Neutrinos Theory

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Plan

I. Neutrinos as Dirac Particles

II. Neutrinos from earth and heavens

III. Neutrinos as Majorana Particles

Plan

I. Neutrinos as Dirac Particles

II. Making Neutrinos

III. Neutrinos as Majorana Particles

Nuclear β-decay

4th December 1930

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^{6} nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...

Neutrinos are left-handed

Helicity of Neutrinos*

M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR

Brookhaven National Laboratory, Upton, New York

(Received December 11, 1957)

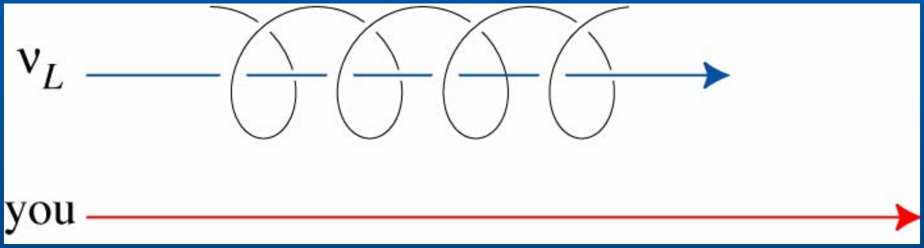
A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] $0-$, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

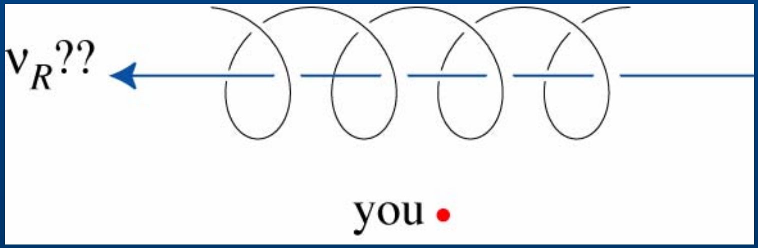
Neutrinos must be massless

Helicity = Chirality (massless)

All neutrinos left handed => massless

If neutrinos are massive





neutrino right-handed => Contradiction!

Standard Model

Exact Lepton number symmetry : terms $m\bar{\nu}^c\nu$ never arise in perturbation theory

Renormalizability : All gauge currents satisfy anomaly cancellation

What about global symmetries : L is broken non perturbatively through the anomaly, but B-L is conserved to all orders : terms $m\bar{\nu}^c\nu$ never arise in the Standard Model

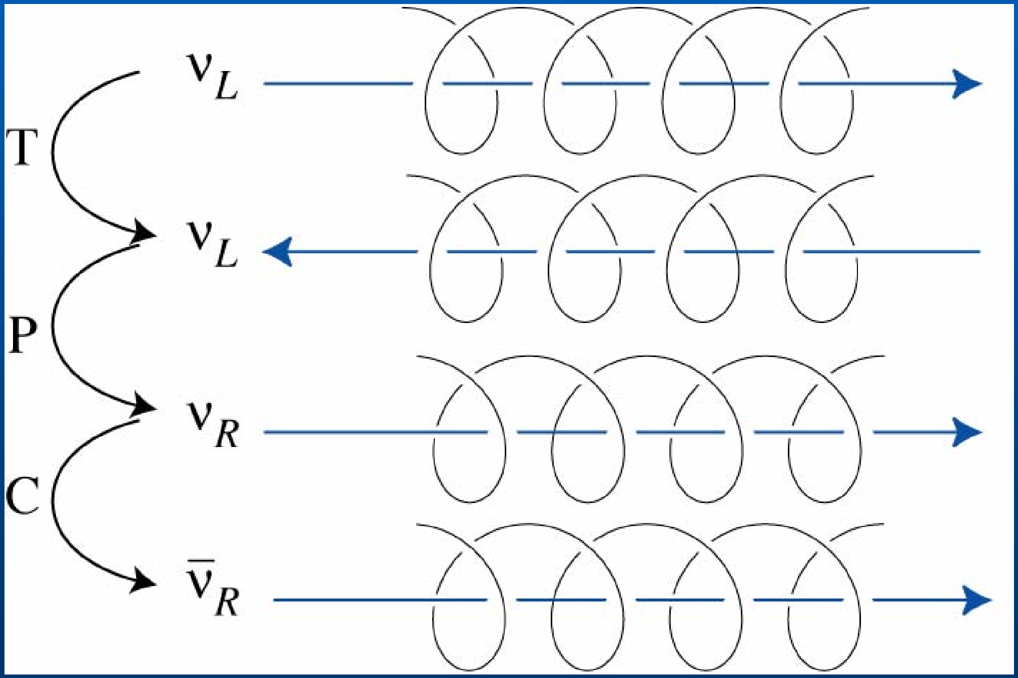
$$\nu^c = C\bar{\nu}^T$$

$\nu, \bar{\nu}^c$ annihilate ν, create $\bar{\nu}$

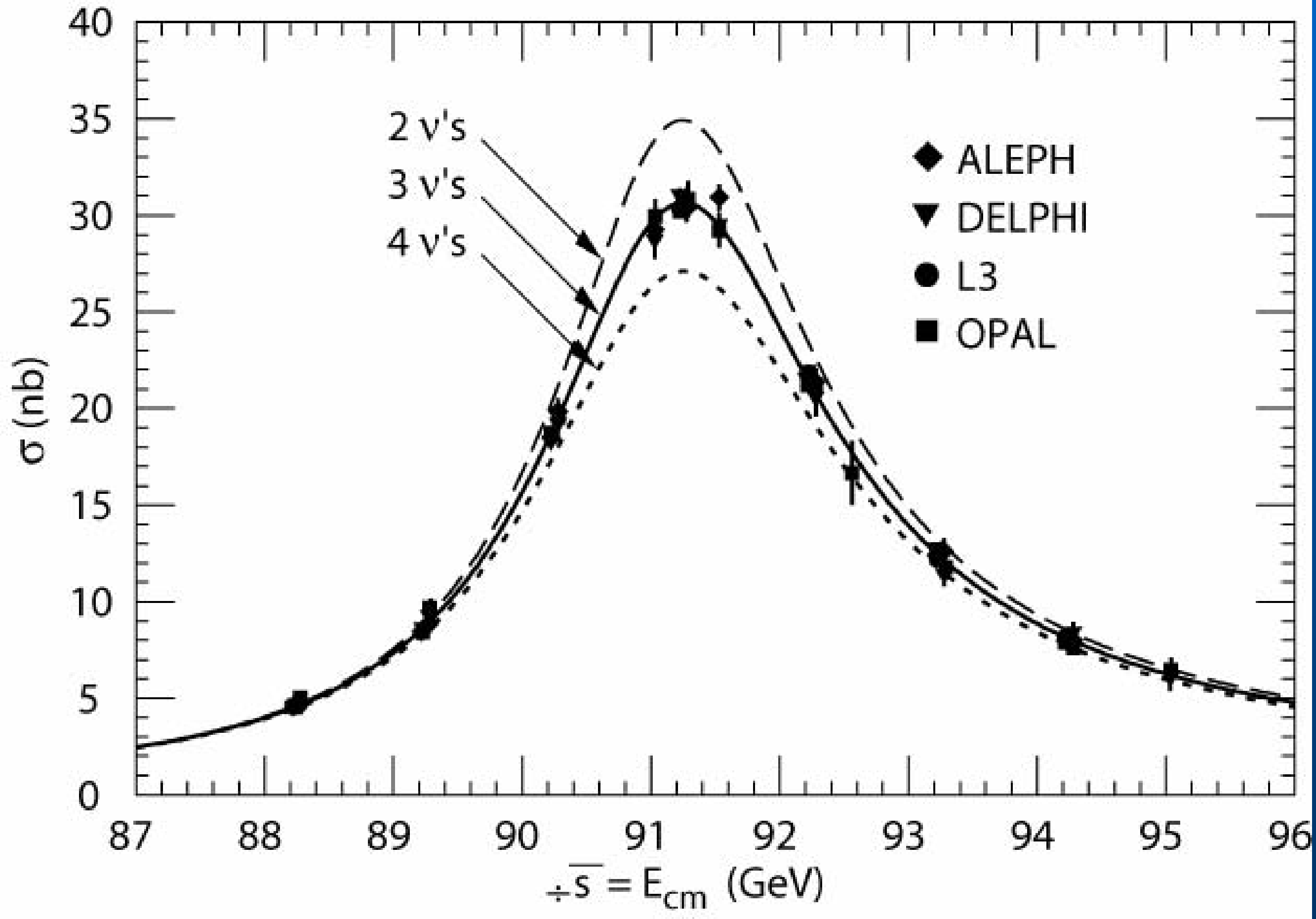
$\bar{\nu}, \nu^c$ create ν, annihilate $\bar{\nu}$

Anti-neutrinos are right-handed

$$\text{CPT}: \ \nu_L \Rightarrow \bar{\nu}_R$$



3 neutrino families (flavors)



Standard Model

$$\mathcal{L}_\nu = i\bar{\nu}_i\gamma^\mu\partial_\mu\nu_i - \frac{g}{2\cos\theta_W}\bar{\nu}_i\gamma^\mu Z^0_\mu\nu_i$$

$$- \frac{g}{\sqrt{2}}\bar{l}_i\gamma^\mu W^-_\mu\nu_i \ + \text{h.c.}$$

$$+ \ldots$$

Global symmetries

$$L_e\left(e^-,\nu_e\right)=+1;\ L_e\left(e^+,\overline{\nu}_e\right)=-1;\ L_e\left(\mu^\pm,\nu_\mu,\ldots\right)=0$$

$$L_\mu\left(\mu^-,\nu_\mu\right)=+1;\ L_\mu\left(\mu^+,\overline{\nu}_\mu\right)=-1;\ L_\mu\left(e^\pm,\nu_e,\ldots\right)=0$$

$$L=L_e+L_\mu+L_\tau$$

The results of charged-current experiments (until recently) were consistent with the existence of three different neutrino species with the separate conservation of L_e, L_μ and L_τ.

Massive Neutrinos : Dirac

$$
\begin{tikzcd}
\nu(\mathrm{p},\mathrm{h}) \arrow[r, leftrightarrow, "CPT"'] \arrow[d, leftrightarrow, "\begin{array}{l}\mathit{Boost,}\\ \mathit{if}\ m \neq 0\end{array}"] & \overline{\nu}(\mathrm{p},-\mathrm{h}) \arrow[d, leftrightarrow, "\begin{array}{l}\mathit{Boost,}\\ \mathit{if}\ m \neq 0\end{array}"'] \\
\nu(\mathrm{p},-\mathrm{h}) \arrow[r, leftrightarrow, "CPT"'] & \overline{\nu}(\mathrm{p},\mathrm{h})
\end{tikzcd}
$$

Standard Model + right handed

$$\mathcal{L}_\nu = \mathrm{i}\,\overline{\nu}_i \gamma^\mu \partial_\mu \nu_i - \frac{g}{2\cos\theta_W}\overline{\nu}_i \gamma^\mu Z^0_\mu \nu_i$$

$$- \frac{g}{2\cos\theta_W}\bar{l}_i \gamma^\mu U_{ij} W^-_\mu \nu_i \; + \mathrm{h.c.}$$

$$+ \mathrm{m_i}\overline{\nu}_i \nu_i + \mathrm{h.c.} + \ldots$$

$$\mathrm{m}\,\overline{\nu}\,\nu = \frac{y\ \mathrm{v}}{\sqrt{2}}(\overline{\nu}_R \nu_L + \overline{\nu}_L \nu_R)$$

$$P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$$

Standard Model + right handed

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$$P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$$

Standard Model + right handed

Masses : Several distinctive features

I am going to concentrate today is neutrino oscillations

Parameters : Dirac

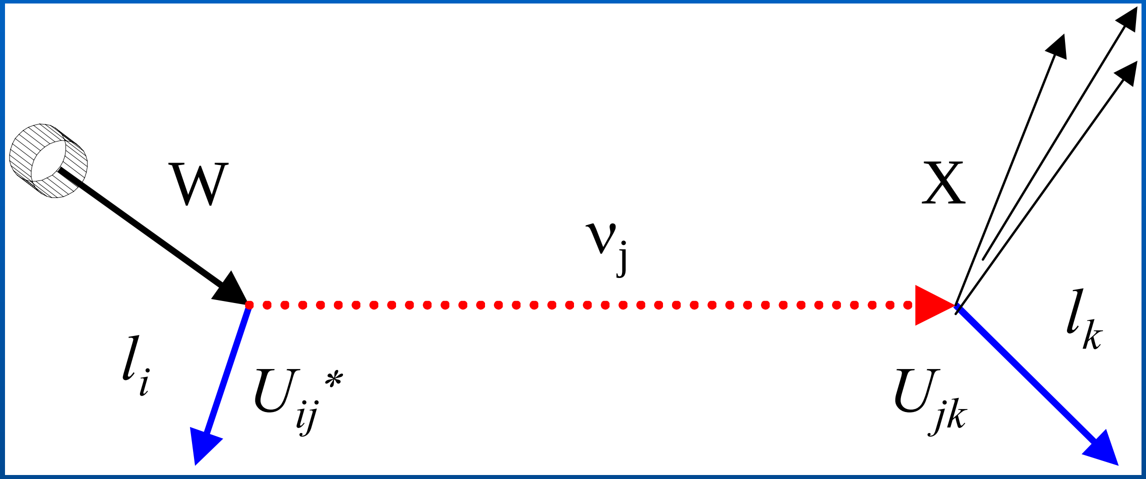
m $_i$

$$U_{\alpha i} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{+i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Exercise

$$m_0 \qquad \Delta m^2 \geq 0 \qquad \Delta M^2 \geq 0 \qquad 0 \leq \theta_{ij} \leq \frac{\pi}{2} \qquad 0 \leq \delta \leq \pi$$

Neutrino Oscillations



$$A\left(l_i \rightarrow \nu_j \rightarrow l_k\right) \propto U_{ij}^{*} U_{jk} e^{-iE_j t}$$

Neutrino Oscillations in Vacuum

$$A\left(l_i \to \nu_j \to l_k\right) \propto U^*_{ij} U_{jk} e^{-iE_j t}$$

Due to different masses, different phase velocities

$$\mathrm{v_i} = \frac{E_i}{p_i} = \sqrt{1 + \frac{m_i^2}{p_i^2}} \cong 1 + \frac{m_i^2}{2E_i^2}$$

$$\Delta \mathrm{v_i} = \frac{\Delta m_i^2}{2E^2}$$

Osc. Length : distance to come back to the initial state

$$\mathrm{l_{osc}} = \frac{4\pi\, E}{\Delta m^2}$$

Production, propagation and detection as a unique process, where neutrinos are virtual particles propagating between production and detection : Neutrinos are described by propagators S $(x_P - x_D)$

Consider finite production and detection regions and finite energy and momentum resolution

If $|x_D - x_P| >> \frac{1}{\Delta p}$

neutrinos can be considered real (on shell) and production, propagation, and detection can be treated separately

Match initial and final conditions

Wave packet formalism

Correct calculation of the phase

Phases calculated in the same space-time point

$$\Delta\phi = \Delta E(t_D - t_P) - \Delta p\,|x_D - x_P|$$

$$\Delta p = \frac{dp}{dE}\Delta E + \frac{dp}{dm^2}\Delta m^2 = \frac{\Delta E}{\mathrm{v}_g} + \frac{\Delta m^2}{2p}$$

$$\Delta\phi \approx \frac{\Delta m^2}{2E}(t_D - t_P) + \Delta E\left[-\frac{|x_D - x_P|}{\mathrm{v}_g} + (t_D - t_P)\right]$$

Second summand is small by either one and/or the other term => standard result

Oscillation effects disappear if neutrino masses are all equal

$$P\left(\nu_\alpha \rightarrow \nu_\beta\right) = |U^*_{\beta i} U_{\alpha i} e^{-iE_i t}|^2$$

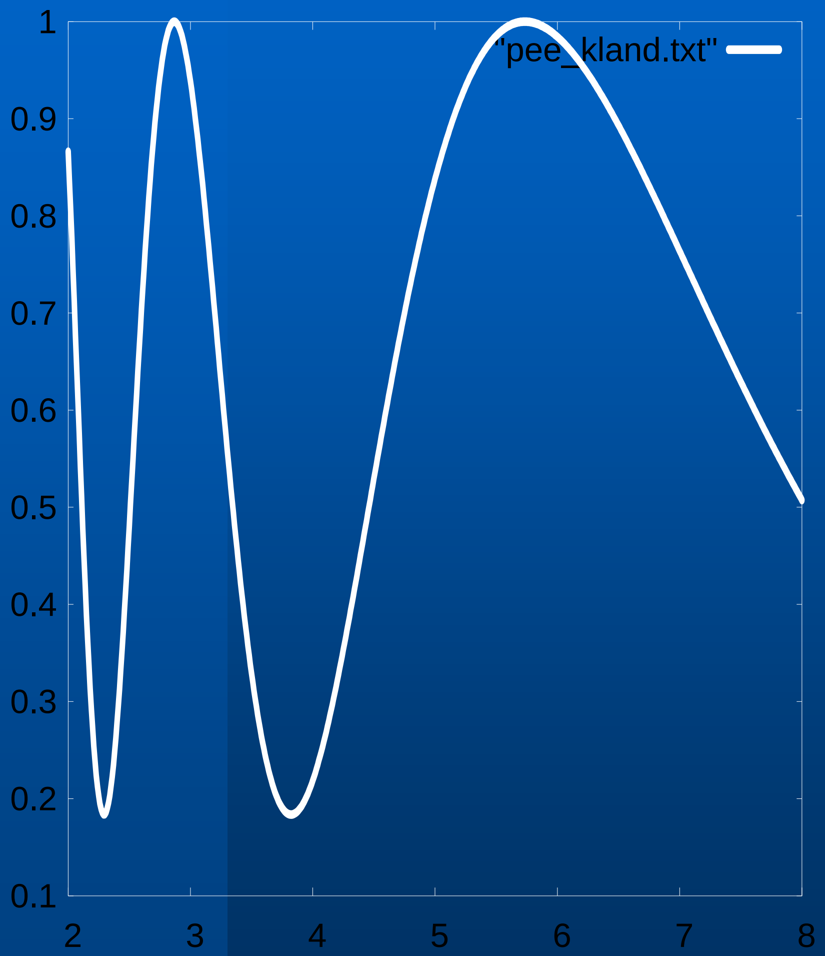
$$= \sum U^*_{\beta i} U_{\alpha i} U^*_{\alpha j} U_{\beta j} e^{-i\frac{\Delta m^2_{ji}}{2E}t}$$

$$U_{\alpha i} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{+i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Exercise : Write P_{ee}, $P_{e\mu}$

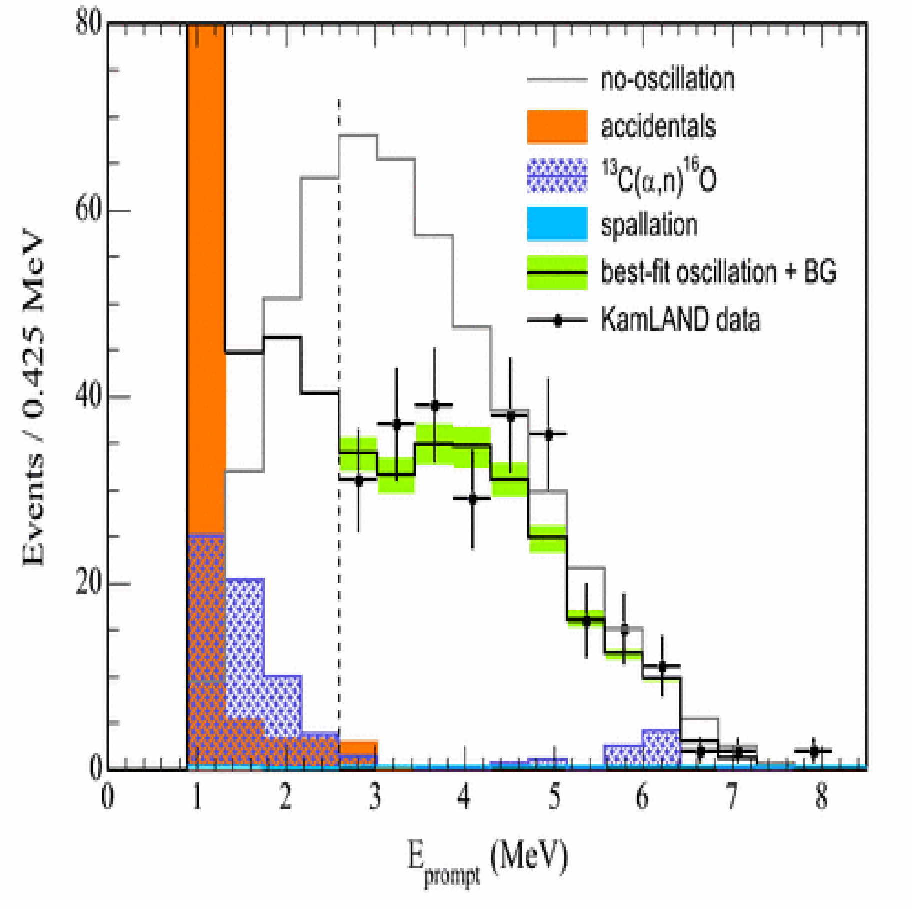
Exercise : Minimal number of $P_{\alpha\beta}$ to know all the others

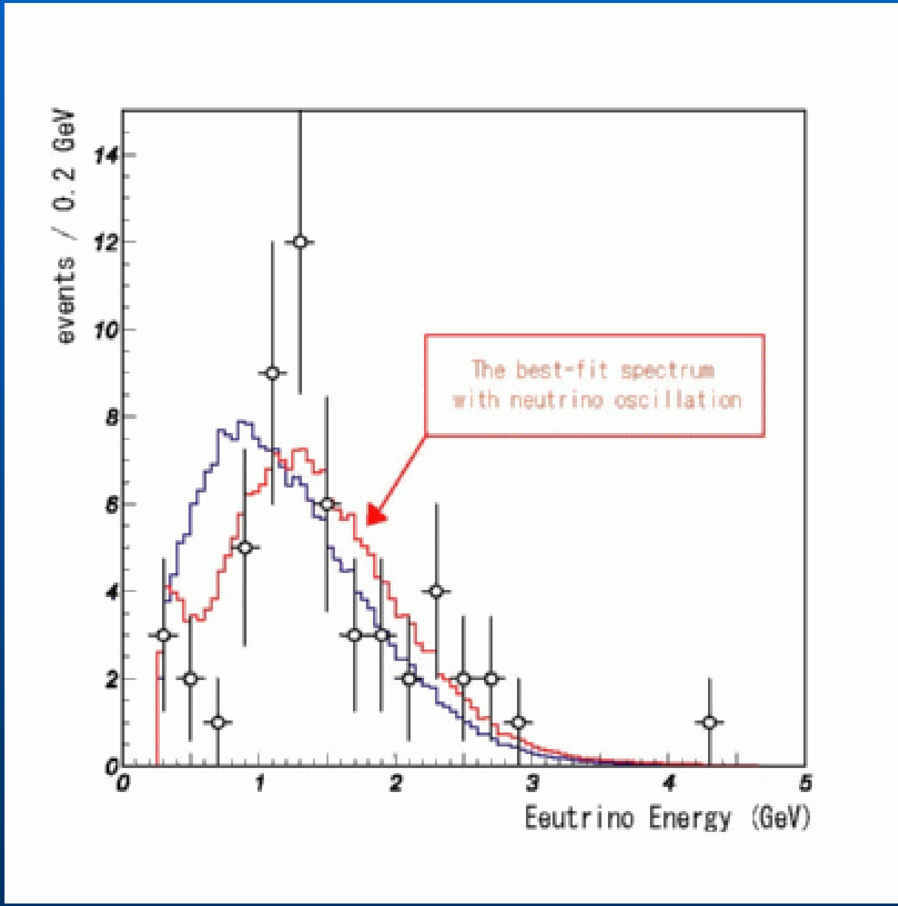
One example :



$$P^{2f} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Some real cases :





Neutrino Oscillations in matter

After a plane wave pass through a slab, the phase is shifted : p (x+(n-1)R)

$$e^{ip(x+(n-1)R)} \approx e^{ipx} + 2\pi\, f(0)\, NR \int_{x-R}^{\infty} dr\, e^{ipx}$$

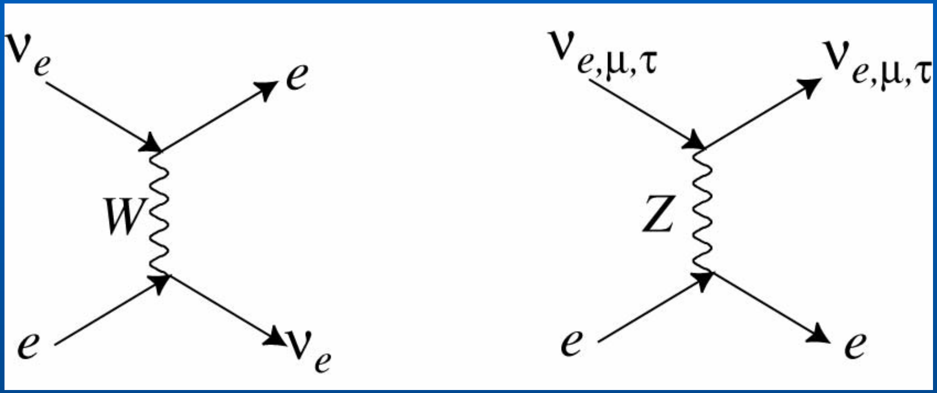
$$= e^{ipx}\left[1 + 1\frac{2\pi\, f(0) NR}{p}\right]$$

Net effect :

$$n - 1 \approx \frac{2\pi\, N\, f(0)}{p^2}$$

Only the difference of potential is relevant

Net effect :



$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \bar{\nu}\gamma^{\mu}(1-\gamma_5)\nu \;\; \bar{e}\gamma^{\mu}(1-\gamma_5)e \Rightarrow \sqrt{2}G_F N_e$$

$$\mathrm{l}_{\text{matt}} = \frac{2\pi}{\sqrt{2}G_F N_e}$$

$$< e\gamma^0 e >= N_e$$

$$< e\gamma^i e >= N_e \mathrm{v}_{\mathrm{i}}$$

Two Neutrino Oscillations in matter

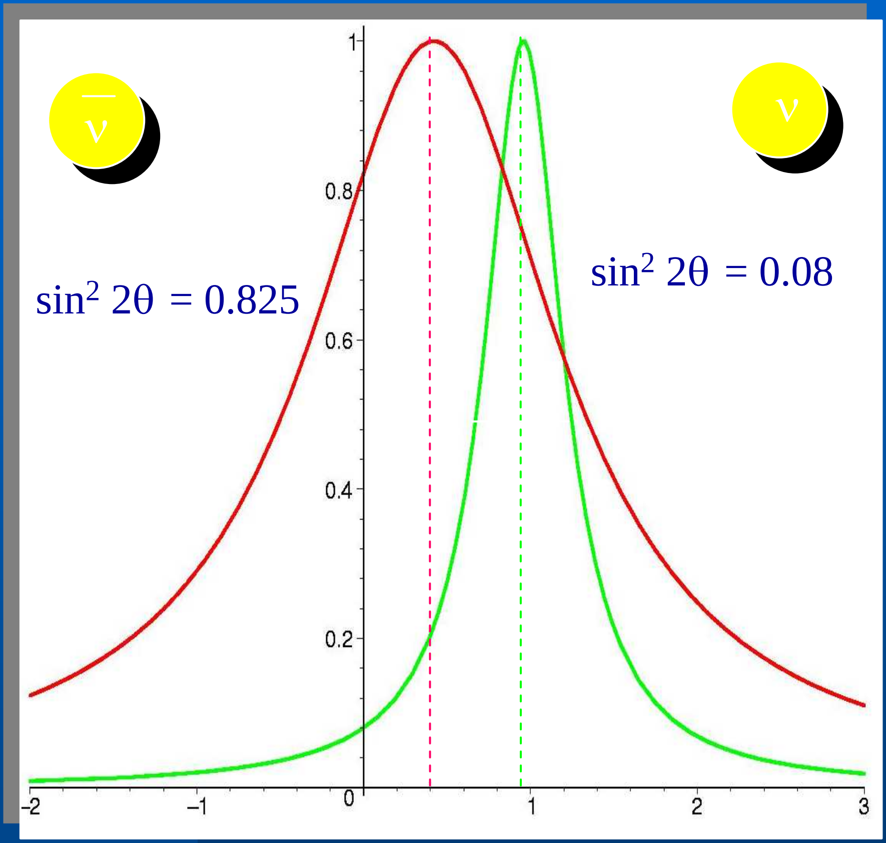
$$i\,\frac{d}{dt}\begin{pmatrix}\nu_e\\ \\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\dfrac{\Delta m^2}{2E}\cos 2\theta + V_e & \dfrac{\Delta m^2}{4E}\sin 2\theta \\ \\ \dfrac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}\begin{pmatrix}\nu_e\\ \\ \nu_\mu\end{pmatrix}$$

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2\sqrt{2}G_F N_e E/\Delta m^2)^2 + \sin^2 2\theta}$$

Difference of the eigenvalues

$$H_2 - H_1 = \frac{\Delta m^2}{2E}\sqrt{(\cos 2\theta - 2\sqrt{2}G_F n_e E/\Delta m^2)^2 + \sin^2 2\theta}$$

Resonance



l_ν / l_0 $\sim n E$

In resonance:

$$\sin^2 2\theta_m = 1$$

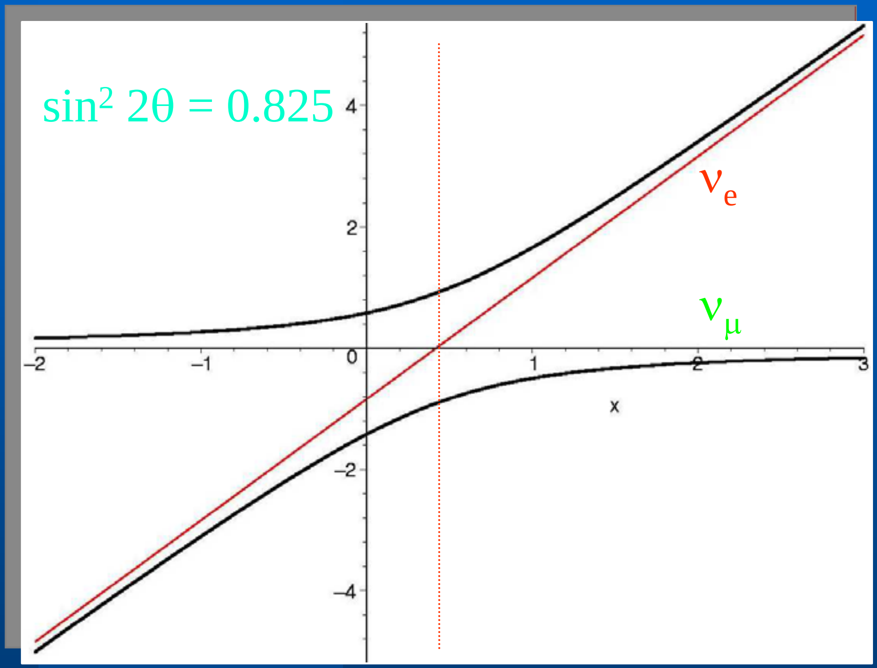
Flavor mixing is maximal
Level split is minimal

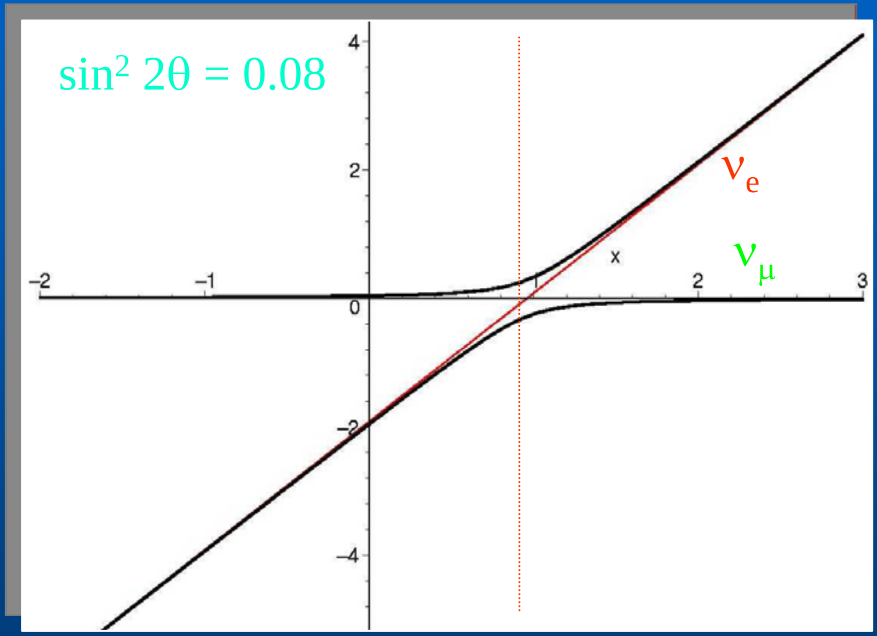
$$l_\nu = l_0 \cos 2\theta$$

Vacuum oscillation length $\approx$ Refraction length

- Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$
- Resonance layer: $n = n_R + \Delta n_R$

Level crossing





$$\nu\,(t) = \cos\theta_a\,\nu_{1m} + \sin\theta_a\,\nu_{2m}\,e^{-i\phi(t)}$$

Graphical interpretation

$\vec{\nu} = (\mathrm{Re}\ \nu_e^{+}\nu_\mu,\ \ \mathrm{Im}\ \nu_e^{+}\nu_\mu,\ \ \nu_e^{+}\nu_e - 1/2)$

elements of density matrix

$\vec{B} = \frac{2\pi}{l_m}(\sin 2\theta_m,\ 0,\ \cos 2\theta_m)$

$l_m = 2\pi/\Delta H$ oscillation length

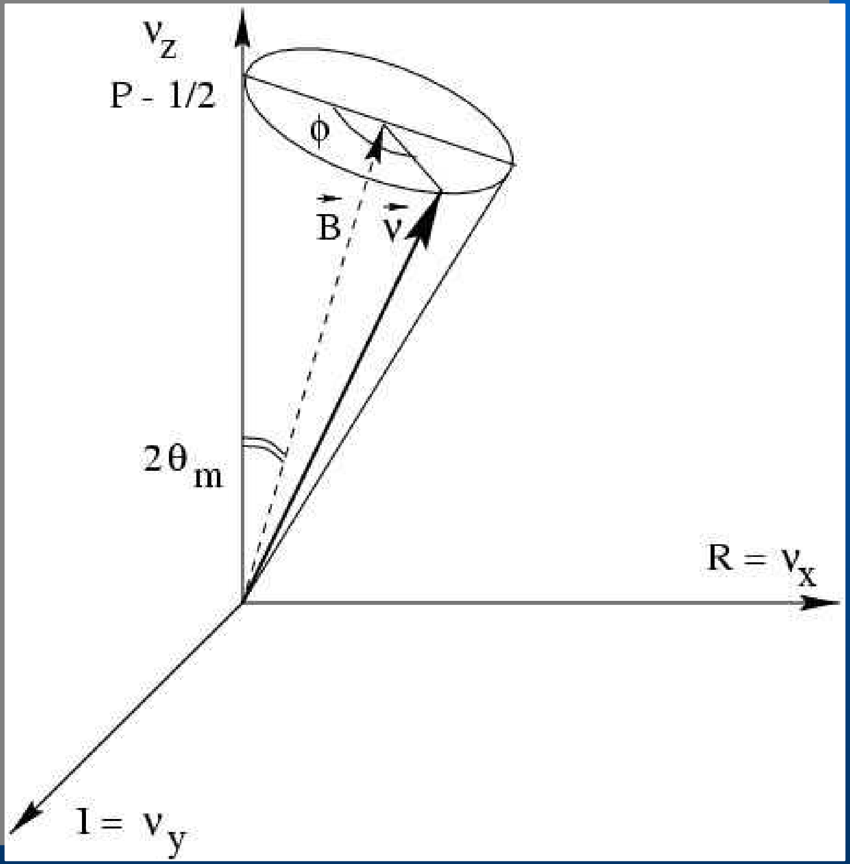
Evolution equation

$$\frac{d\vec{\nu}}{dt} = (\vec{B} \times \vec{\nu})$$

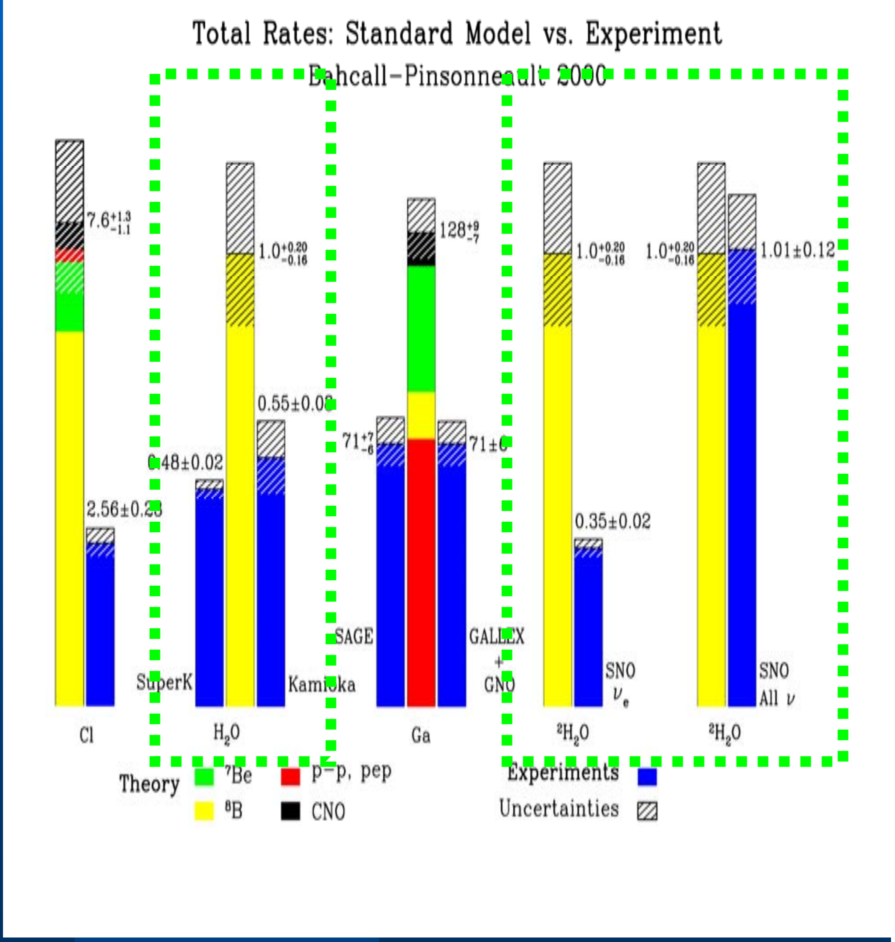
Coincides with equation for the electron spin precession in the magnetic field

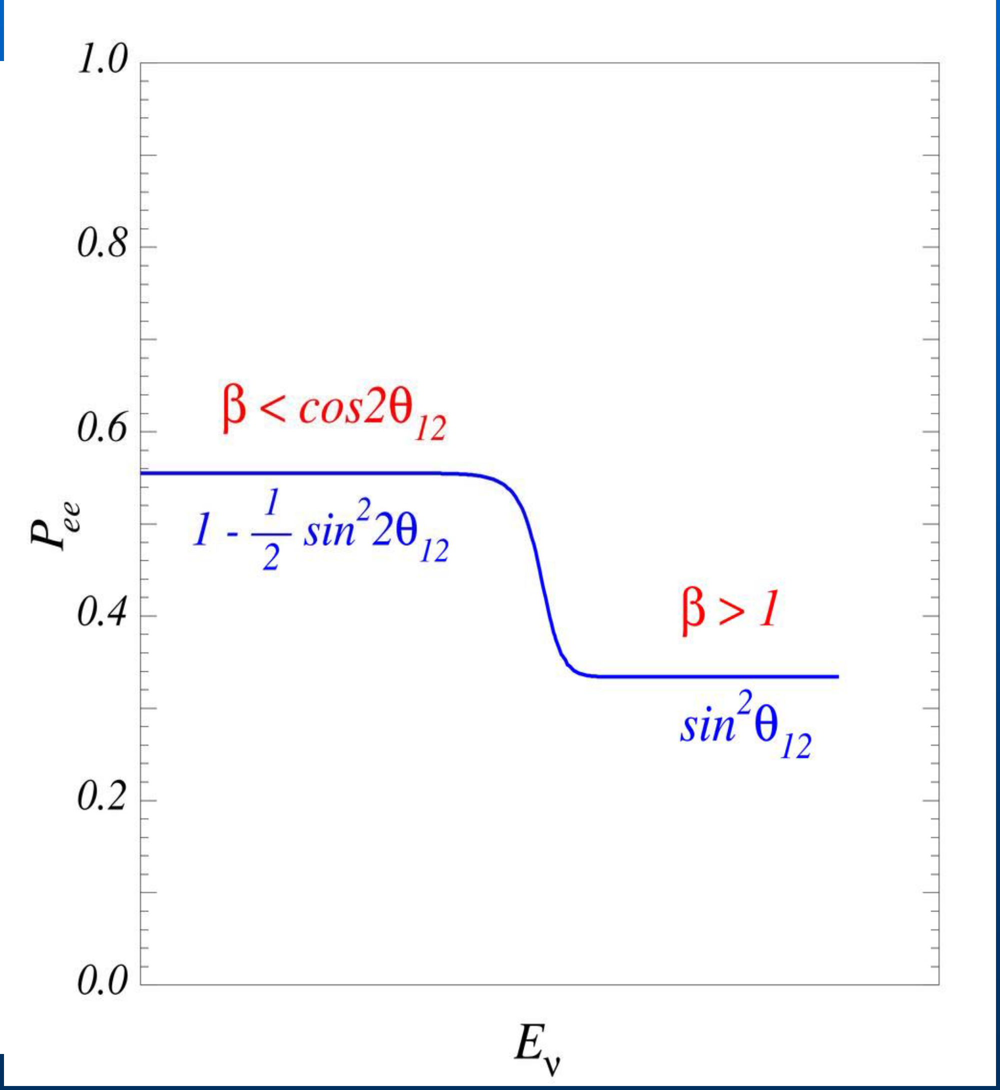
$\phi = 2\pi t/ l_m$ - phase of oscillations

$P = \nu_e^{+}\nu_e = \nu_Z + 1/2 = \cos^2\theta_Z/2$



A real case: solar neutrinos





The Neutrino Matrix :SM + ν mass

ALL oscillation neutrino data BUT LSND

3 σ ranges:

$$|U_{\alpha i}| = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & 0.00-0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}$$

$$7.3 \leq \frac{\Delta \mathrm{m}^2}{10^{-5}\,\mathrm{eV}^2} \leq 9.3\,(8.2)$$

$$1.6 \leq \frac{\Delta \mathrm{M}^2}{10^{-3}\,\mathrm{eV}^2} \leq 3.6\,(2.2)$$

$$0.28 \leq \tan^2\theta_{12} \leq 0.60 \;\; (0.39)$$

$$0.51 \leq \tan^2\theta_{23} \leq 2.1 \;\; (1.0)$$

$$\sin^2\theta_{13} \leq 0.041 \;\; (0.005)$$