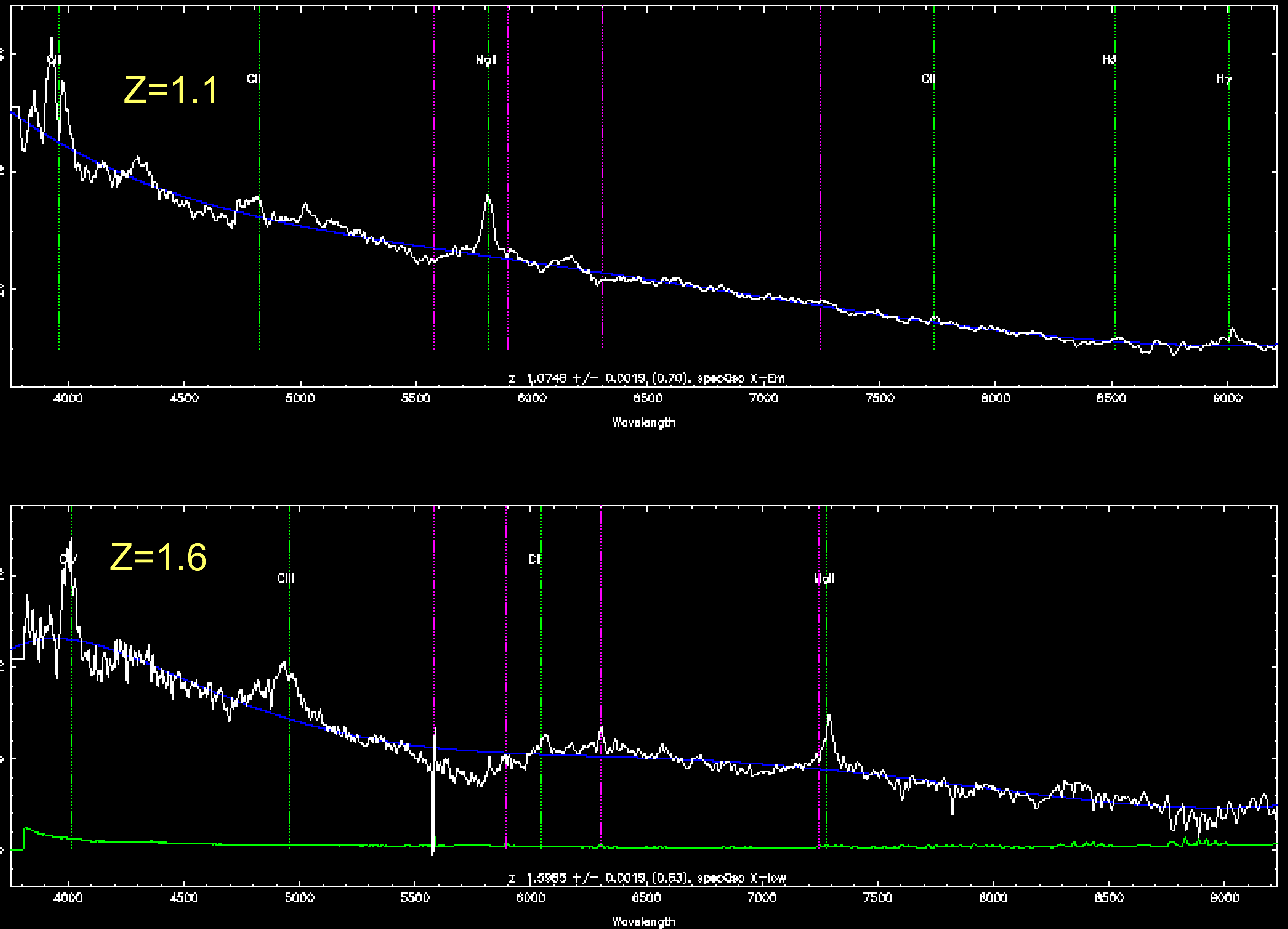




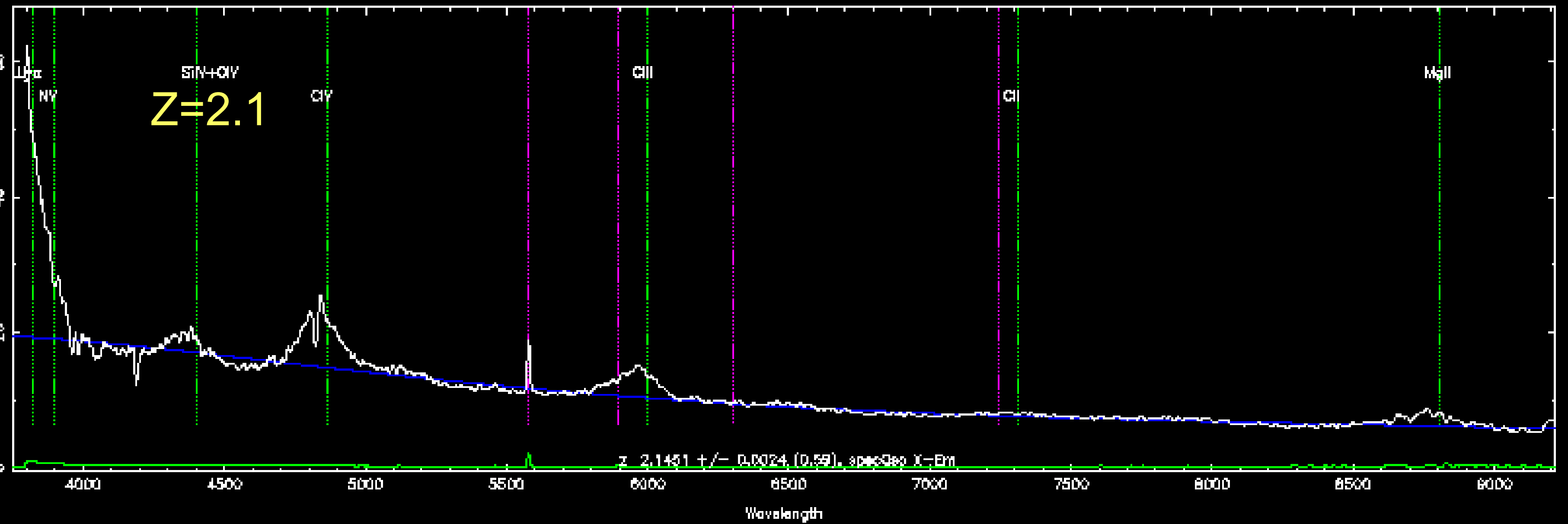
Z=0.1
OII
Hγ
OIII
Hβ
OIII
z 0.1031 +/- 0.0007 (1.00), specQso X-Em
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
Wavelength

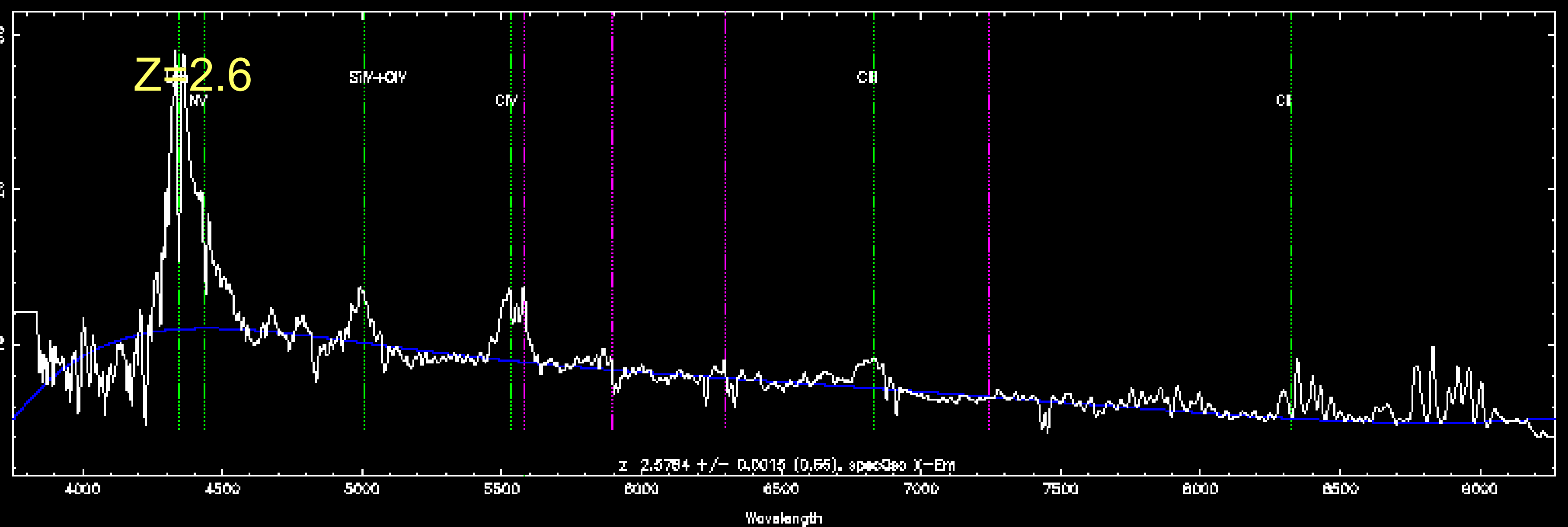


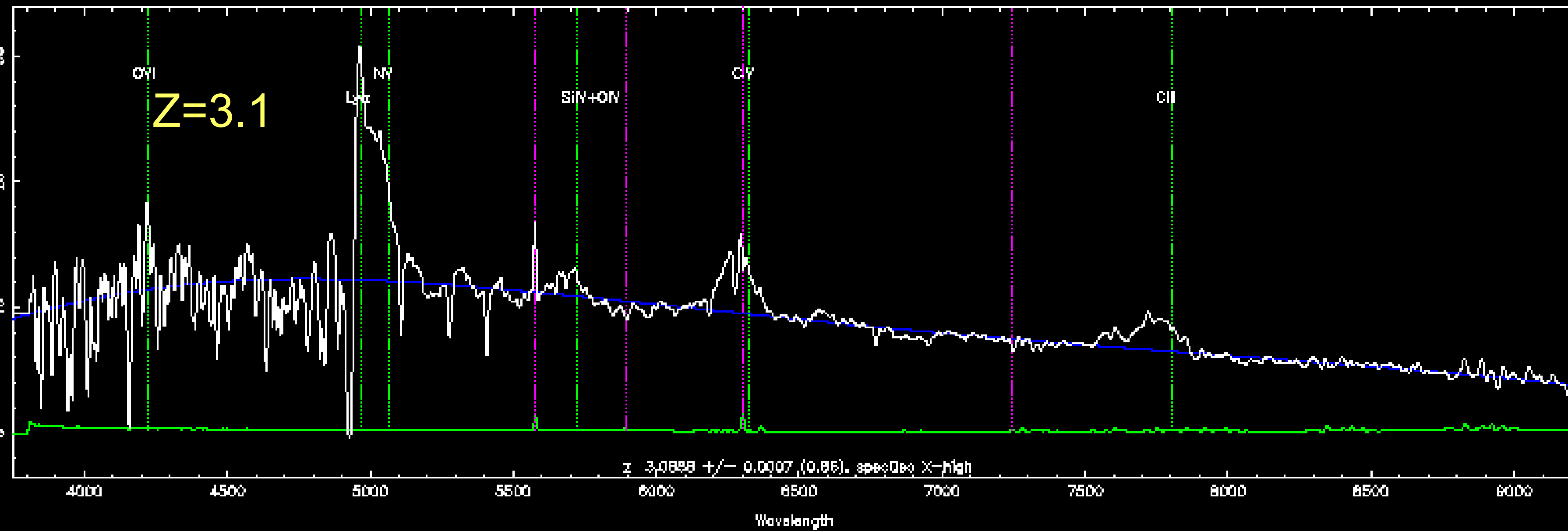
Z=0.6
OII
Hδ
Hγ
Hβ
OIII
OIII
z 0.5675 +/- 0.0010 (0.88), specQso X-Em
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
Wavelength

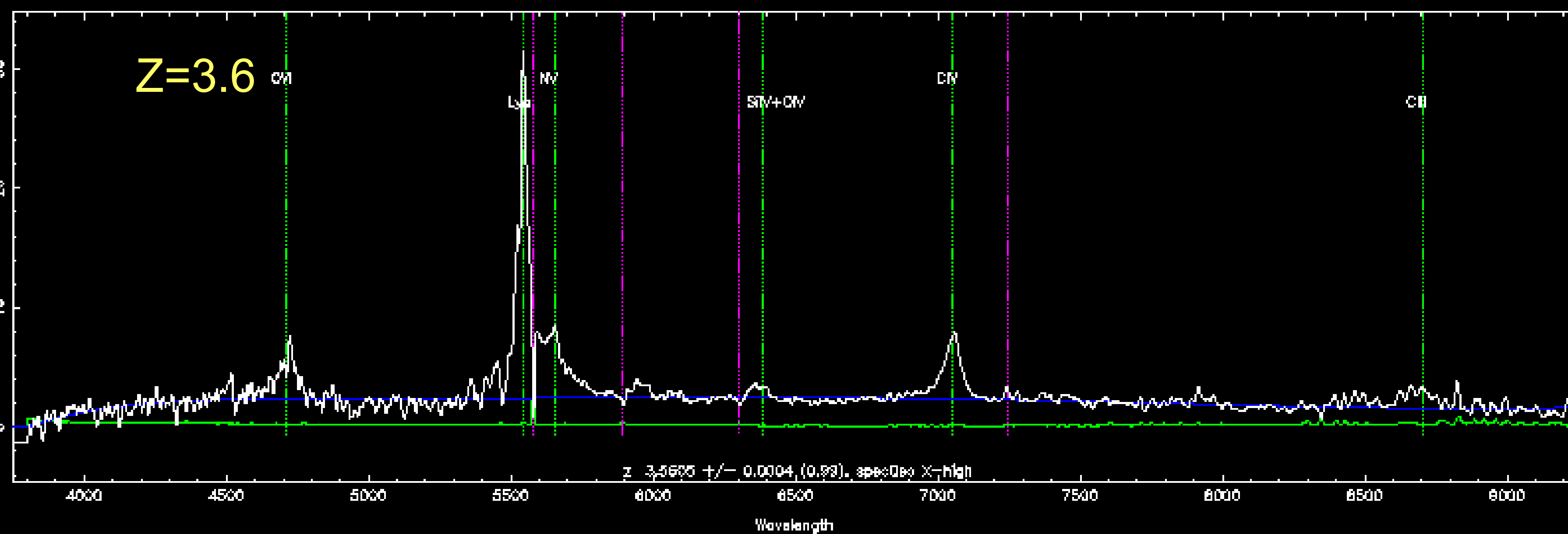


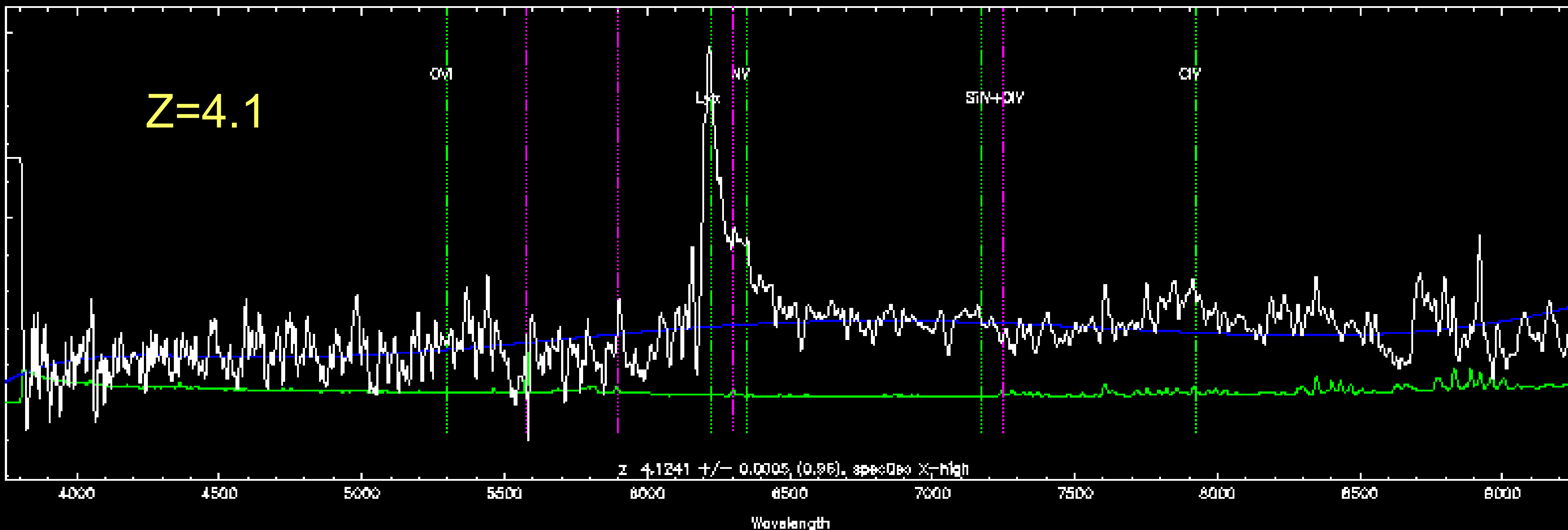
Z=1.1
CIII
CII
MgII
OII
Hδ
Hγ
z 1.0748 +/- 0.0019 (0.70). specQso X-Em
Wavelength
Z=1.6
CIV
CIII
CII
MgII
z 1.5985 +/- 0.0019 (0.63). specQso X-low
Wavelength


Z=2.1
Lyα
NV
SiIV+OIV
CIV
CIII
CII
MgII
z 2.1451 +/- 0.0024 (0.59), specQso X-Em
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
Wavelength

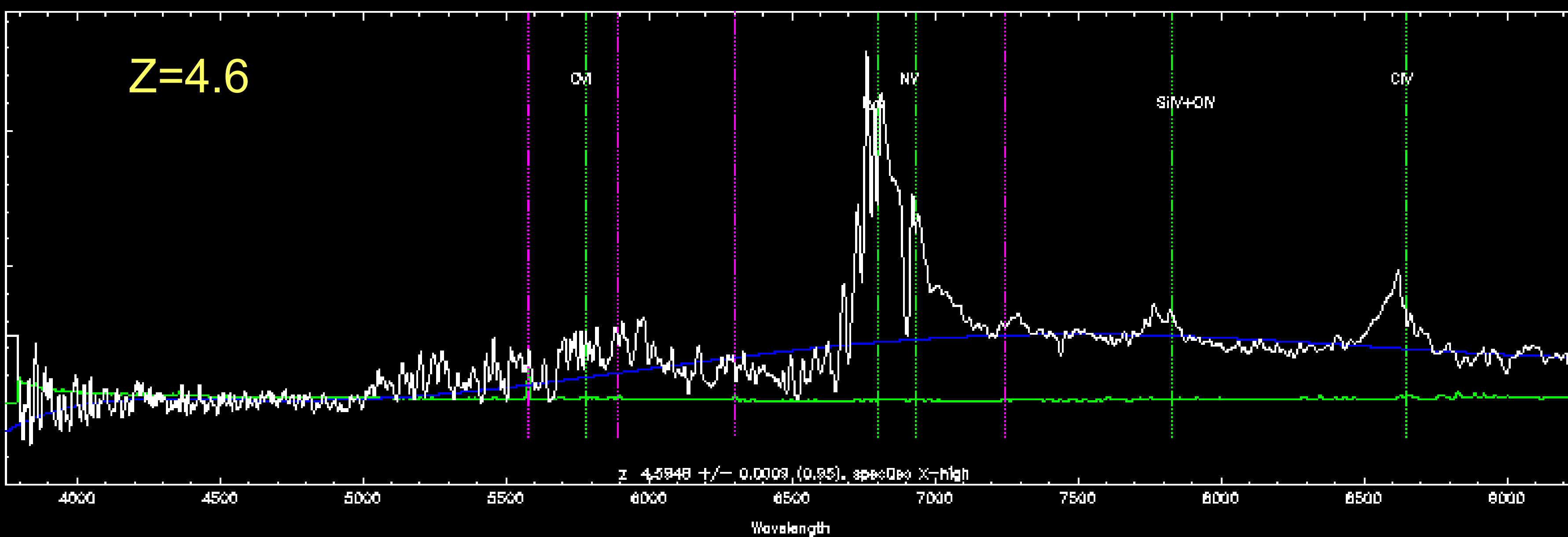

Z=2.6
NV
SiIV+OIV
CIV
CIII
CII
z 2.5784 +/- 0.0015 (0.55), specQso X-Em
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
Wavelength


Z=3.1
OVI
Lyα
NV
SiIV+OIV
CIV
CIII
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
Wavelength

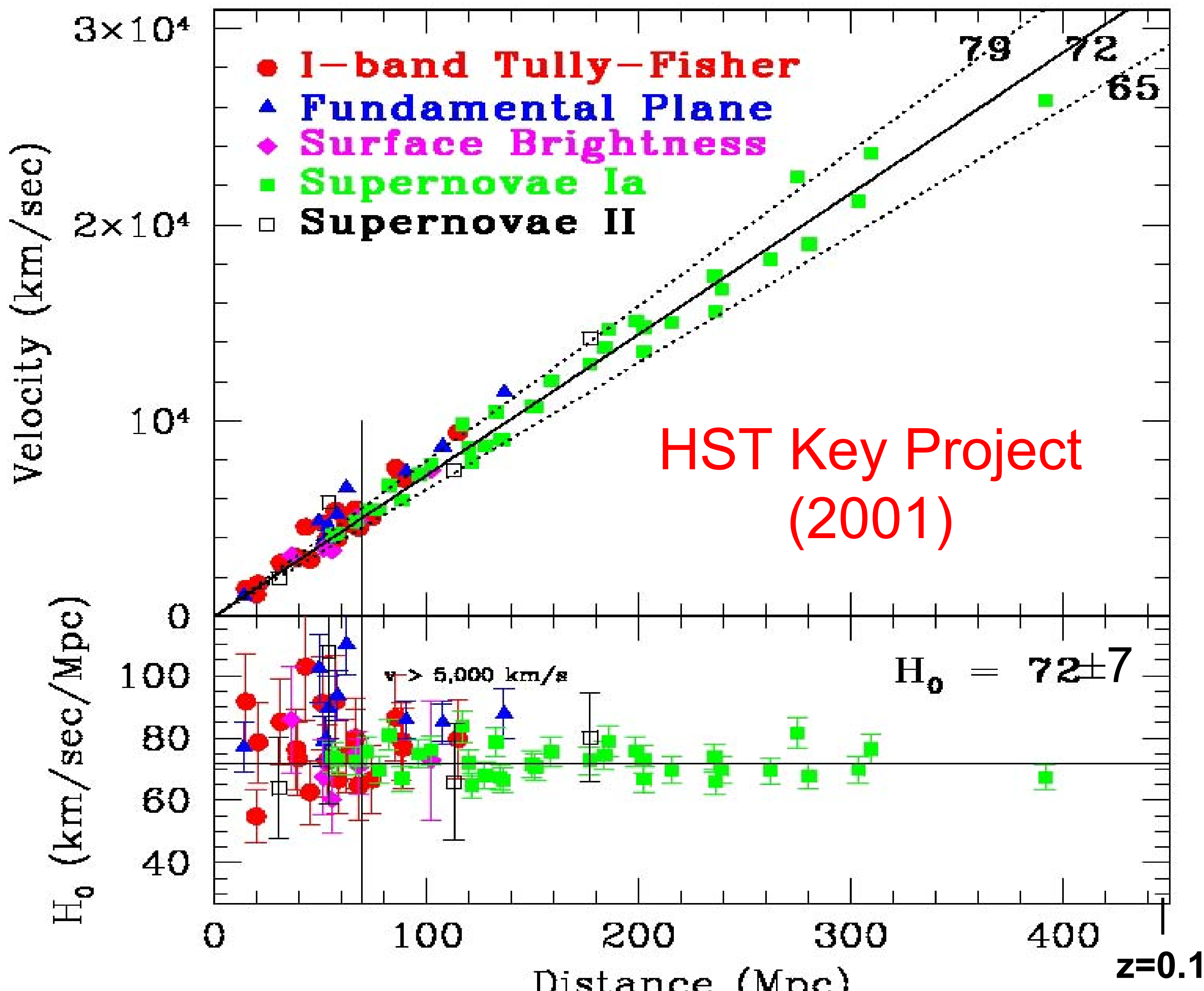

Z=3.6
OVI
Lyα
NV
SiIV+OIV
CIV
CIII
4000
4500
5000
5500
6000
6500
7000
7500
8000
8500
9000
Wavelength



Z=4.1
OVI
Lyα
NV
SiIV+OIV
CIV
z 4.1241 +/- 0.0005
Wavelength

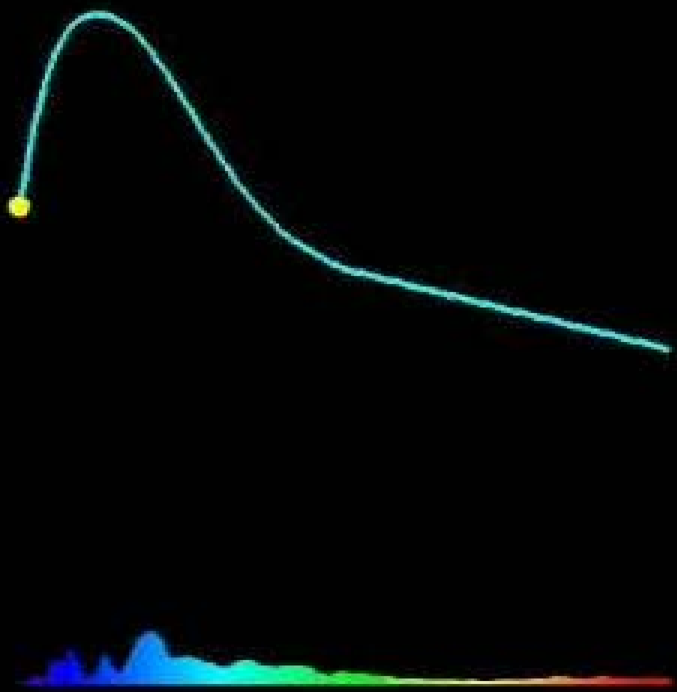


Z=4.6
OVI
NV
SiIV+OIV
CIV
z 4.5948 +/- 0.0009
Wavelength

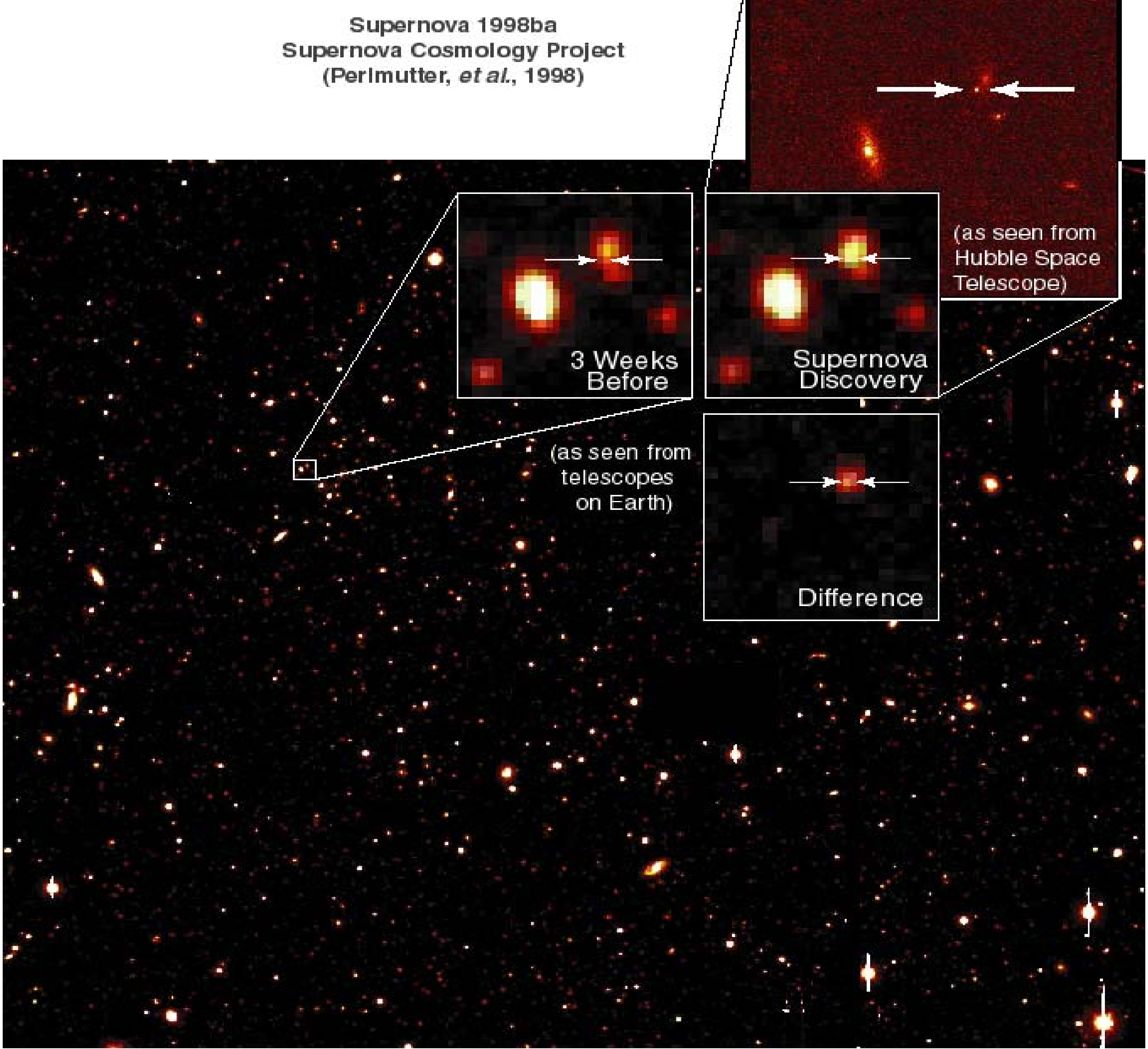


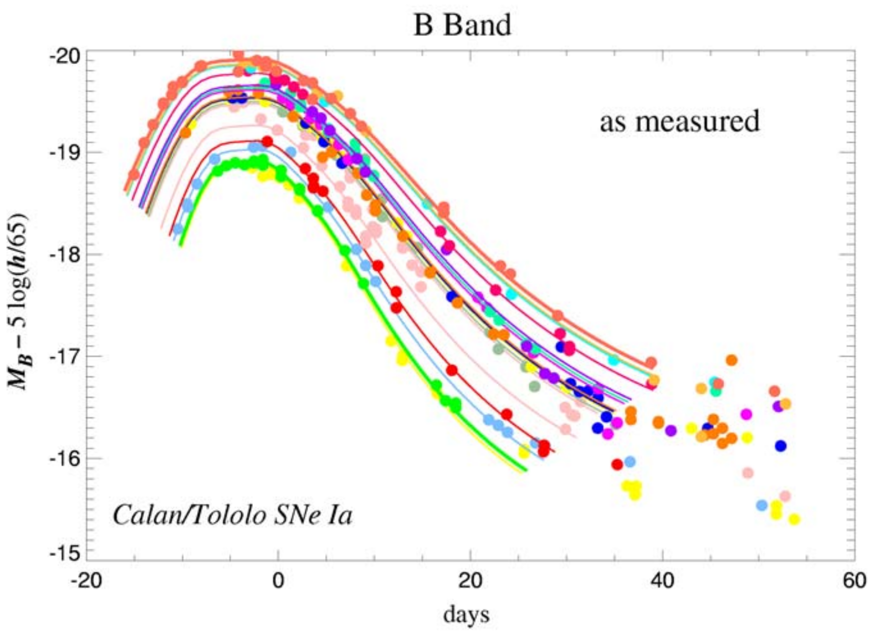
I-band Tully-Fisher
Fundamental Plane
Surface Brightness
Supernovae Ia
Supernovae II
79
72
65
HST Key Project
(2001)
Velocity (km/sec)
3×10⁴
2×10⁴
10⁴
0
v > 5,000 km/s
H0 = 72±7
H0 (km/sec/Mpc)
100
80
60
40
0
100
200
300
400
z=0.1
Distance (Mpc)

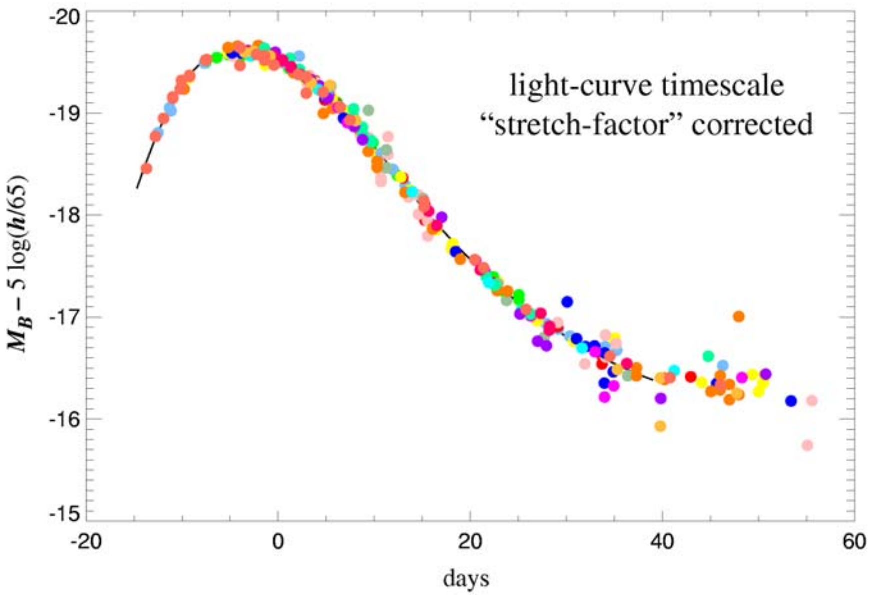
The Accelerating Universe



Supernova 1998ba
Supernova Cosmology Project
(Perlmutter, *et al.*, 1998)







Kim, *et al.* (1997)

Supernovae Ia lightcurves & stretch-factor

SNIa as standard candles

Luminosity distance

L Absolute Luminosity of source

F Measured Flux at detector

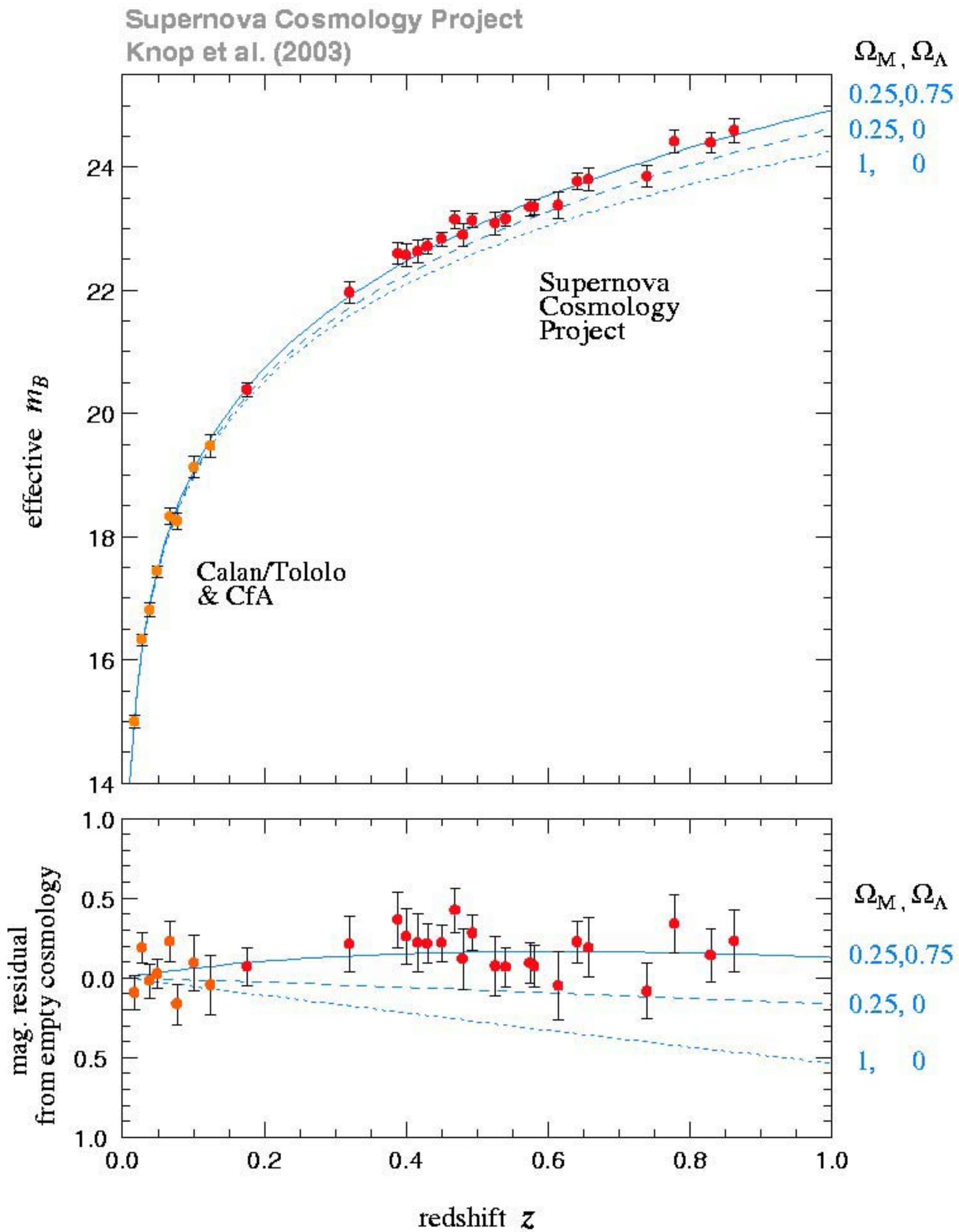
$$F = \frac{L}{4\pi\,(1+z)^2 a_0^2 r^2(z)} \equiv \frac{L}{4\pi\, d_L^2(z)}$$

$$H_0 d_L(z) = (1+z)\left|\Omega_K\right|^{-1/2} \mathbf{sinn}\left[\int_0^z \frac{\left|\Omega_K\right|^{1/2} dz'}{H(z')}\right]$$

Effective magnitude

$$m(z) \equiv M + 5\,\mathbf{log}_{10}\left(\frac{d_L(z)}{\text{Mpc}}\right) + 25$$

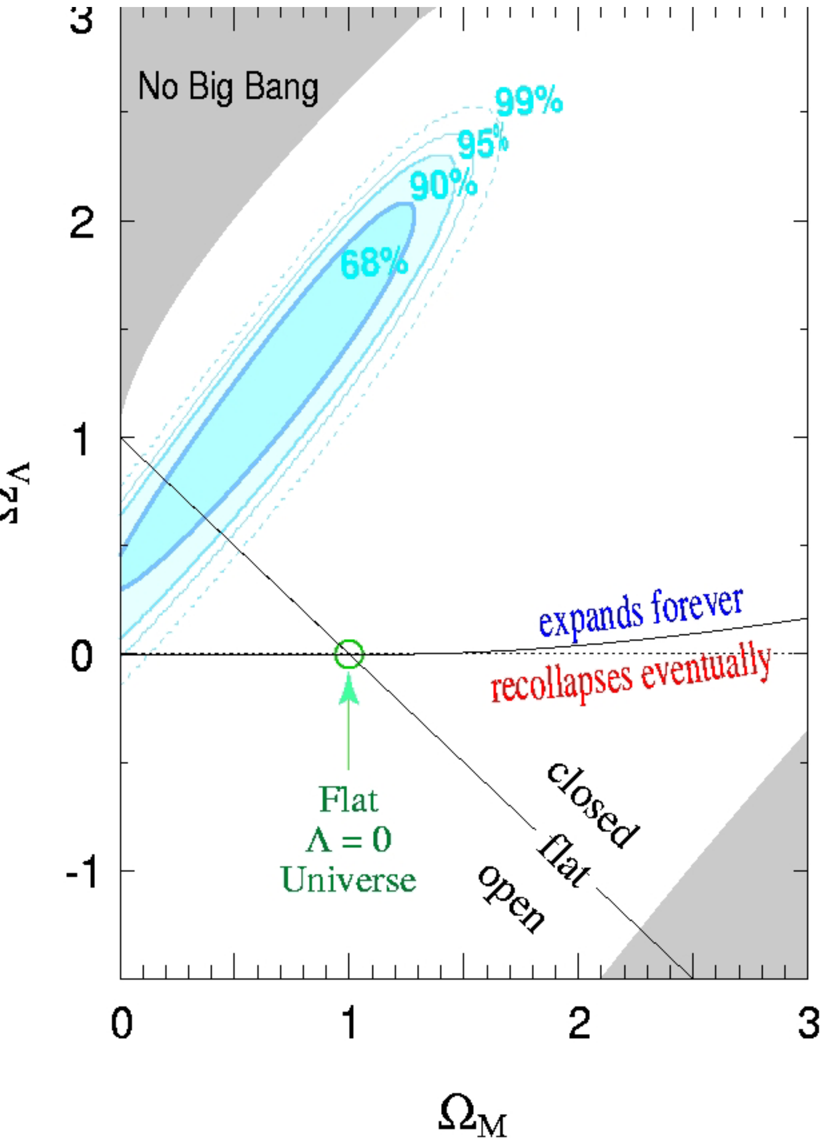
$$= \overline{M} + 5\,\mathbf{log}_{10}[H_0 d_L(z)]$$

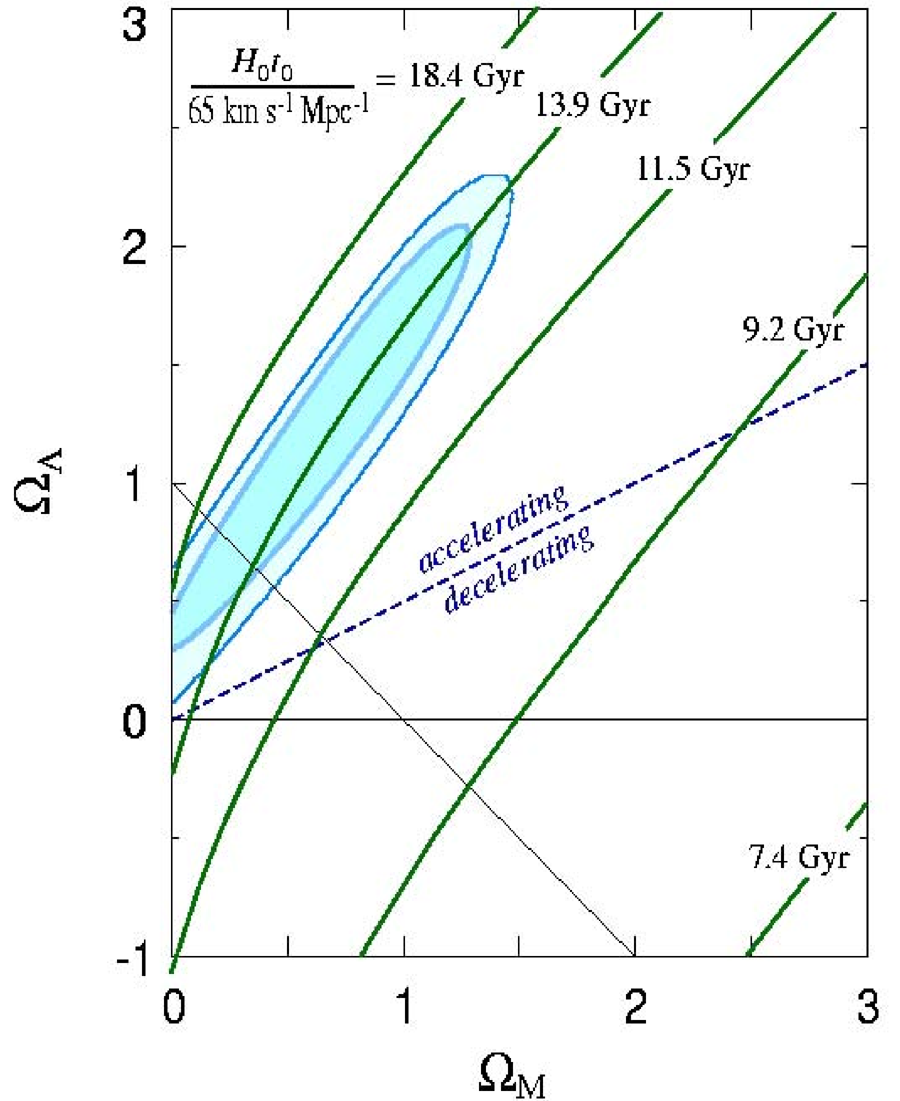


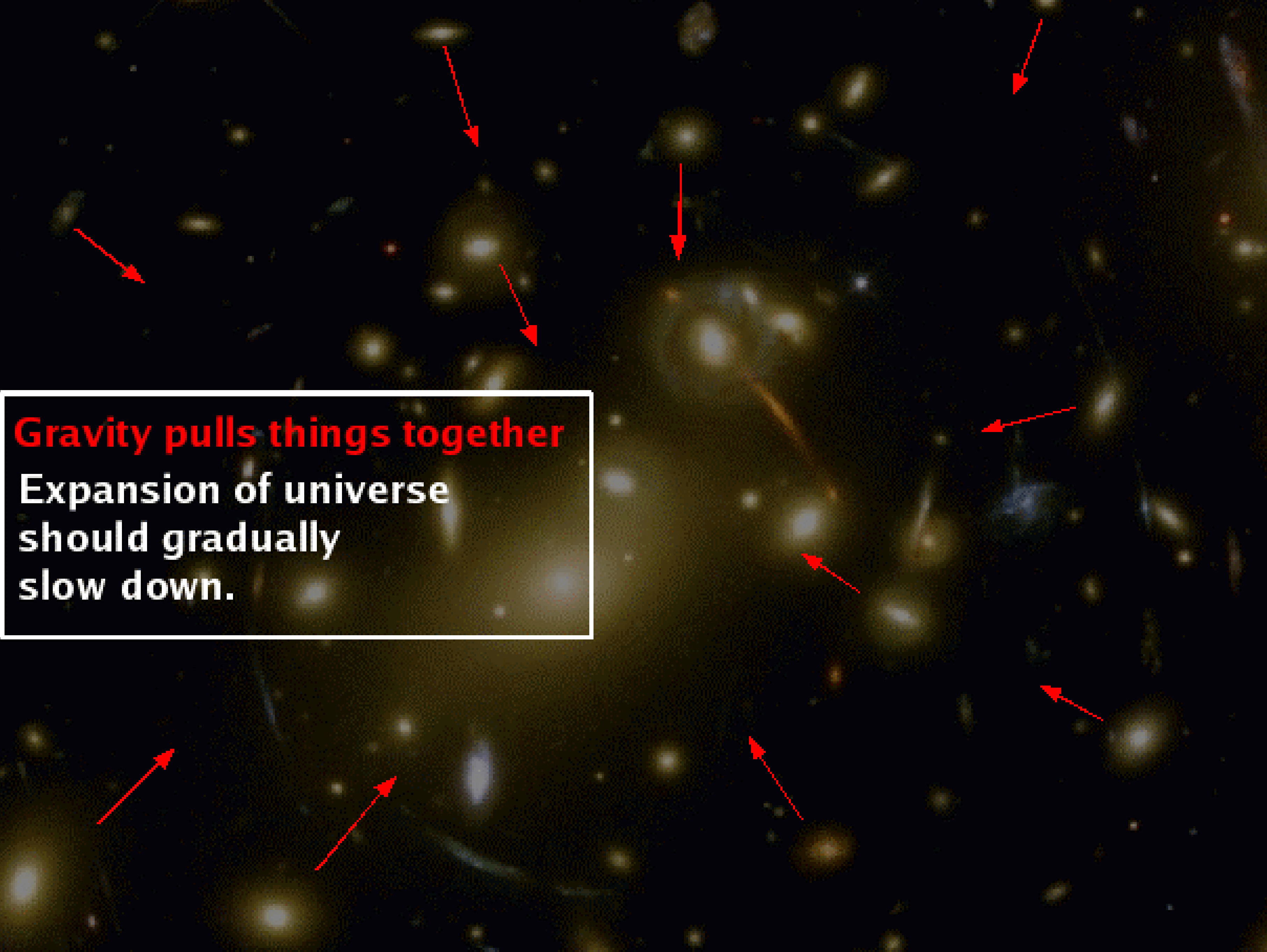
The Accelerating Universe (2003)

$$0.8\Omega_M - 0.6\Omega_\Lambda = -0.16 \pm 0.05$$

SuperNovae Cosmology Project (1998-2003)





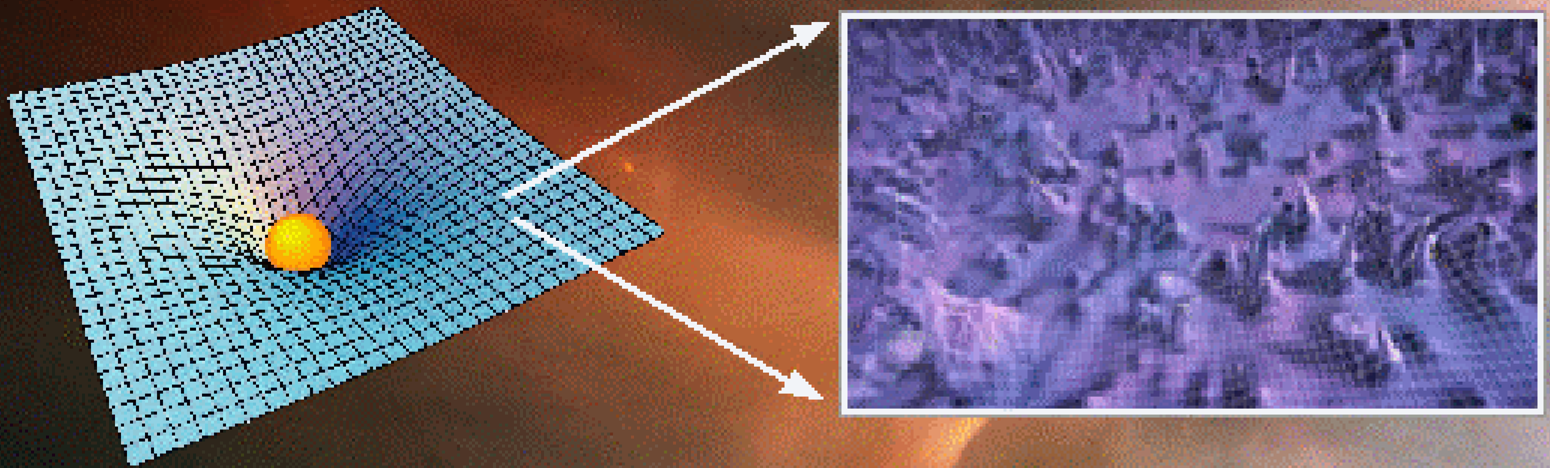

Gravity pulls things together
Expansion of universe
should gradually
slow down.


Something is pushing the galaxies apart
DARK ENERGY

The Physics of Nothing

How can *nothing* be most of *everything* in the universe?

The answer (maybe) is quantum uncertainty:
"empty space" is a sea of virtual particles winking
in and out of existence:



Nothing is something!

Cosm. Const. = Vacuum Energy

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$T_{\mu\nu} = p_v g_{\mu\nu} = -\rho_v g_{\mu\nu} \quad \Rightarrow \quad \Lambda = 8\pi G \rho_v$$

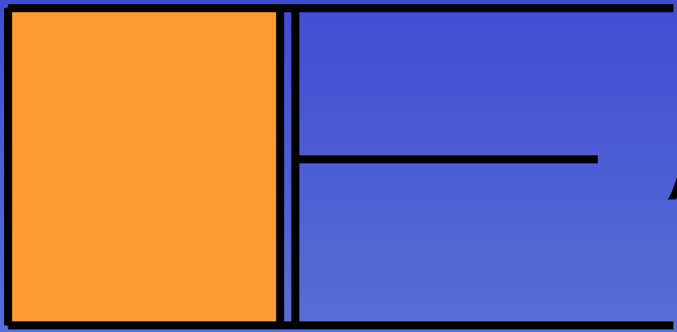
$\rho_v =$  $+ \ \ldots$

$$\rho_v = \sum_i \int_0^{\Lambda_{UV}} \frac{d^3k}{(2\pi)^3} \frac{\eta \varpi_i(k)}{2} = \frac{\eta \Lambda_{UV}^4}{16\pi^2} \sum (-1)^{F_i} N_i + \ldots$$

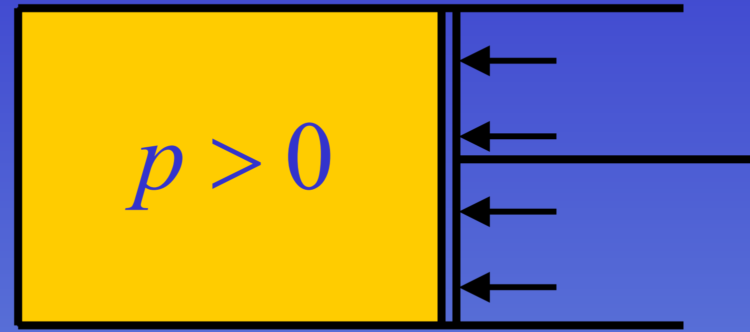
$$\Lambda_{UV} \approx M_{Pl} \quad \Rightarrow \quad \rho_v^{th} \approx 10^{120} \rho_v^{obs} = 10^{120} \left(2 \cdot 10^{-3} eV\right)^4$$

$$\Lambda_{UV} \approx M_{EW} \quad \Rightarrow \quad \rho_v^{th} \approx 10^{65} \rho_v^{obs}$$

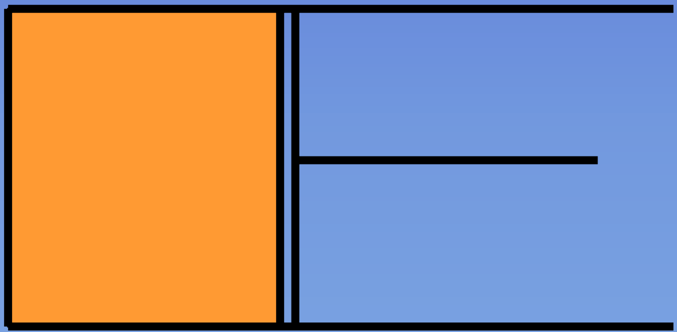
Normal Matter



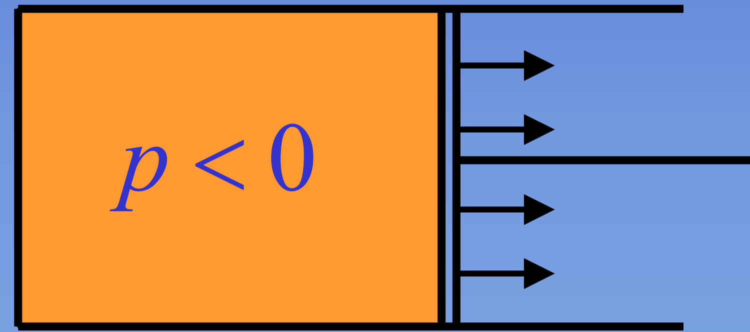
$$\rho_1 > \rho_2$$



$$d(\rho V) + p dV = TdS \approx 0$$



$$\rho_1 = \rho_2$$



Vacuum Energy

Nature of Dark Energy?

$$\Lambda = 8\pi G\rho_v = \text{const.} \overset{\text{P.F.}}{\Rightarrow} \quad w_v = \frac{p_v}{\rho_v} = -1$$

$$\rho_x \neq \text{const.} \quad \Rightarrow \quad w_x \neq -1$$

$$H^2(z) = H_0^2\left[\Omega_M(1+z)^3 + \Omega_x e^{\int_0^z (1+w_x(u))\frac{3\,du}{1+u}} + \Omega_K(1+z)^2\right]$$

$$q(z) = -1 + (1+z)\frac{d}{dz}\ln H(z)$$

$$= \frac{1}{2}\Omega_0 + \frac{3}{2}w_x(z)\Omega_x(z)$$

Coasting Point

Assuming $w_x = w = \text{const.} < 0$

$$q(z) = \frac{1}{2}\left[\frac{\Omega_M + (1+3w)\Omega_x(1+z)^{3w}}{\Omega_M + \Omega_x(1+z)^{3w} + \Omega_K(1+z)^{-1}}\right] = 0$$

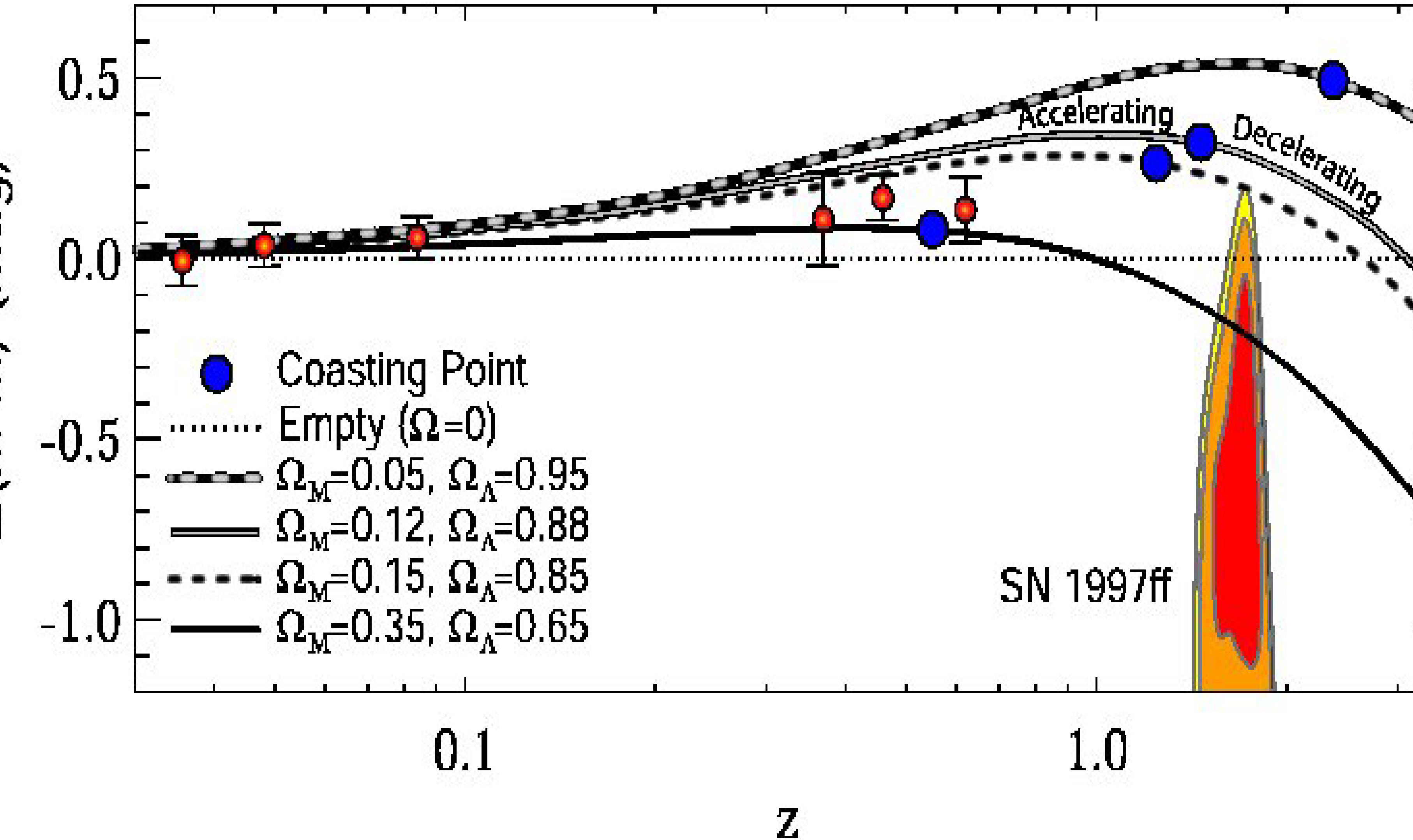
$$\Rightarrow \quad z_c = \left(\frac{(3|w|-1)\Omega_x}{\Omega_M}\right)^{\frac{1}{3|w|}} - 1$$

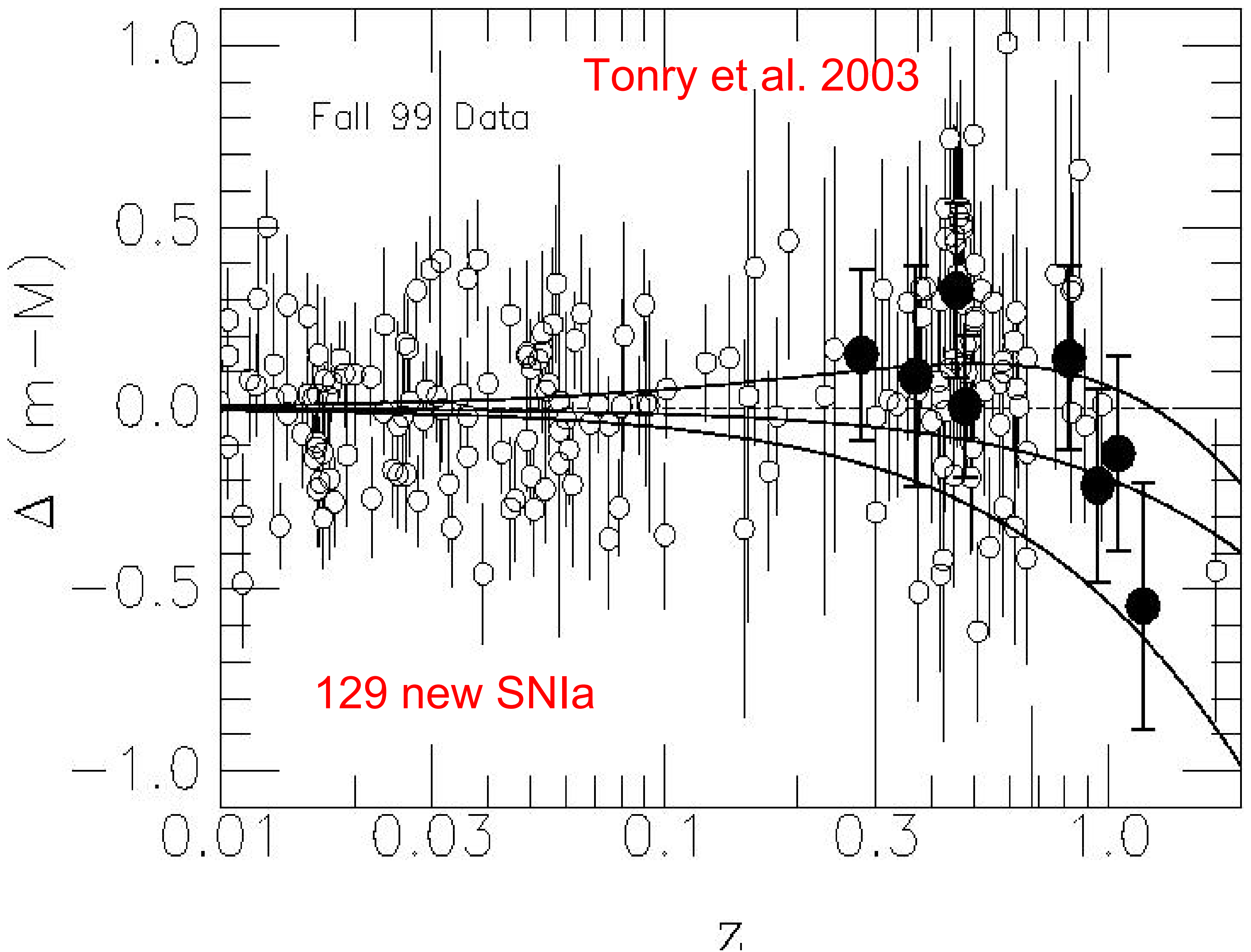
$z > z_c$ universe decelerating

$z < z_c$ universe accelerating

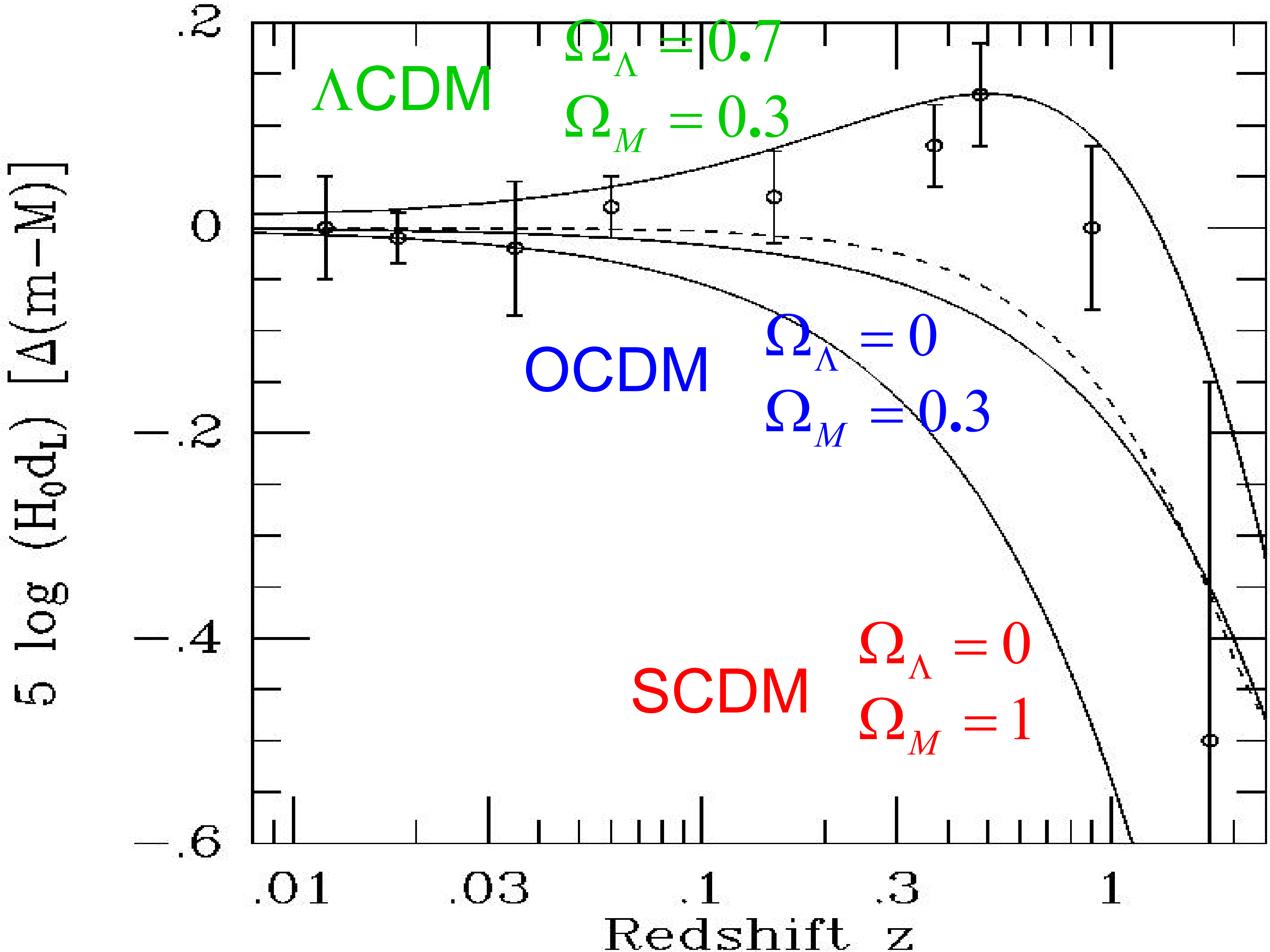
e.g. $w = -1 \quad \Rightarrow \quad z_c = \left(\frac{2\Omega_\Lambda}{\Omega_M}\right)^{\frac{1}{3}} - 1 \approx 0.5$

Perlmutter et al. 2002



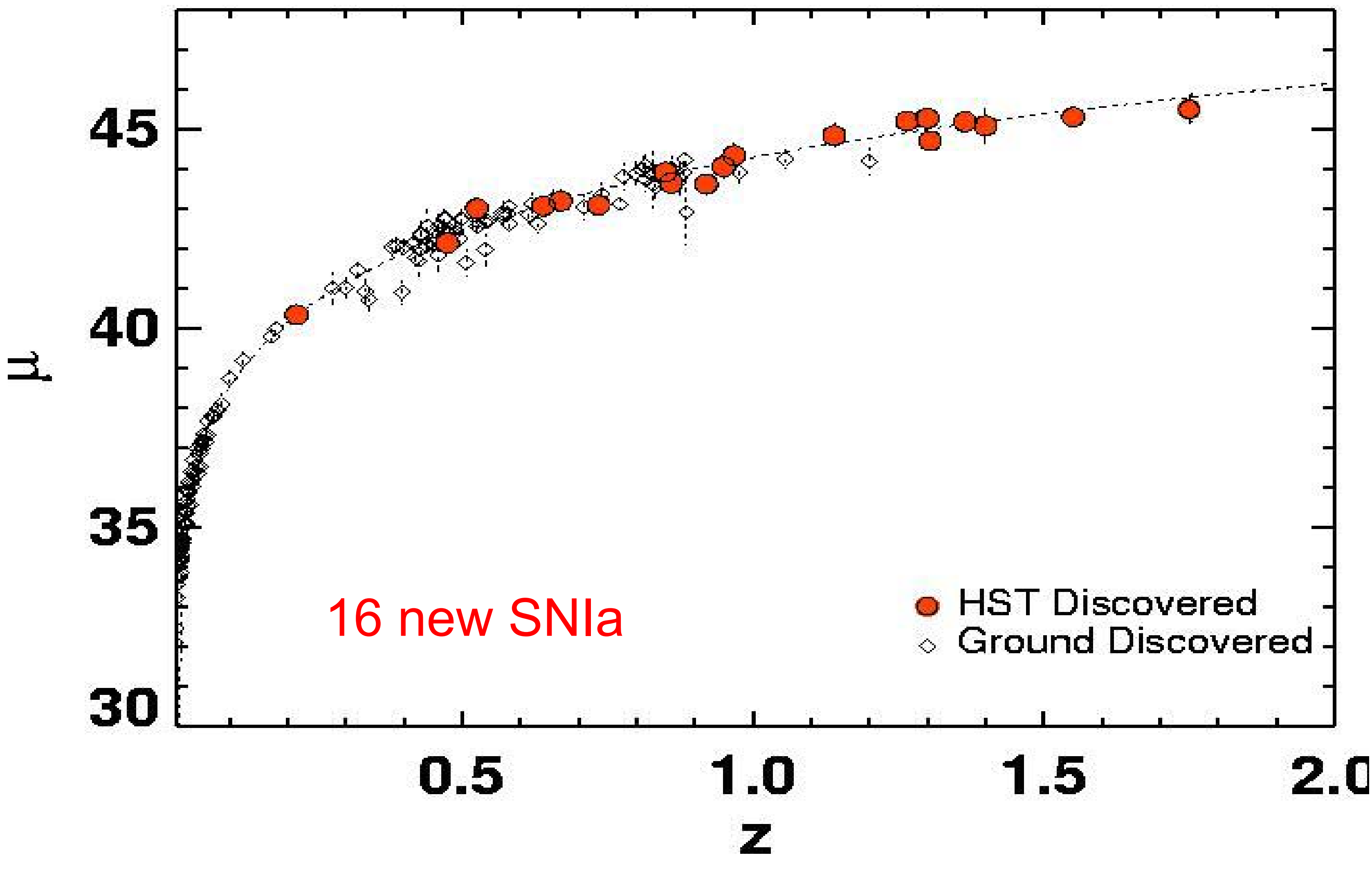


Tonry et al. 2003
Fall 99 Data
129 new SNIa
Δ (m−M)
1.0
0.5
0.0
−0.5
−1.0
0.01
0.03
0.1
0.3
1.0
z



Freedman & Turner (2003)

Riess et al. (2004)



Taylor expansion to higher order

$$\frac{a(t)}{a_0} = 1 + H_0(t - t_0) - \frac{q_0}{2!} H_0^2 (t - t_0)^2 + \frac{j_0}{3!} H_0^3 (t - t_0)^3 + \ldots$$

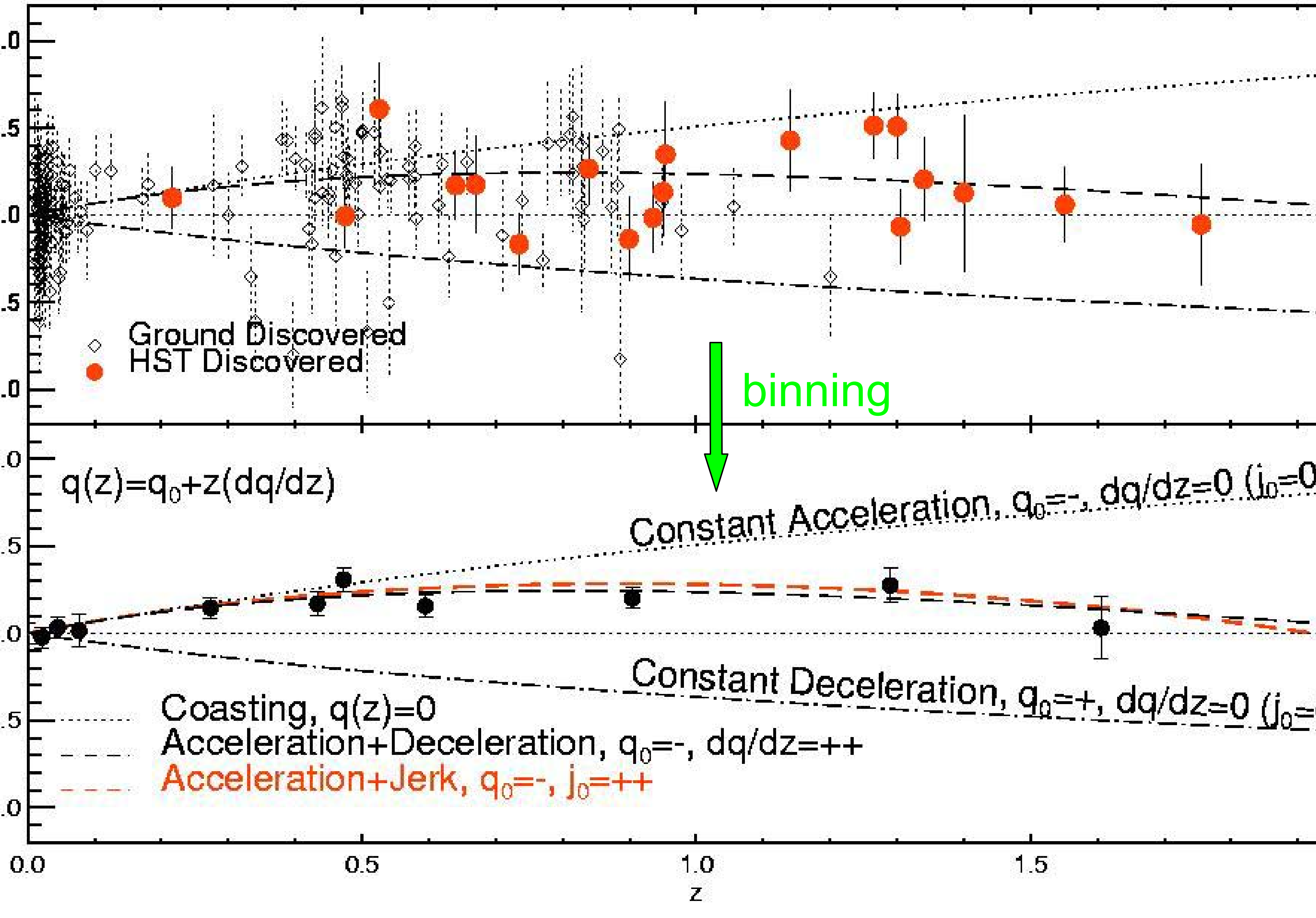
$$q_0 = -\frac{\ddot{a}}{aH^2}(t_0) = \frac{1}{2}\sum_i (1 + 3w_i)\,\Omega_i = \frac{1}{2}\Omega_M - \Omega_\Lambda$$

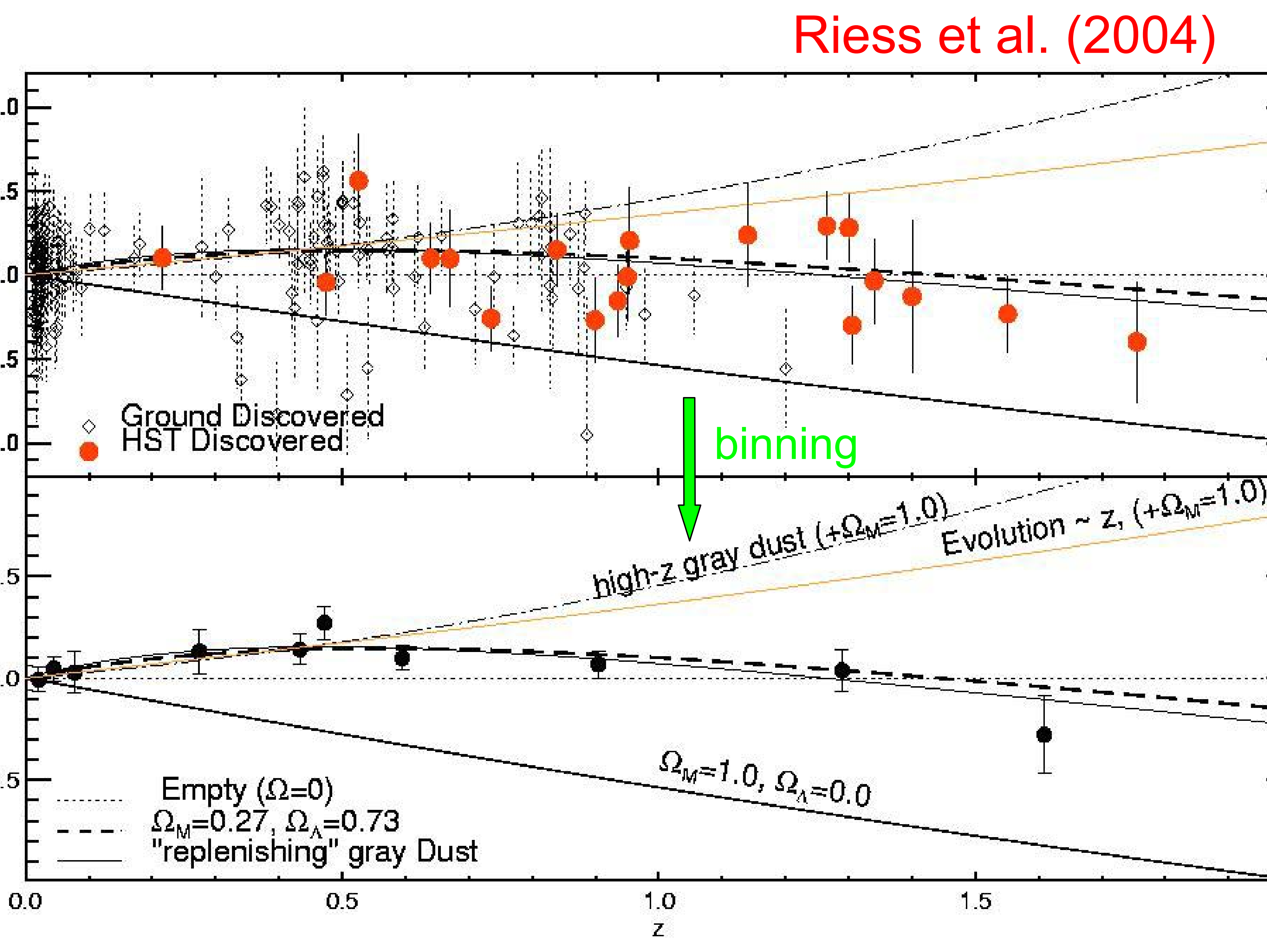
$$j_0 = \frac{\dddot{a}}{aH^3}(t_0) = \frac{1}{2}\sum_i (1 + 3w_i)(2 + 3w_i)\,\Omega_i = \Omega_M + \Omega_\Lambda$$

To good approximation:

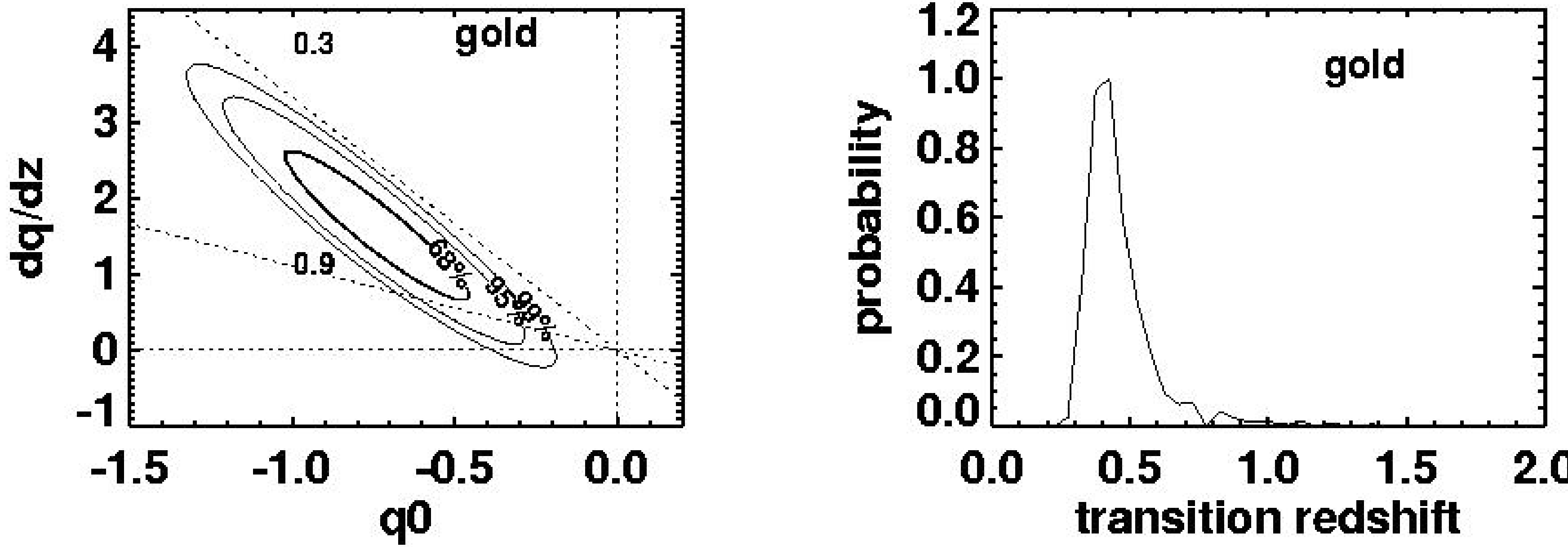
$$d_L(z) = \frac{cz}{H_0}\left\{1 + \frac{1}{2}[1 - q_0]z - \frac{1}{6}[1 - q_0 - 3q_0^2 + j_0]z^2 + \ldots\right\}$$

Riess et al. (2004)

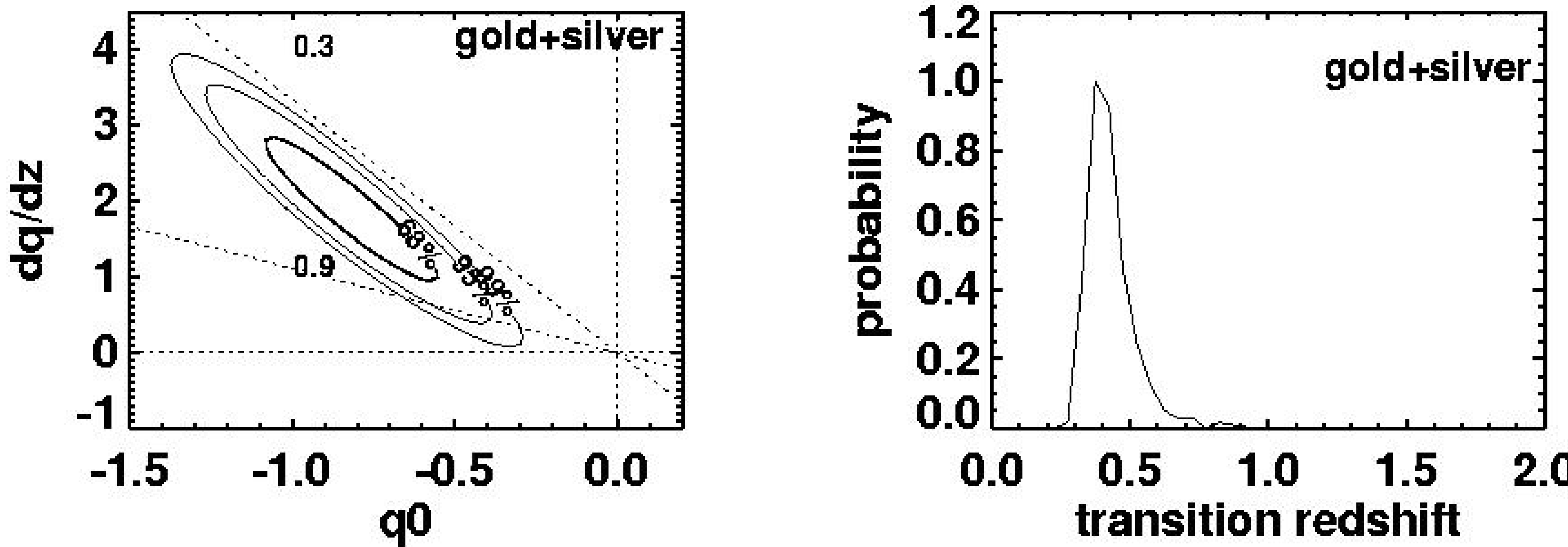


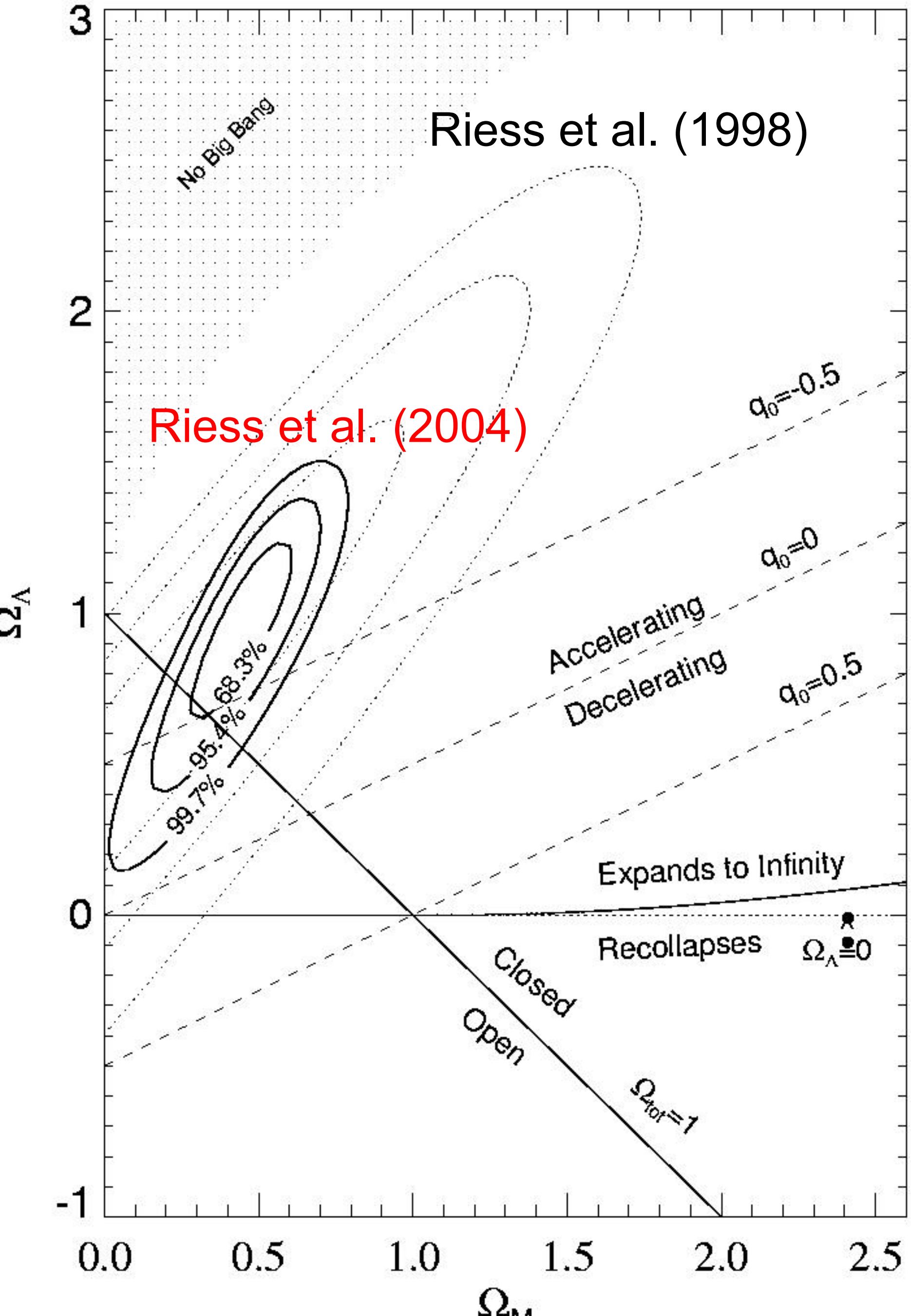


Riess et al. (2004)
Ground Discovered
HST Discovered
binning
high-z gray dust (+ΩM=1.0)
Evolution ~ z, (+ΩM=1.0)
ΩM=1.0, ΩΛ=0.0
Empty (Ω=0)
ΩM=0.27, ΩΛ=0.73
"replenishing" gray Dust
0.0
0.5
1.0
1.5
z


gold
0.3
0.9
68%
95%
99%
dq/dz
q0
-1.5
-1.0
-0.5
0.0
probability
transition redshift
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.5
2.0

$$z_c = 0.46 \pm 0.13$$


gold+silver
0.3
0.9
68%
95%
99%
dq/dz
q0
-1.5
-1.0
-0.5
0.0
probability
transition redshift
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.5
2.0



Riess et al. (2004)
192+16 SNIa

Flat Universe

$$\Omega_M = 0.29 \pm 0.05$$

$$\Omega_\Lambda = 0.71 \pm 0.05$$

Model Building

SCDM $H(z) = H_0(1+z)^{3/2}$ ruled out!

ΛCDM $H(z) = H_0[\Omega_M(1+z)^3 + 1 - \Omega_M]^{1/2}$

$$\Rightarrow \quad \Omega_M = 0.29 \pm 0.04$$

ΛCDMw $H(z) = H_0[\Omega_M(1+z)^3 + \Omega_\Lambda(1+z)^{3(1+w)}]^{1/2}$

$$\Rightarrow \quad \Omega_M = 1 - \Omega_\Lambda = 0.3\,, \quad w = -1.02 \pm 0.10$$

ΛCDM - w(z)

$$H(z) = H_0[\Omega_M(1+z)^3 + \Omega_\Lambda \exp[3\int_0^z (1+w(u))\tfrac{du}{1+z}]]^{1/2}$$

Linear Ansatz: $w(z) = w_0 + w_1 z$

best fit : $w_0 = -1.2,\ w_1 = 2.0$

Linder Ansatz: $w(z) = w_0 + \dfrac{w_1 z}{1+z}$

best fit : $w_0 = -1.3,\ w_1 = 2.8$

Chaplygin gas: $p_c = A/\rho_c$ matter $\rightarrow \Lambda$

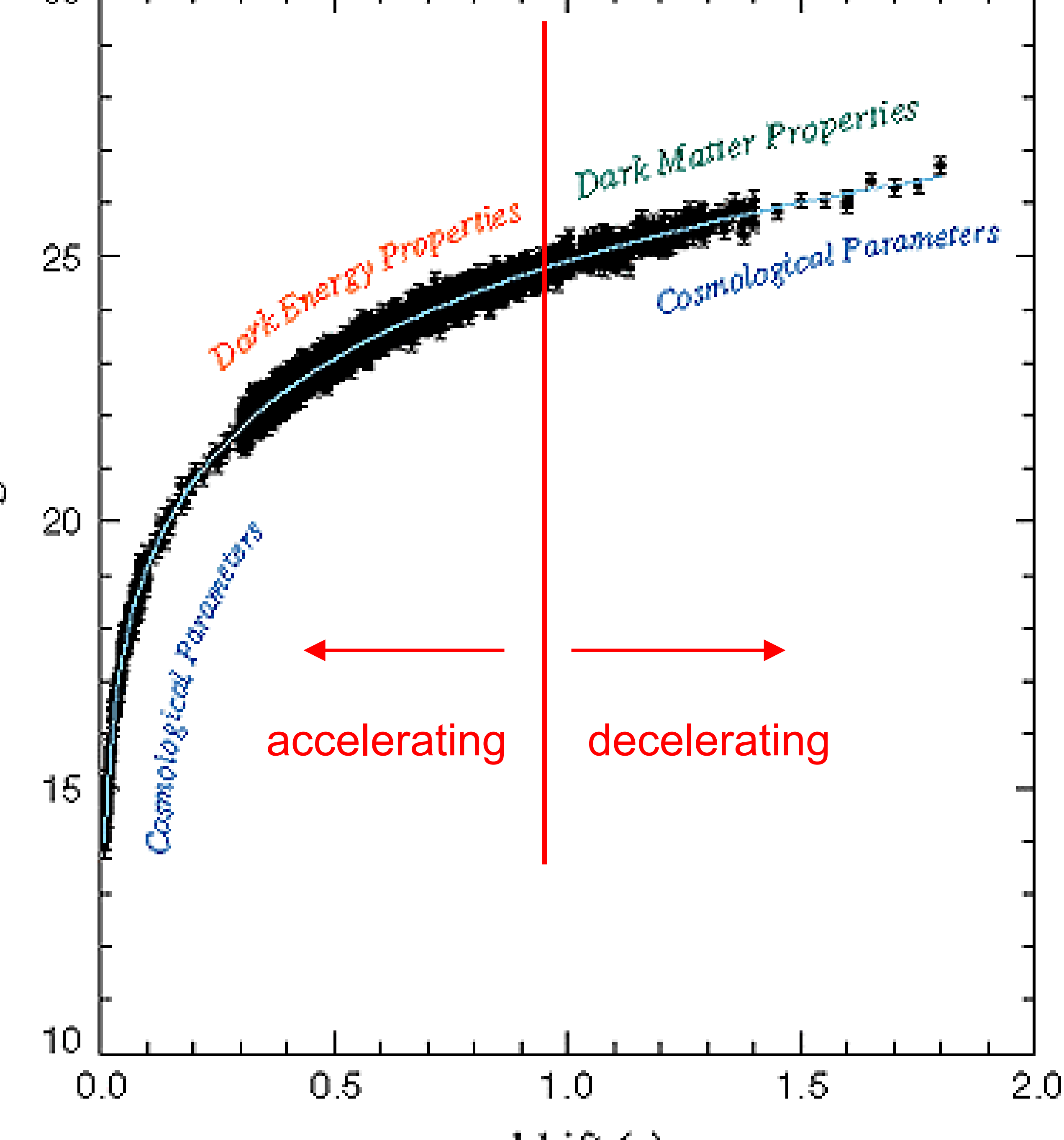
best fit : $A = 0.96$

Cardassian Ansatz: $w = n - 1 \neq -1 \quad const.$

best fit : $n = 0.07$

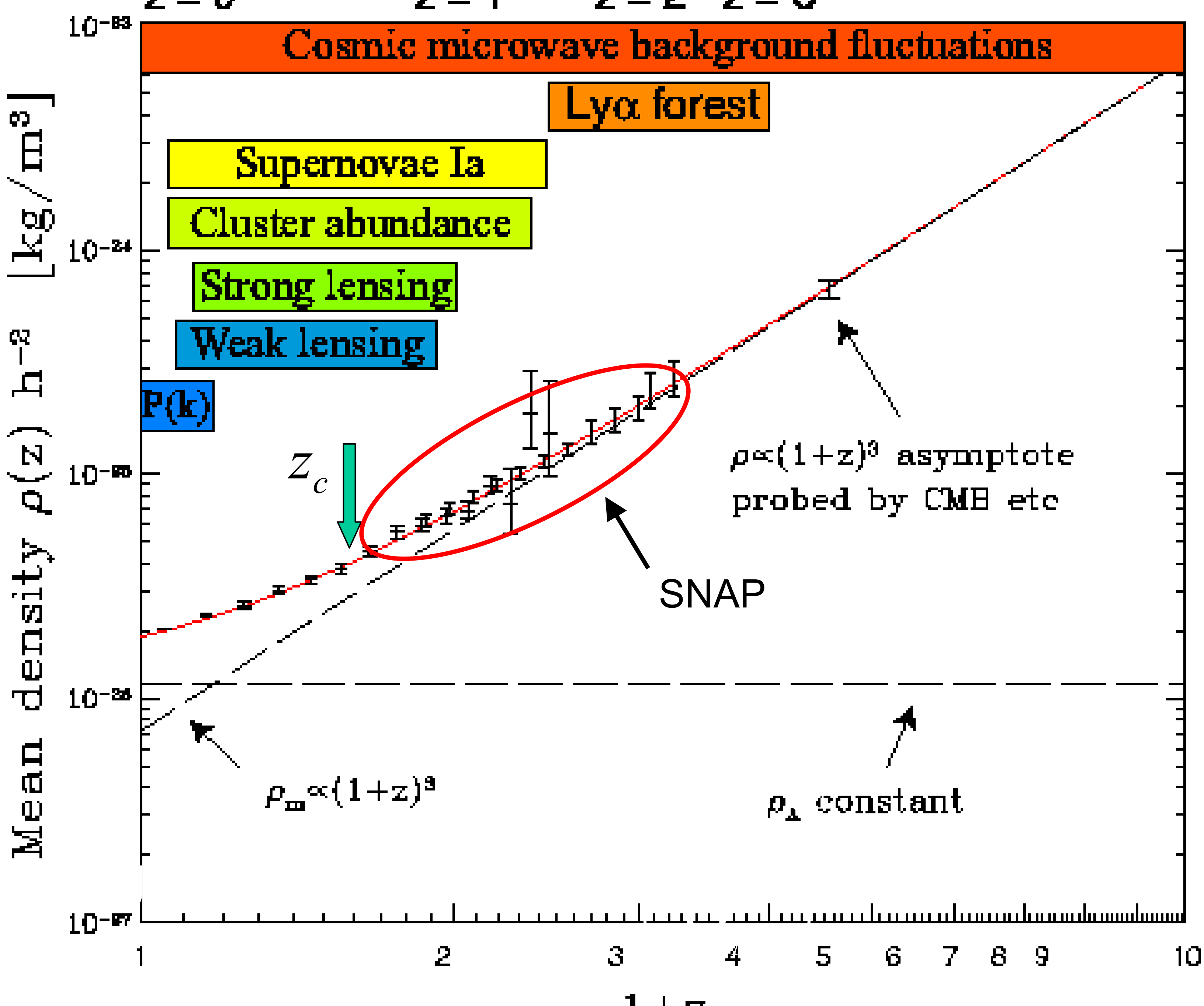
Ghost condensate: Dark matter $\rightarrow \Lambda$

SNAP satellite

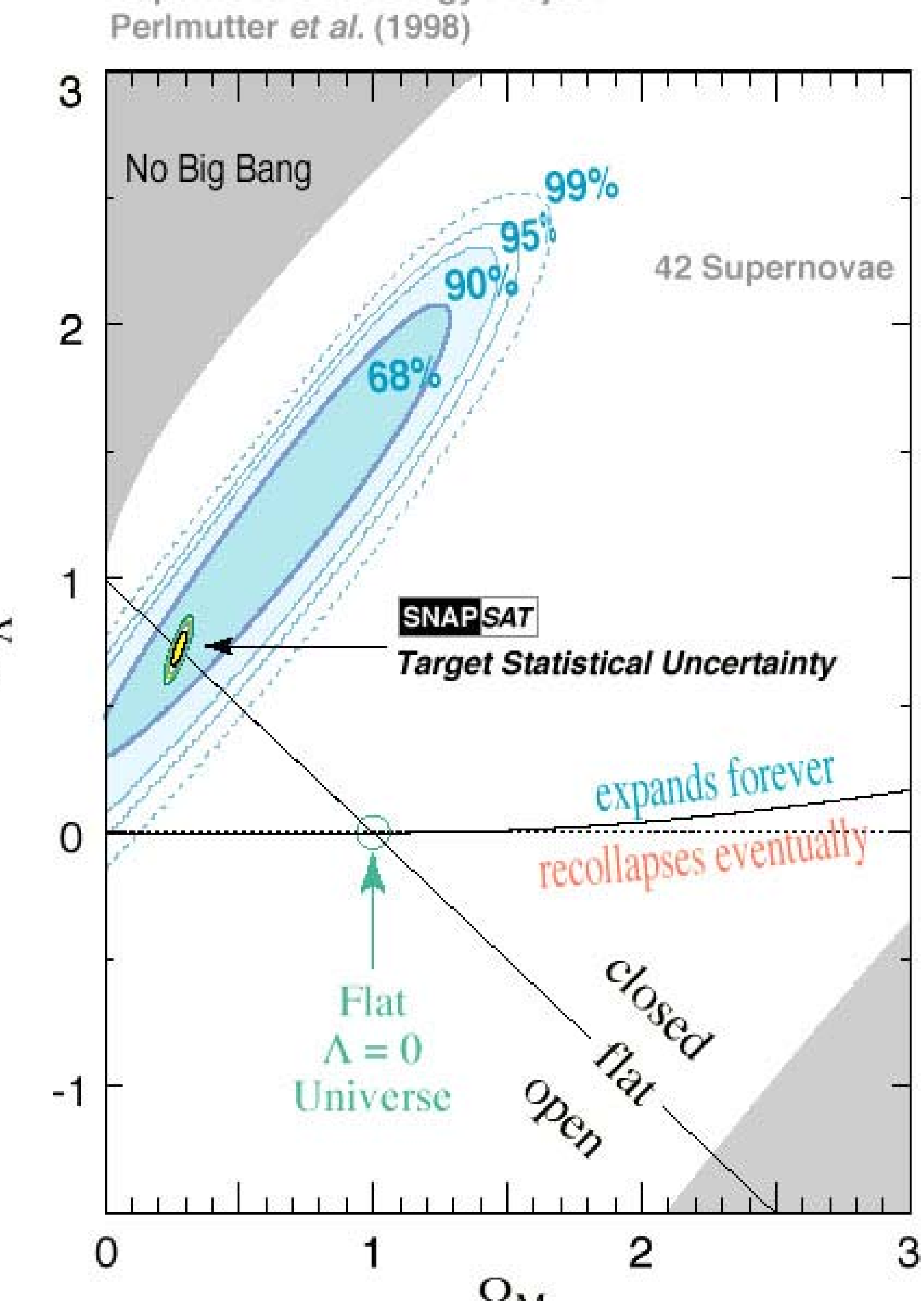


SNAP satellite:
2000 SNIa
up to z=2

$$\Omega_{\Lambda}(z)$$
$$w(z)$$



Cosmic microwave background fluctuations
Lyα forest
Supernovae Ia
Cluster abundance
Strong lensing
Weak lensing
P(k)
Z c
SNAP
ρ∝(1+z)³ asymptote
probed by CMB etc
ρm∝(1+z)³
ρΛ constant
Mean density ρ(z) h⁻² [kg/m³]
1
2
3
4
5
6
7
8
9
10

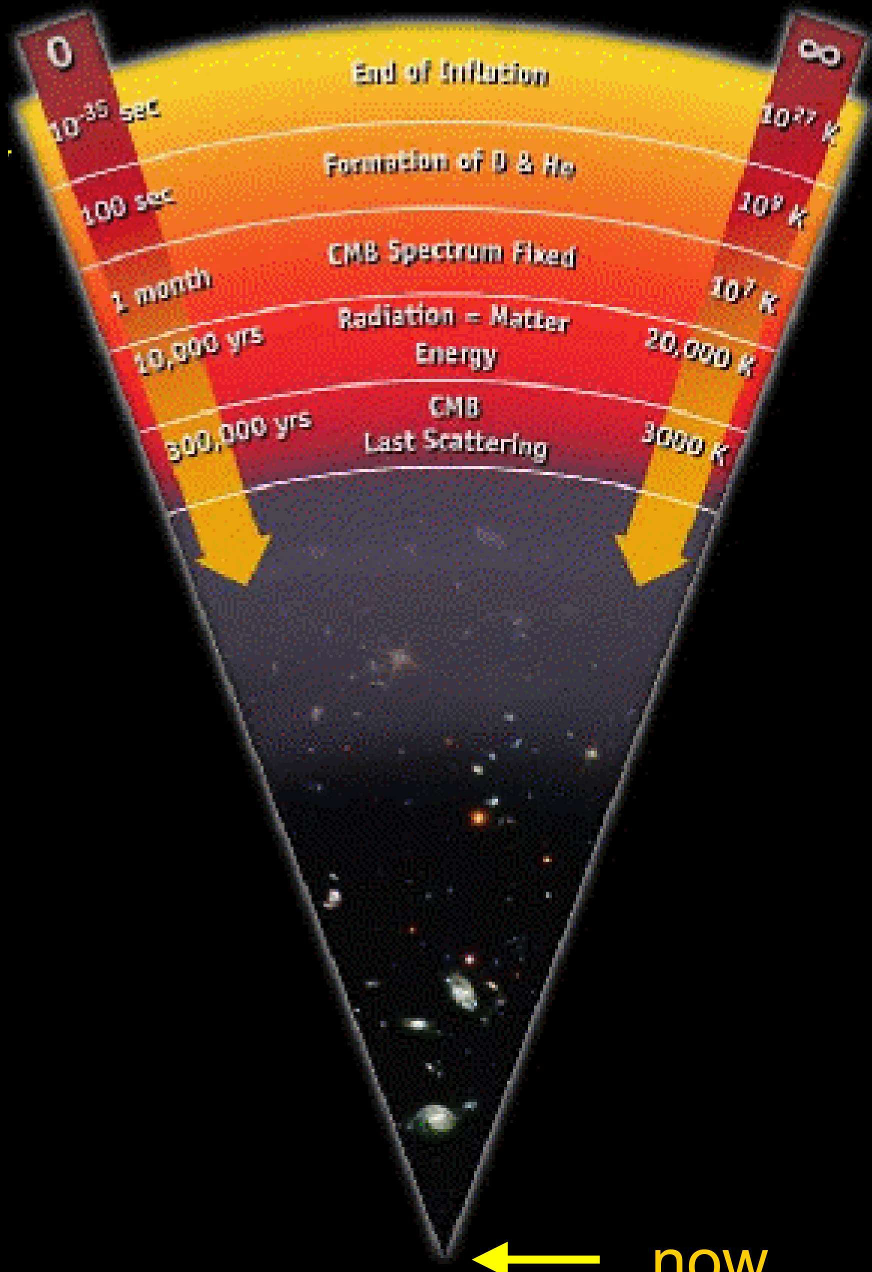


SNAP satellite
2000 SNIa
to z=2

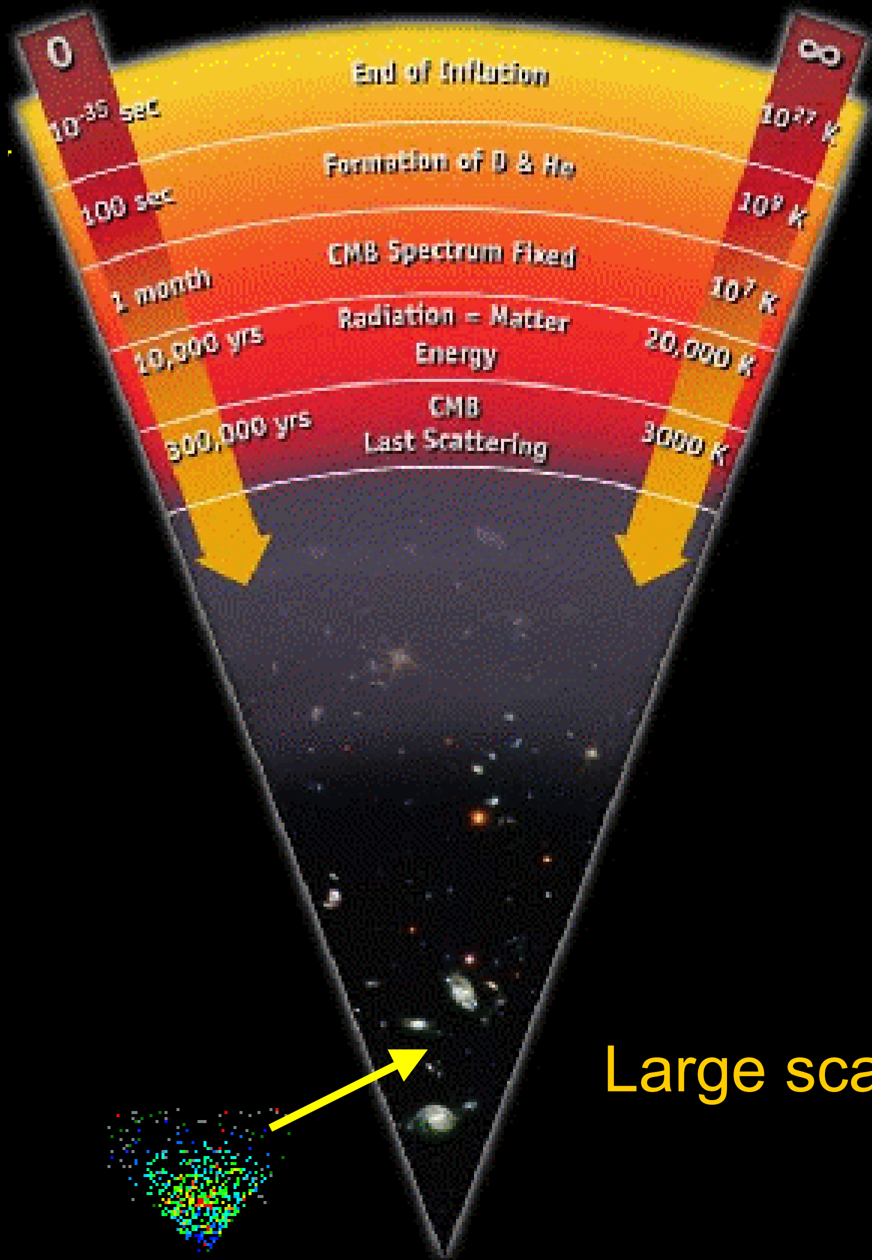
The Aging Universe

If the universe is expanding, necessarily it must have been denser and hotter in the past

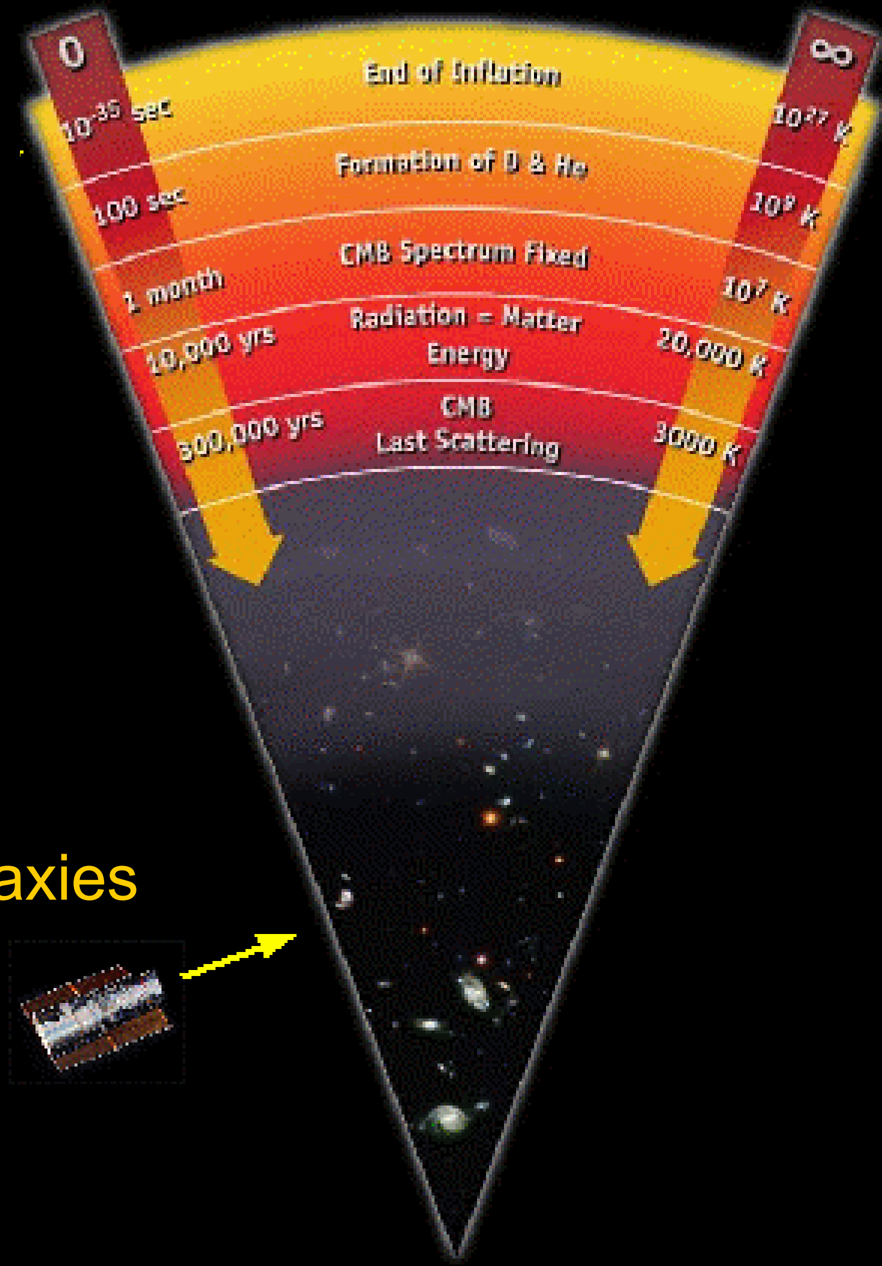
Tracing the history of the universe, we reach the realm of high energy physics and particle accelerators


0
∞
End of Inflation
10^{-35} sec
10^{27} K
Formation of D & He
100 sec
10^{9} K
CMB Spectrum Fixed
1 month
10^{7} K
Radiation = Matter Energy
10,000 yrs
20,000 K
CMB Last Scattering
300,000 yrs
3000 K

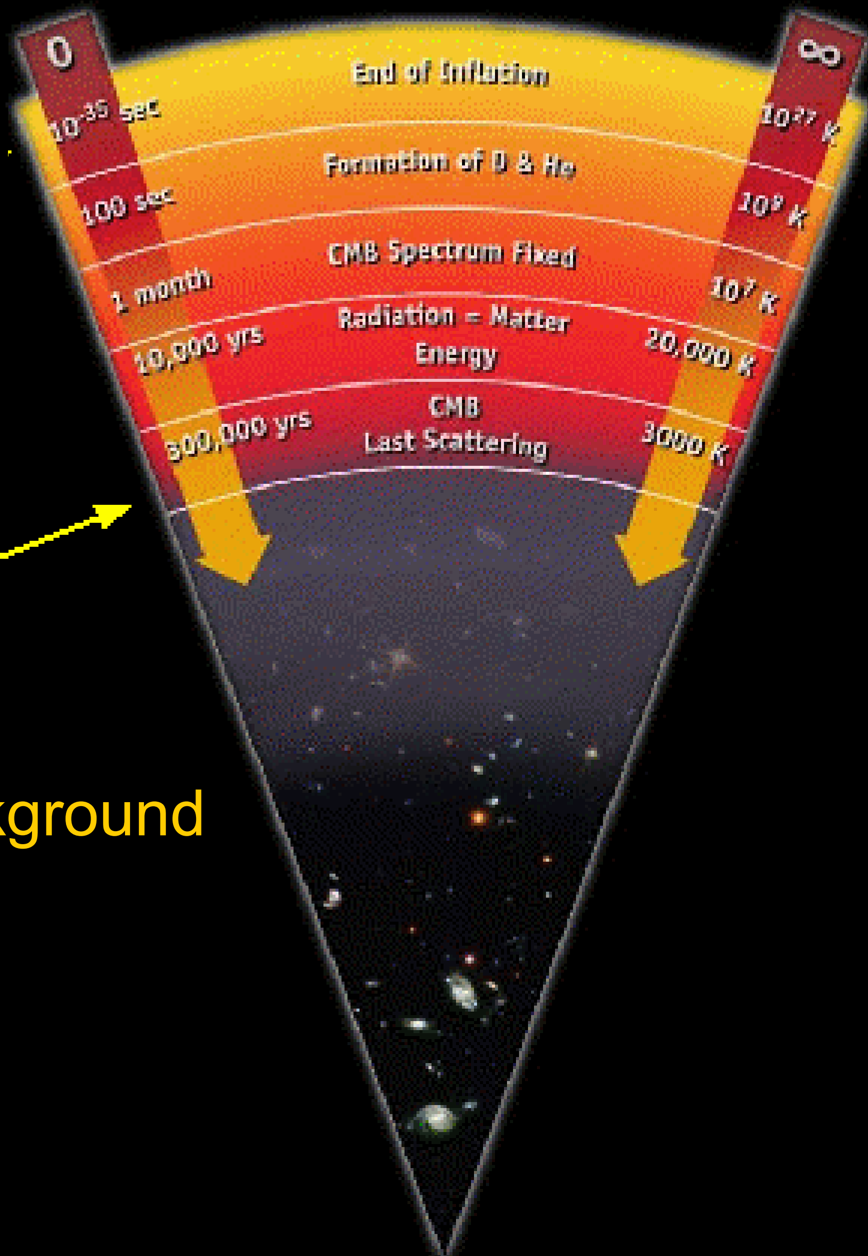
now



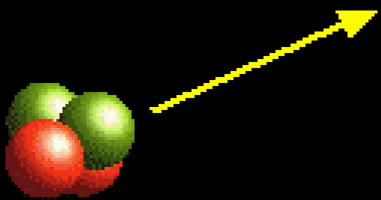
Large scale structure



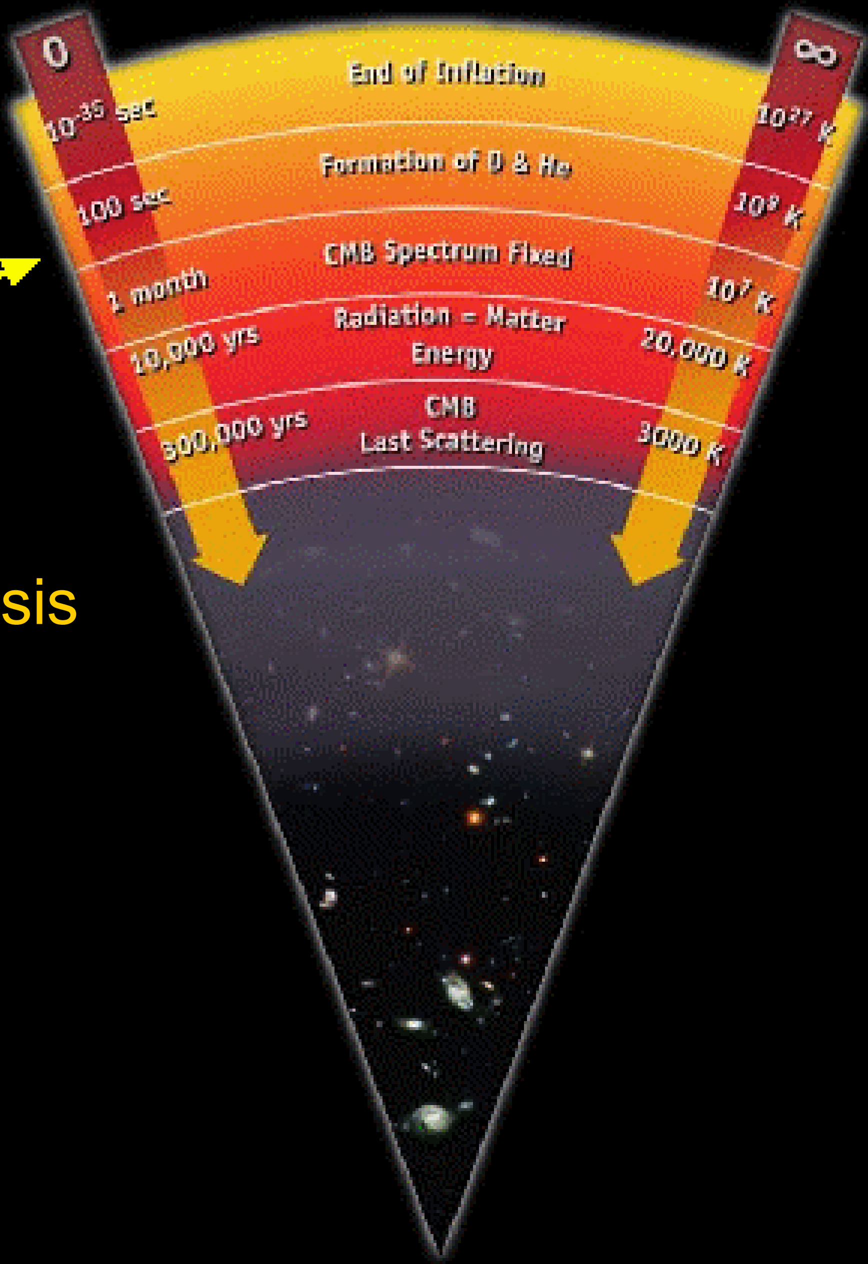
first galaxies

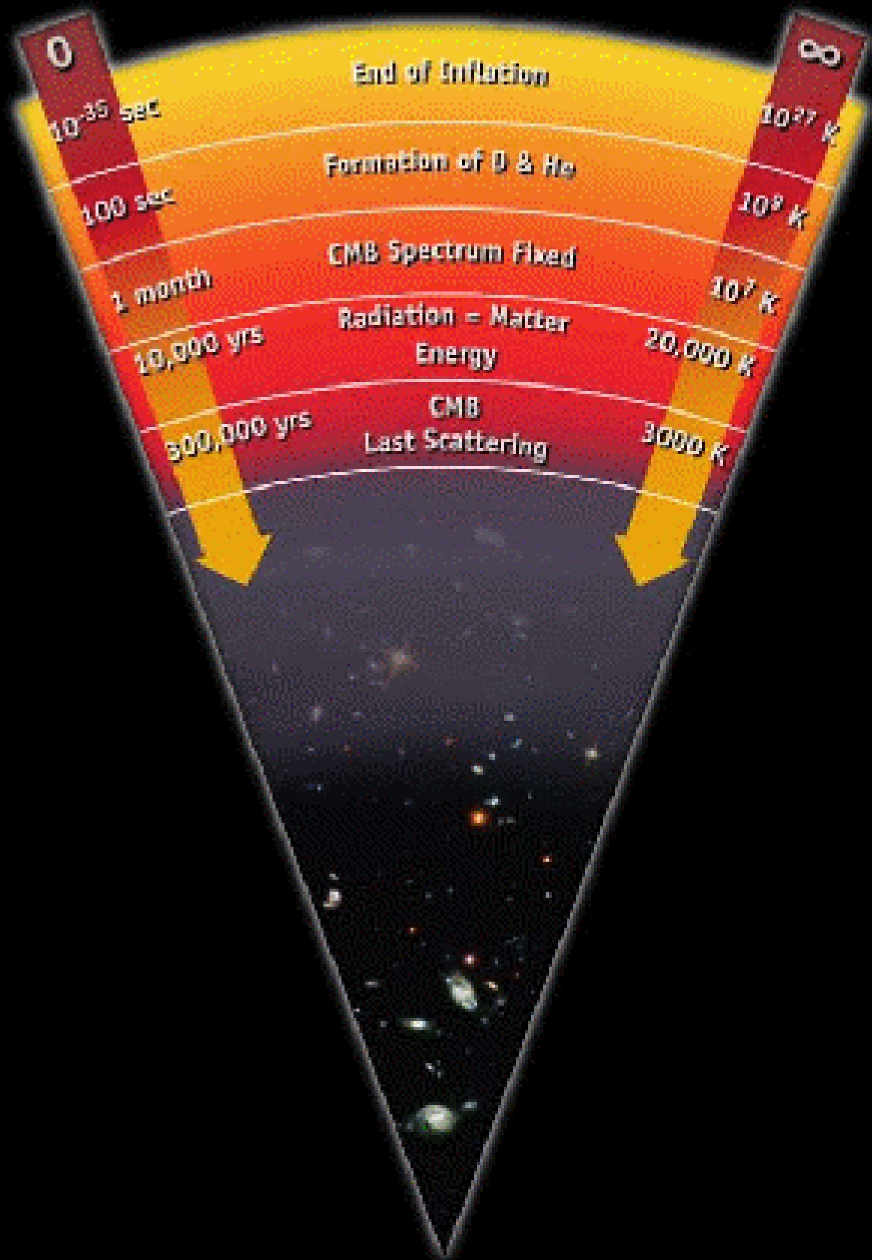


microwave background



primordial nucleosynthesis

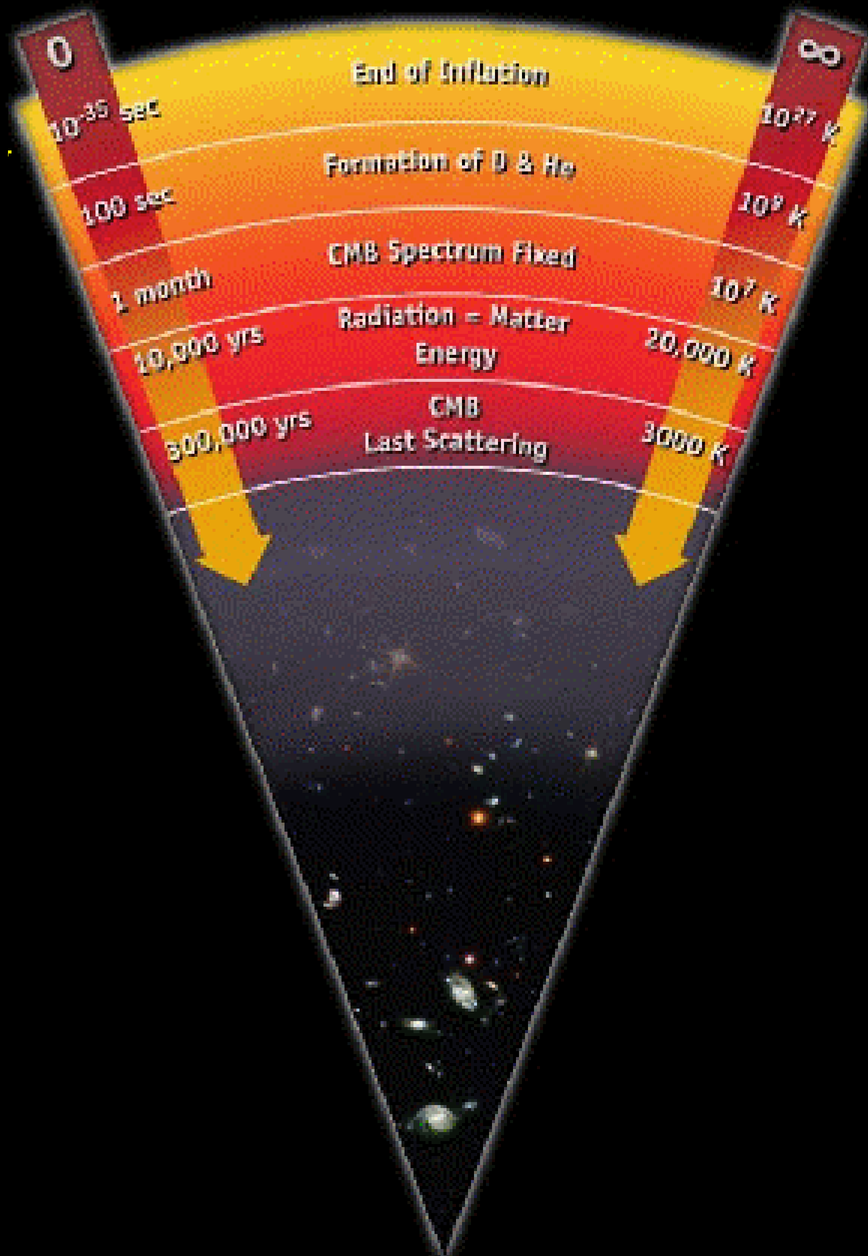


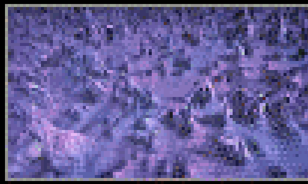


baryogenesis


Fermilab's
Primordial
SOUP


CAUTION:
Contents are extremely dense and are under enormous pressure.
INGREDIENTS
Quarks.....58%
Force Carriers.....29%
Electron-like Particles.....9%
Neutrinos.....5%
Higgs Bosons.....1%
INSPECTED BY U.S. Department of Energy


0
∞
End of Inflation
10^-35 sec
10^27 K
Formation of D & He
100 sec
10^9 K
CMB Spectrum Fixed
1 month
10^7 K
Radiation = Matter
Energy
10,000 yrs
20,000 K
CMB
Last Scattering
300,000 yrs
3000 K



inflation