Neutrinos Theory

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References

Neutrino Unbound

http://www.nu.to.infn.it/

Massive Neutrinos : Dirac

$$\nu(p,h) \xleftrightarrow{CPT} \bar{\nu}(p,-h)$$

$\updownarrow$ *Boost, if $m \neq 0$* $\qquad$ *Boost, if $m \neq 0$* $\updownarrow$

$$\nu(p,-h) \xleftrightarrow{CPT} \bar{\nu}(p,h)$$

Standard Model + right handed

$$\mathcal{L}_\nu = \mathrm{i}\bar{\nu}_i\gamma^\mu\partial_\mu\nu_i - \frac{g}{2\cos\theta_W}\bar{\nu}_i\gamma^\mu Z^0_\mu\nu_i$$

$$- \frac{g}{2\cos\theta_W}\bar{l}_i\gamma^\mu U_{ij}W^-_\mu\nu_i + \mathrm{h.c.}$$

$$+ \mathrm{m_i}\bar{\nu}_i\nu_i + \mathrm{h.c.} + \ldots$$

$$\mathrm{m}\,\bar{\nu}\nu = \frac{y\,\mathrm{v}}{\sqrt{2}}(\bar{\nu}_R\nu_L + \bar{\nu}_L\nu_R)$$

$$P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$$

Standard Model + right handed

$$\mathcal{L}_\nu = i\bar{\nu}_i\gamma^\mu\partial_\mu\nu_i - \frac{g}{2\cos\theta_W}\bar{\nu}_i\gamma^\mu Z^0_\mu\nu_i$$

$$-\frac{g}{2\cos\theta_W}\bar{l}_i\gamma^\mu U_{ij} W^-_\mu\nu_i + \text{h.c.}$$

$$+ m_i\bar{\nu}_i\nu_i + \text{h.c.} + \ldots$$

$$m_i\bar{\nu}_i\nu_i = \frac{y\,v}{\sqrt{2}}(\bar{\nu}_R\nu_L + \bar{\nu}_L\nu_R)$$

$$P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$$

Parameters : Dirac

m $_i$

$$U_{\alpha i}=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\cdot\begin{pmatrix} c_{13} & 0 & s_{13}e^{+i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix}\cdot\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Exercise

$$m_0 \qquad \Delta m^2 \geq 0 \qquad \Delta M^2 \geq 0 \qquad 0 \leq \theta_{ij} \leq \frac{\pi}{2} \qquad 0 \leq \delta \leq \pi$$

Wave packets : Neutrino osc.

$$\psi_i(x,t) = \int \frac{dp}{2\pi\sigma_p} e^{-\frac{(p-<p_i>)^2}{4\sigma_p^2}} e^{-i(px-E_it)}$$

Relativistic neutrinos :

$$P(\nu_\alpha \to \nu_\beta) \sim \sum U^*_{\beta i} U_{\alpha i} U^*_{\alpha j} U_{\beta j} e^{-i\frac{\Delta m^2_{ji}}{2E}L} e^{-\frac{L^2}{32\sigma_x^2}\frac{(\Delta m^2_{ji})^2}{E^2}}$$

$$\text{If } l_{osc} = \frac{4\pi E}{\Delta m^2} < l_{coh} = 4\sqrt{2}\sigma_x \frac{E^2}{\Delta m^2} \Rightarrow \text{Osc.}$$

$$\left.\begin{cases} \text{if } l_{osc} << L \Rightarrow \text{average oscillations} \\ \text{if } l_{coh} << L \Rightarrow \text{Incoherent propagation} \end{cases}\right\} \text{Same result}$$

Size of the wave packet

Coherent neutrino emission in plasma :

Interrupted by e.m interactions of the source particles (pressure broadening)

$$\sigma_x \sim \frac{1}{\mathrm{v}} \sim \frac{\frac{1}{\pi N}\left(\frac{3T}{2Z_1^2 Z_2^2 e^2}\right)^2}{\sqrt{\frac{3T}{m}}} \sim 1.4 \cdot 10^{17} \frac{\sqrt{m}\left(\frac{T}{MeV}\right)^{3/2}}{Z_1^2 Z_2^2\left(\frac{N}{cm^{-3}}\right)} \mathrm{Km} \approx 10^{-12} - 10^{-11}\, Km\ (\mathrm{Sun})$$

Reactor : e.m. interactions of produced e^-

$$\approx 10^{-9}\ \mathrm{Km}\ (\mathrm{Reactor})$$

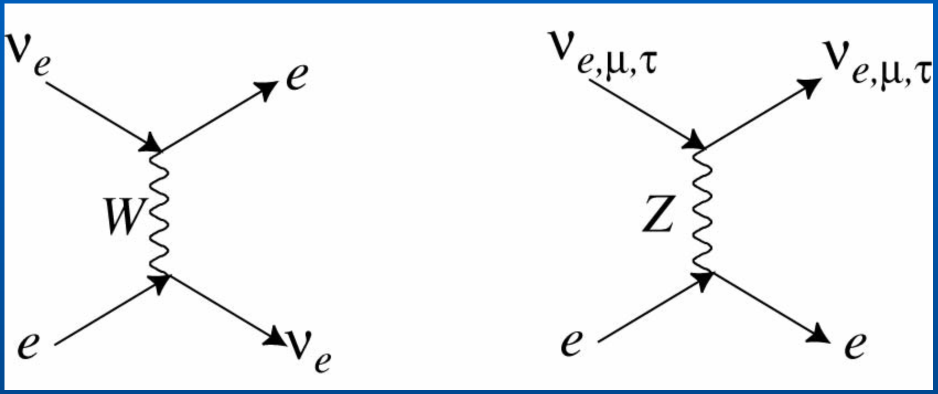
Accelerator : time scale of weak interactions

$$\sim c\tau \approx 10^{-3}\ \mathrm{Km}\ (\mathrm{Accelerator})$$

Matter effects

Only the difference of potential is relevant

Net effect :



$$H_{\text{int}} = \frac{G_F}{\sqrt{2}} \bar{\nu}_e \gamma^\mu (1-\gamma_5) \nu_e \int d^3 p_e \, \mathrm{f}(\mathrm{E_e}, \mathrm{T}) \, \bar{e} \gamma^\mu (1-\gamma_5) e \Rightarrow \sqrt{2} G_F N_e$$

$$\mathrm{l}_{\text{matt}} = \frac{2\pi}{\sqrt{2} G_F N_e}$$

$$< e\gamma^0 e >= N_e$$

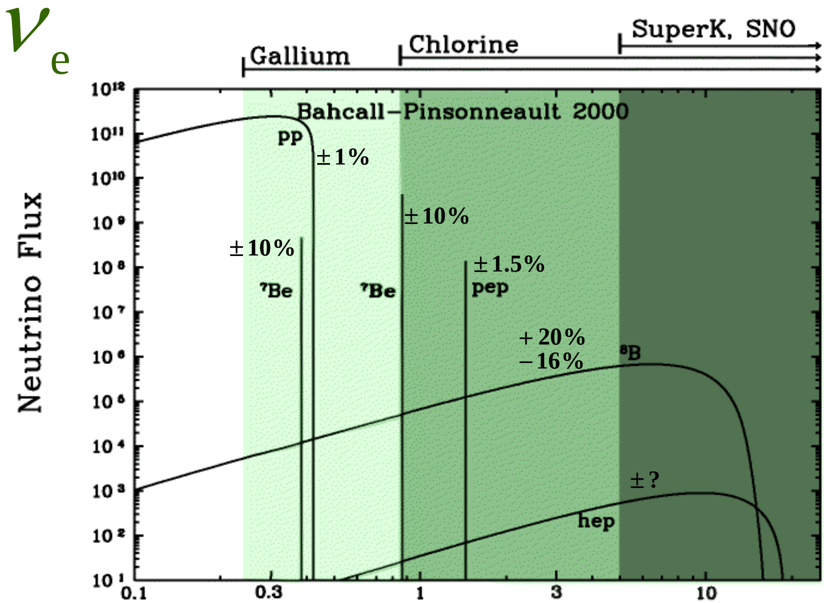
$$< e\gamma^i e >= N_e \mathrm{v}_\mathrm{i}$$

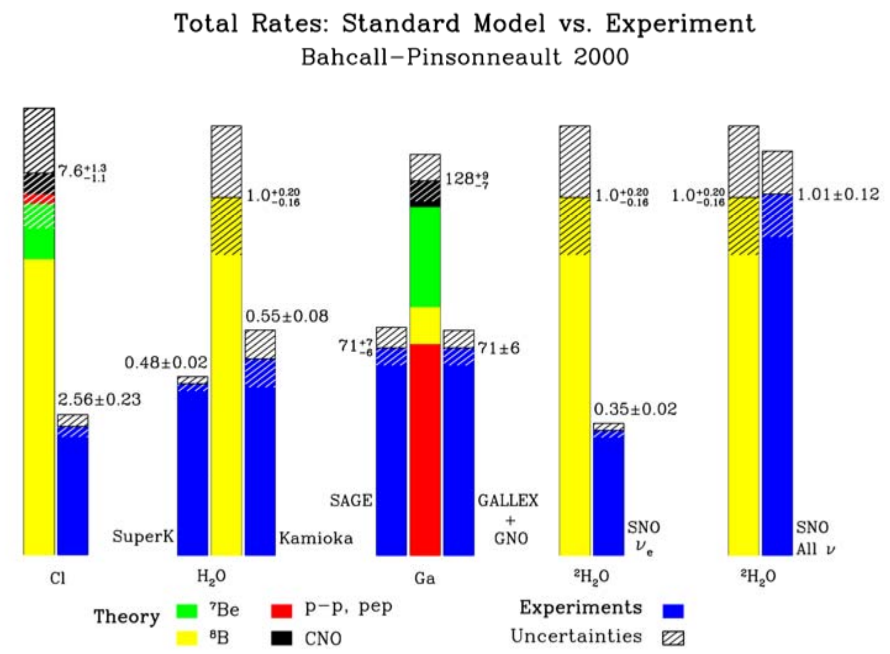
Some Neutrino Sources

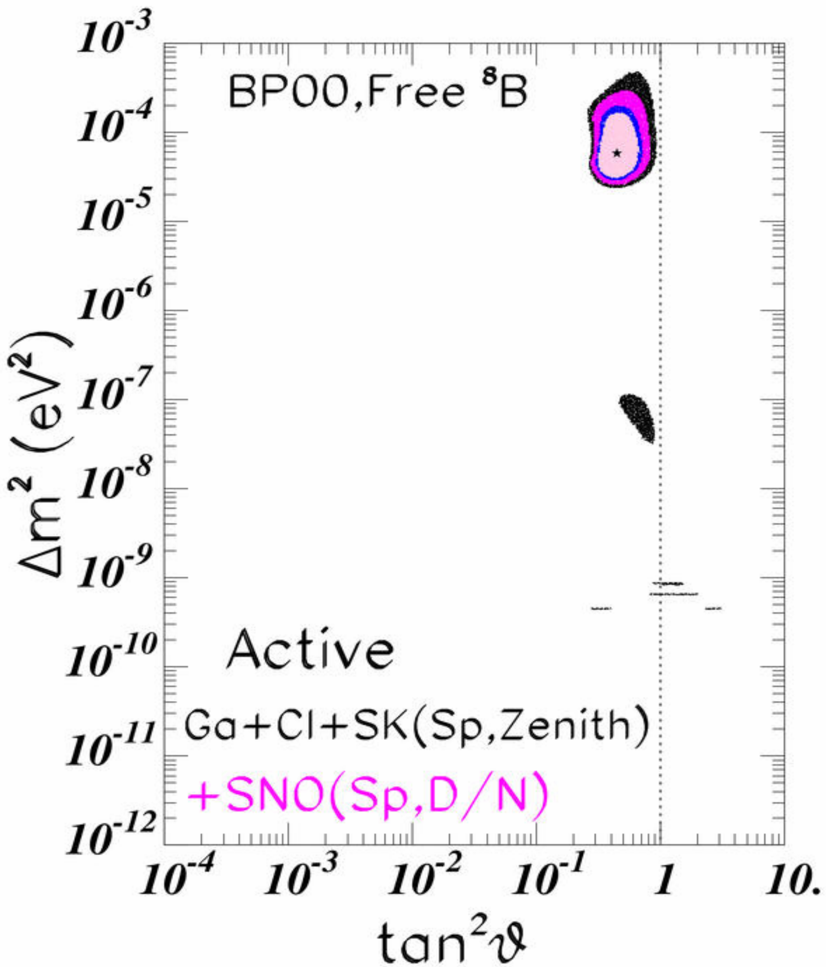
	$l_{osc} = \frac{4\pi E}{\Delta m^2}$	$l_{coh} = 4\sqrt{2}\sigma_x \frac{E^2}{\Delta m^2}$	$l^e_{matt} = \frac{2\pi}{\sqrt{2} G_F N_e}$
Solar	$10 - 10^3$ Km	$10^5 - 10^7$ Km	10^2 Km
Snova	$10^{-11} - 10^{-7}$ Km	$10^{-8} - 10^{-4}$ Km	10^{-10} Km
React	$1, 10^2$ Km	$10^6, 10^8$ Km	10^4 Km
Atmos	$10 - 10^5$ Km	$10^{14} - 10^{22}$ Km	$10^3 - 10^4$ Km
Accel	$10^2 - 10^3$ Km	$10^{16} - 10^{18}$ Km	10^4 Km

Solar

$$P(\nu_e \rightarrow \nu_a) = P(\Delta m^2_{sol}, \theta_{sol})$$

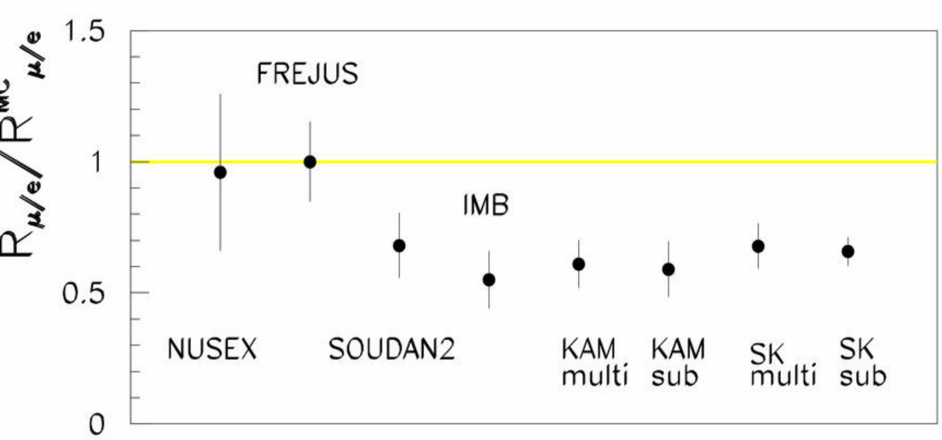


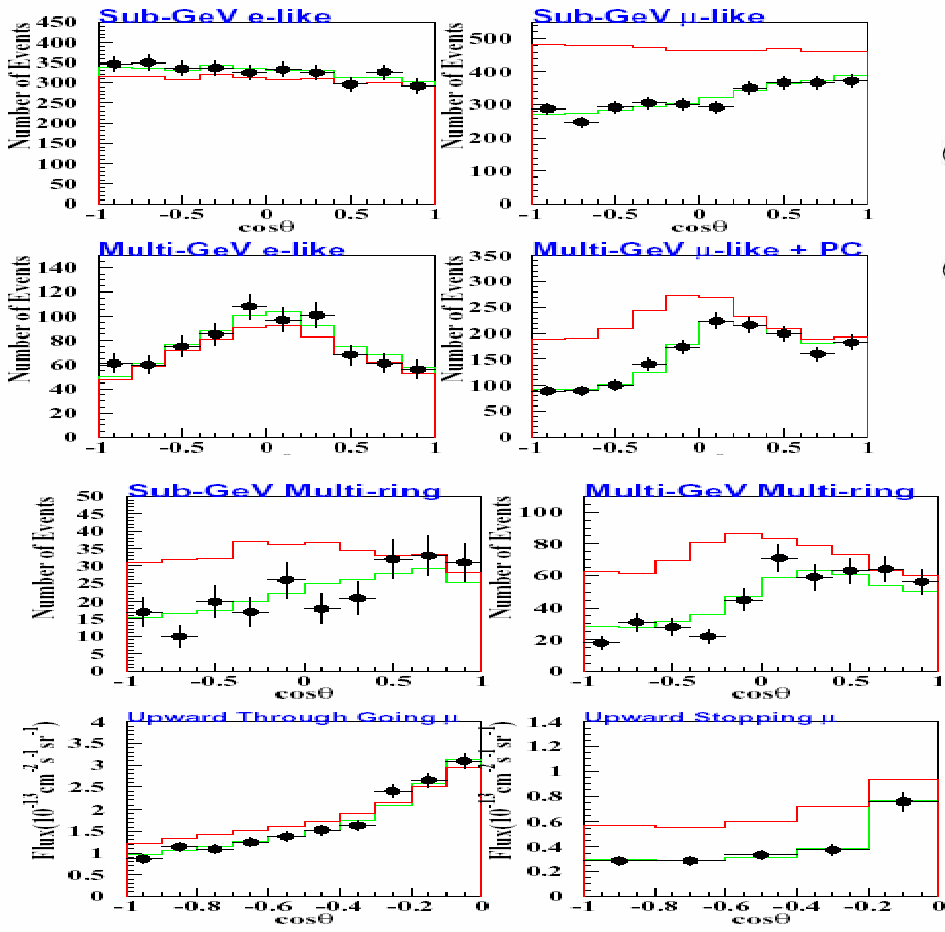


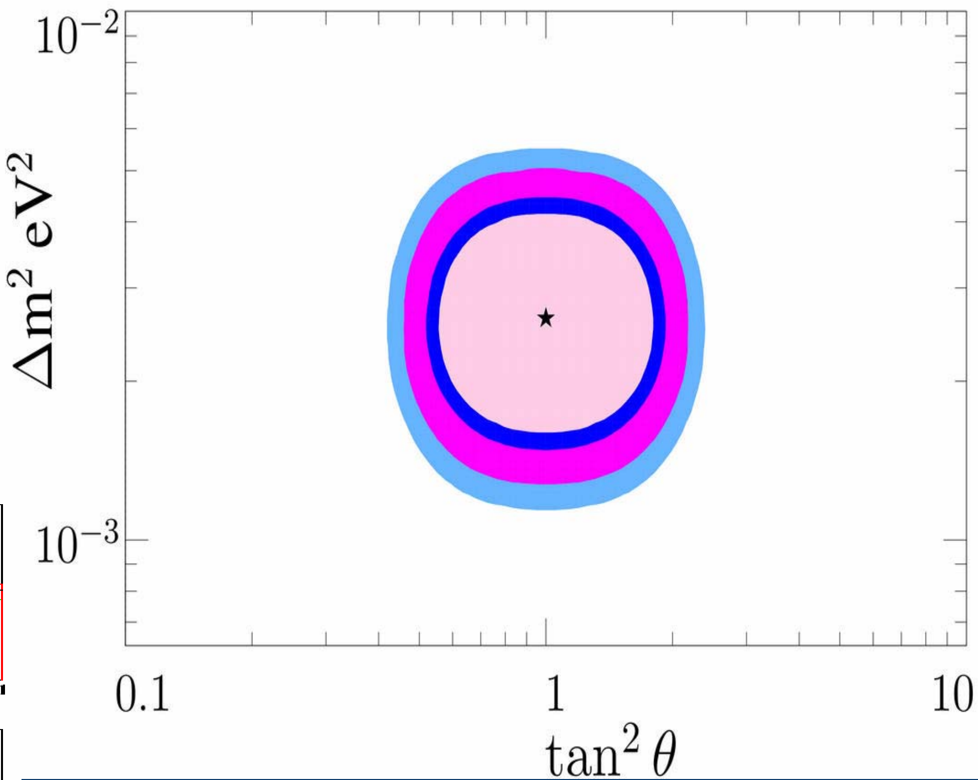


Atmospheric

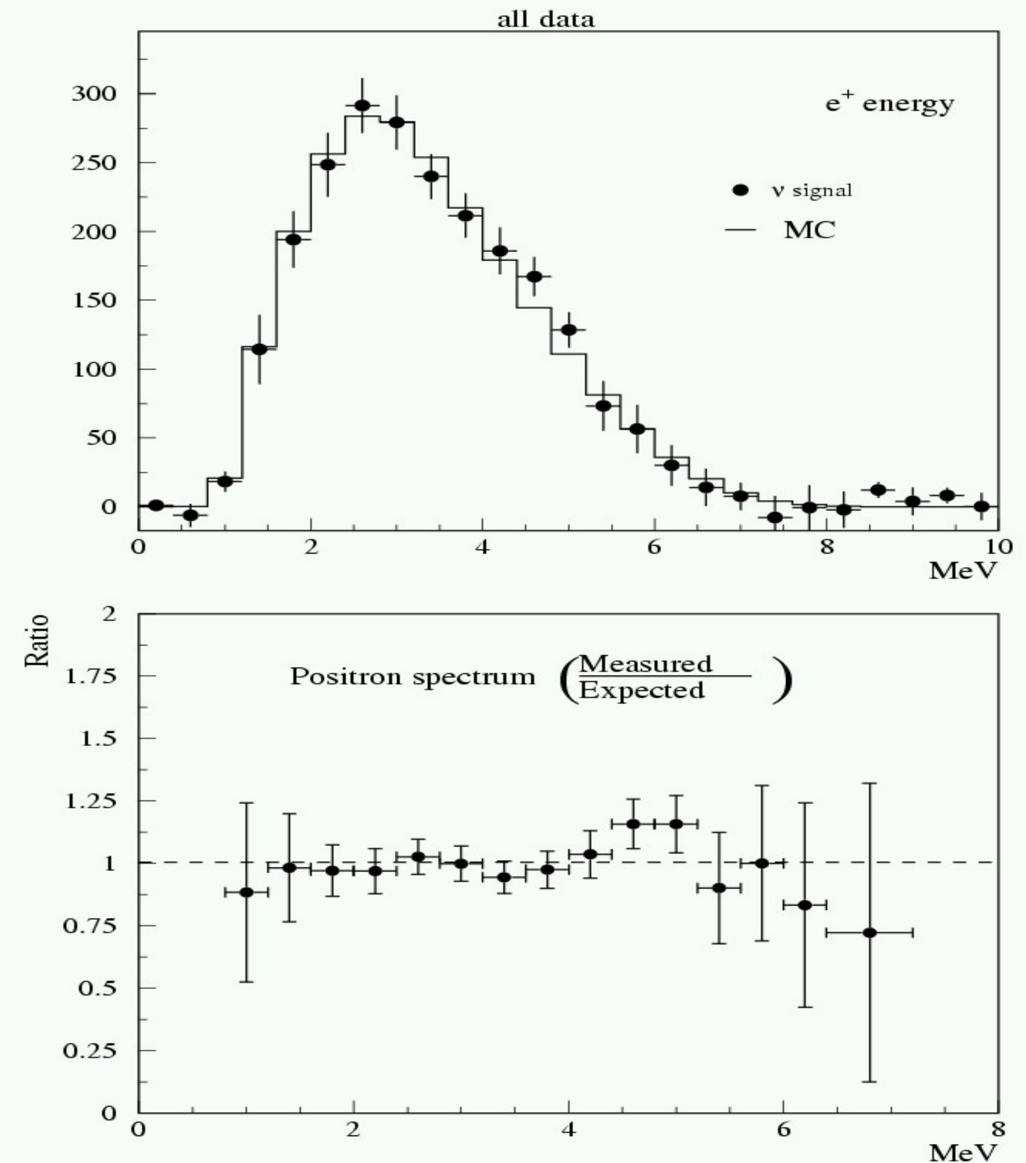
$$P(\nu_\mu \to \nu_\tau) = P(\Delta M^2_{ATM}, \theta_{ATM})$$





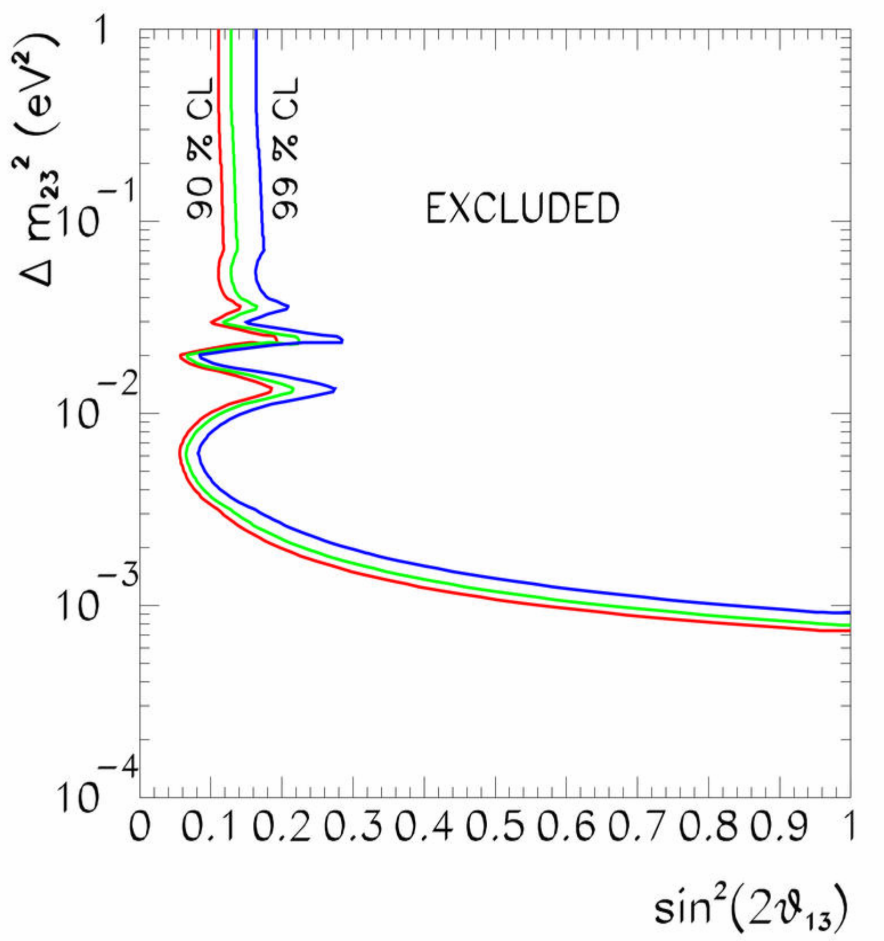


Reactor : …,Chooz/Palo Verde,…



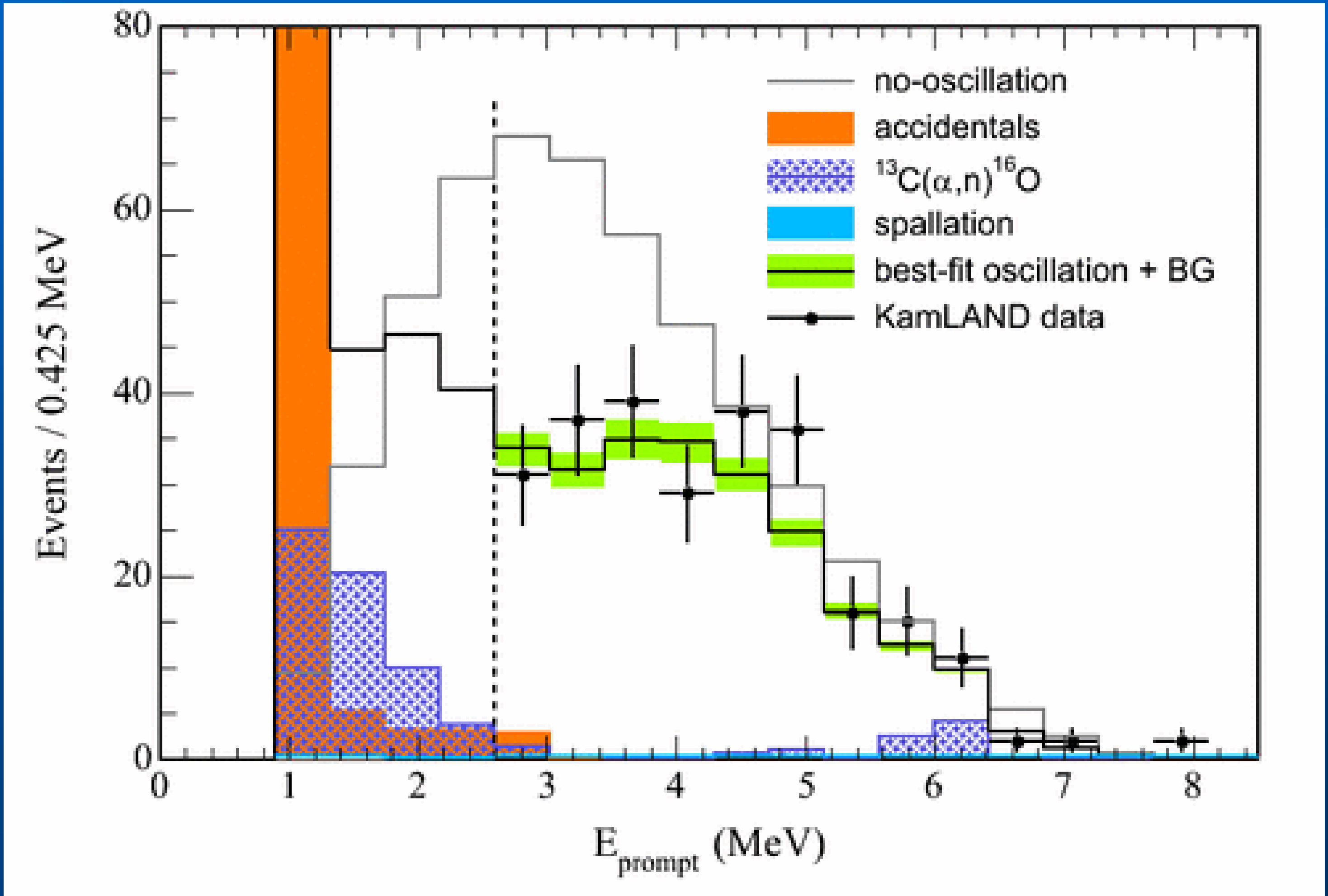
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P(\Delta M^2_{atm}, \theta_{chooz})$$

$$R = 1.01 \pm 2.8\% \pm 2.7\%$$



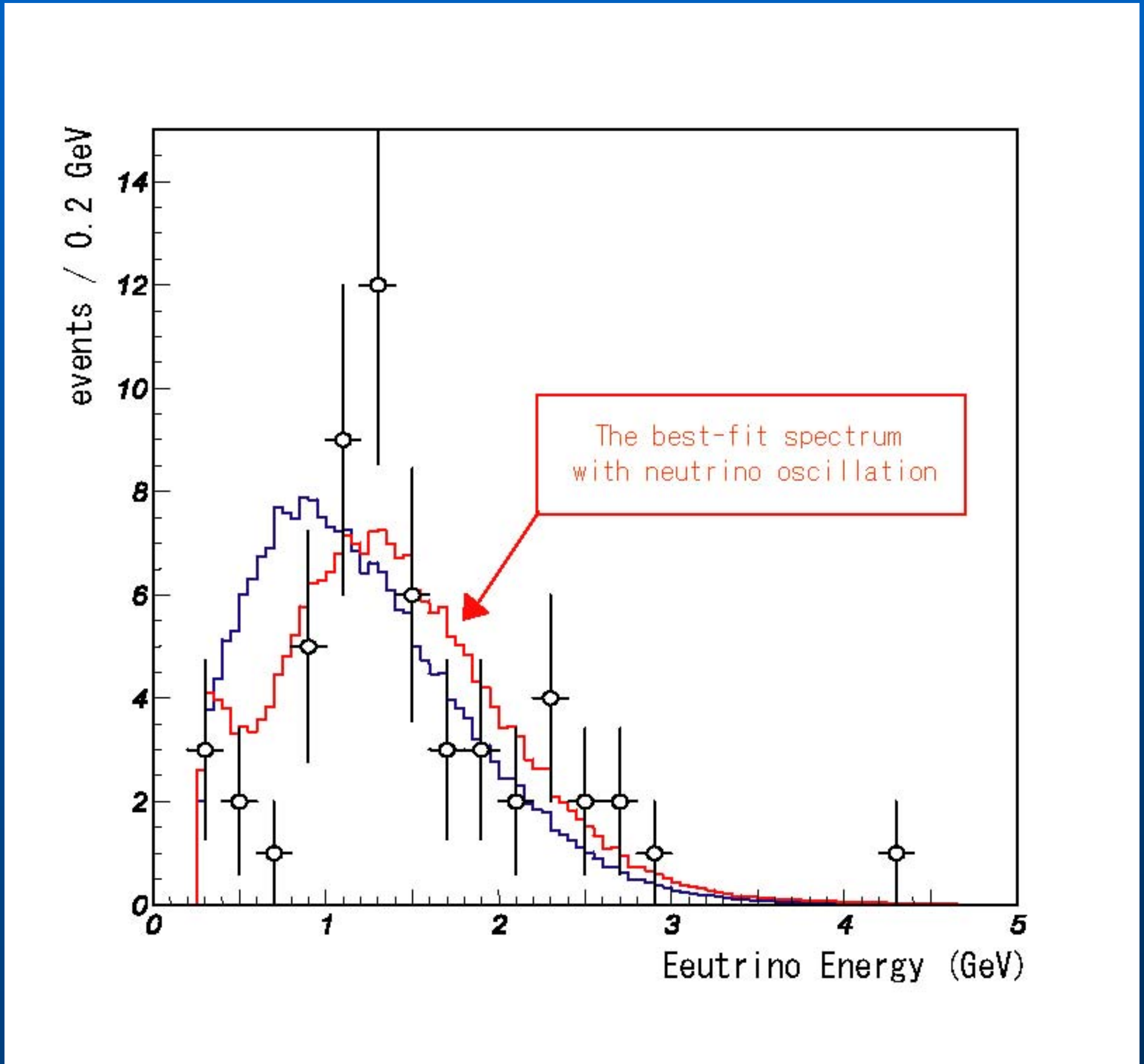
Reactor : …,KamLAND

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = P(\Delta m^2_{sun}, \theta_{sun})$$



Accelerator : K2K

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\Delta M^2_{ATM}, \theta_{ATM})$$



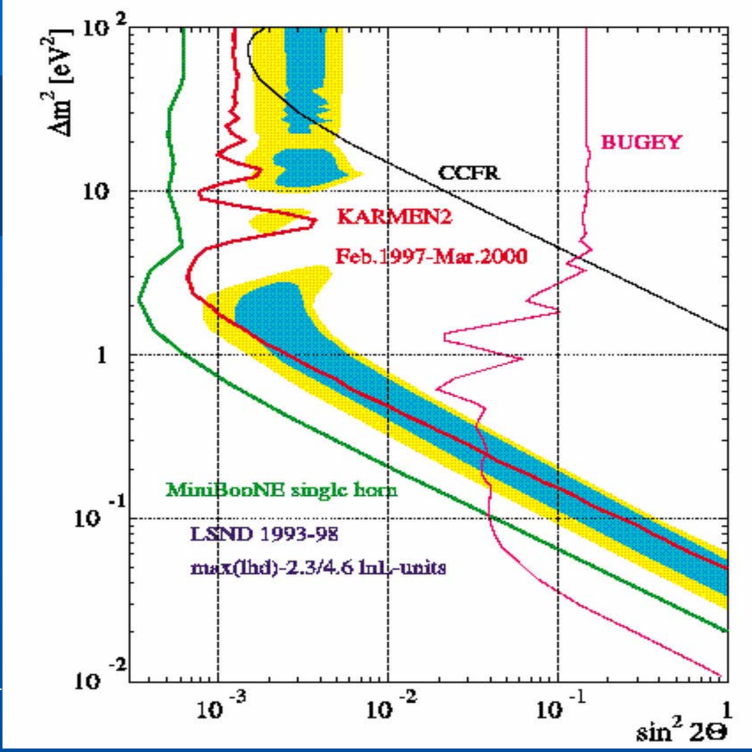
LSND, Miniboone

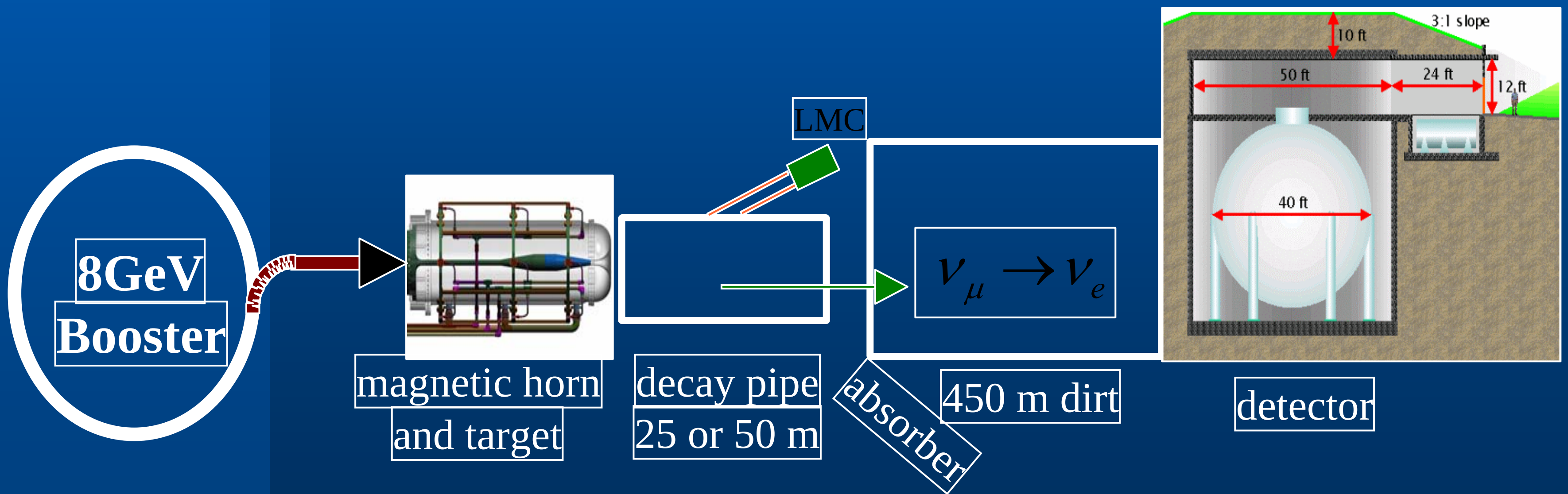
$$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

Proton beam producing π^+

$$N(\bar{\nu}_e) = 87.9 \pm 22.4 \pm 6.0$$

$$d = 30m \quad 20MeV < E < 60MeV$$





$$0.5GeV < E < 1GeV \quad d = 500m$$

The Neutrino Matrix :SM + ν mass

ALL oscillation neutrino data BUT LSND

3 σ ranges:

$$|U_{\alpha i}| = \begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} = \begin{pmatrix} 0.79-0.88 & 0.47-0.61 & 0.00-0.20 \\ 0.19-0.52 & 0.42-0.73 & 0.58-0.82 \\ 0.20-0.53 & 0.44-0.74 & 0.56-0.81 \end{pmatrix}$$

$$7.3 \le \frac{\Delta m^2}{10^{-5}\,\mathrm{eV}^2} \le 9.3\,(8.2)$$

$$1.6 \le \frac{\Delta M^2}{10^{-3}\,\mathrm{eV}^2} \le 3.6\,(2.2)$$

$$0.28 \le \tan^2\theta_{12} \le 0.60 \;\; (0.39)$$

$$0.51 \le \tan^2\theta_{23} \le 2.1 \;\; (1.0)$$

$$\sin^2\theta_{13} \le 0.041 \;\; (0.005)$$

The Neutrino Matrix : SM + ν mass

ALL oscillation neutrino data BUT LSND

$$|U_{\alpha i}| \sim \begin{pmatrix} \frac{1}{\sqrt{2}}(1+\lambda) & \frac{1}{\sqrt{2}}(1-\lambda) & \varepsilon \\ \frac{1}{2}(1-\lambda+\Delta+\varepsilon\cos\delta) & \frac{1}{2}(1+\lambda+\Delta-\varepsilon\cos\delta) & \frac{1}{\sqrt{2}}(1-\Delta) \\ \frac{1}{2}(1-\lambda-\Delta-\varepsilon\cos\delta) & \frac{1}{2}(1+\lambda-\Delta+\varepsilon\cos\delta) & \frac{1}{\sqrt{2}}(1+\Delta) \end{pmatrix}$$

1 σ ranges:

$$\lambda = 0.23 \pm 0.04$$

$$\Delta = 0.00 \pm 0.06$$

$$\varepsilon \leq 0.12$$

$$-1 \leq \cos\delta \leq 1$$

SM + ν mass

Neutrinos: Cosmological dark matter to be discovered

If lowest mass is negligible, and normal hierarchy

$$\Omega_\nu = 0.0009 \pm 0.0001$$

If highest mass is 1 eV

$$\Omega_\nu \sim 0.03 (1\,\text{eV})$$

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GRAVITY OF NEUTRINOS OF NONZERO MASS IN ASTROPHYSICS

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Received 1972 July 24

ABSTRACT

If neutrinos have a rest mass of a few eV/c^2, then they would dominate the gravitational dynamics of the large clusters of galaxies and of the Universe. A simple model to understand the virial mass discrepancy in the Coma cluster on this basis is outlined.

Subject headings: cosmology — galaxies, clusters of — neutrinos

The possibility of a finite rest mass for the neutrinos has fascinated astrophysicists (Kuchowicz 1969). A recent discussion of such a possibility has been in the context of the solar-neutrino experiments (Bahcall, Cabibbo, and Yahil 1972). Here we wish to point out some interesting consequences of the gravitational interactions of such neutrinos. These considerations become particularly relevant in the framework of big-bang cosmologies which we assume to be valid in our discussion here.

In the early phase of such a Universe when the temperature was ~1 MeV, several

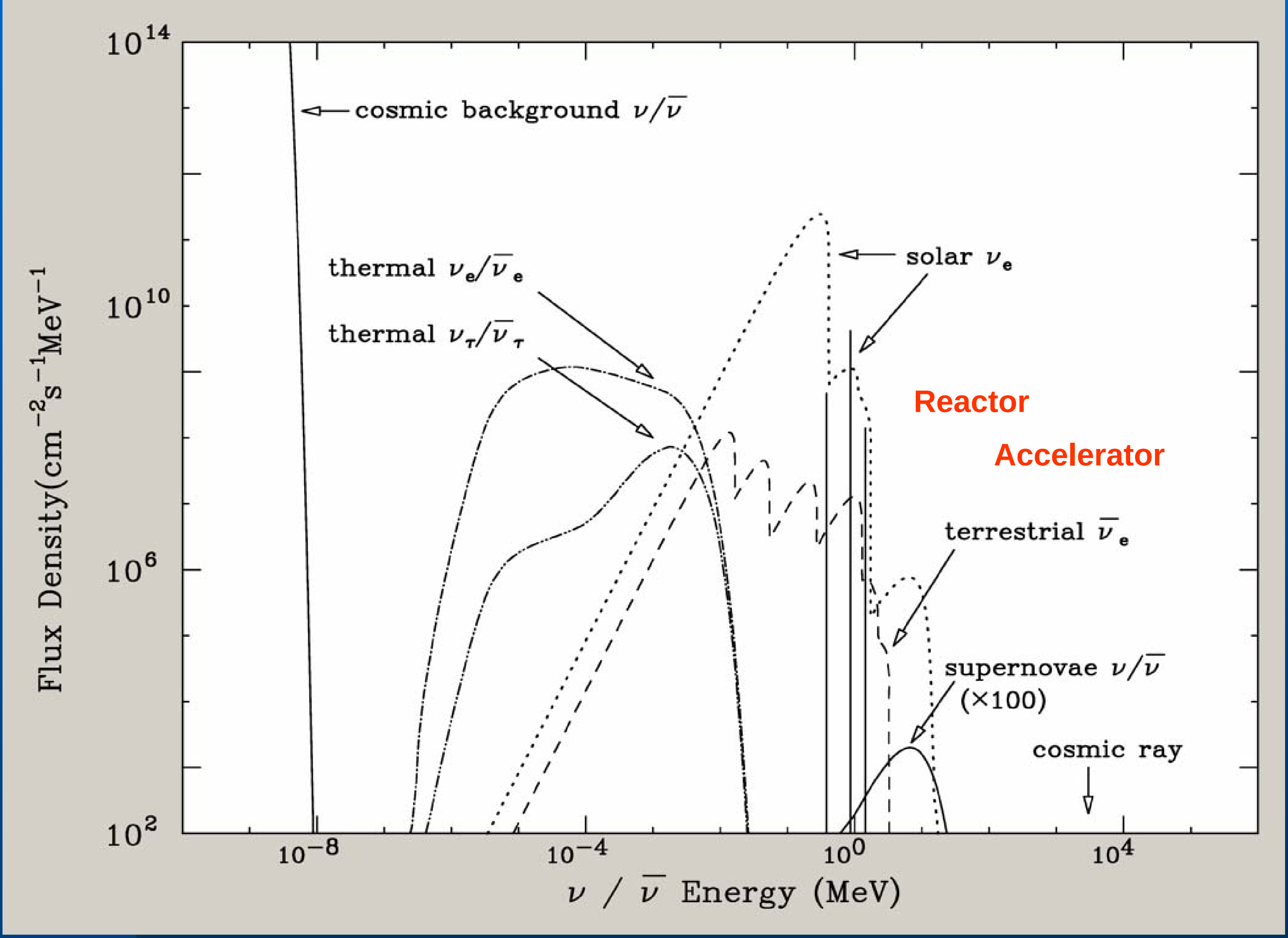
Plan

I. **Neutrinos as Dirac Particles**

II. **Making Neutrinos**

III. **Neutrinos as Majorana Particles**

Neutrino Sources



Cosmic Neutrino Background

56 cm^{-3} at 1.9K (0.17 meV)

Possible mechanical effect : torque of order G_F if target and neutrino background are polarized (Stodolsky effect) and net neutrino-antineutrino asymmetry

Still far from observability, awaiting for future technology

Neutrinos from Thermal Plasma

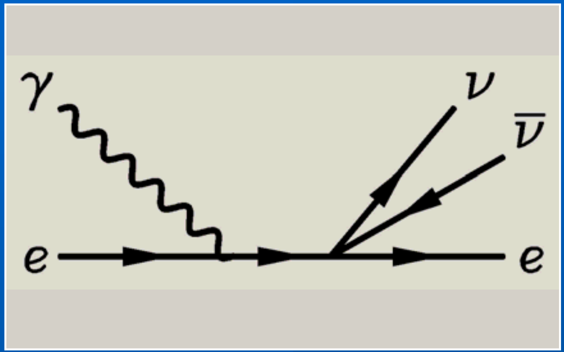
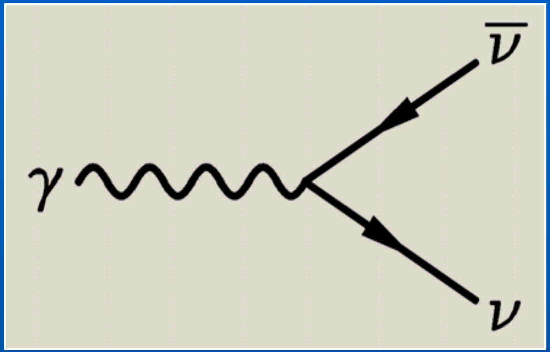
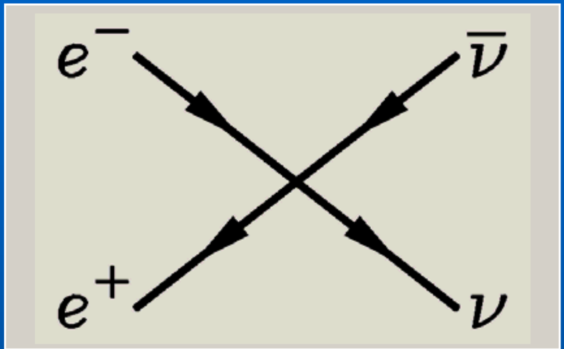


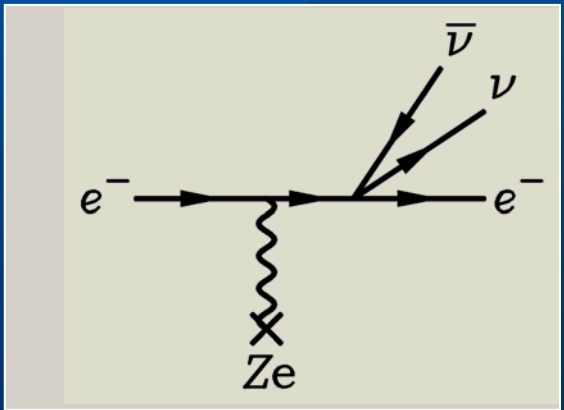
Photo (Compton)



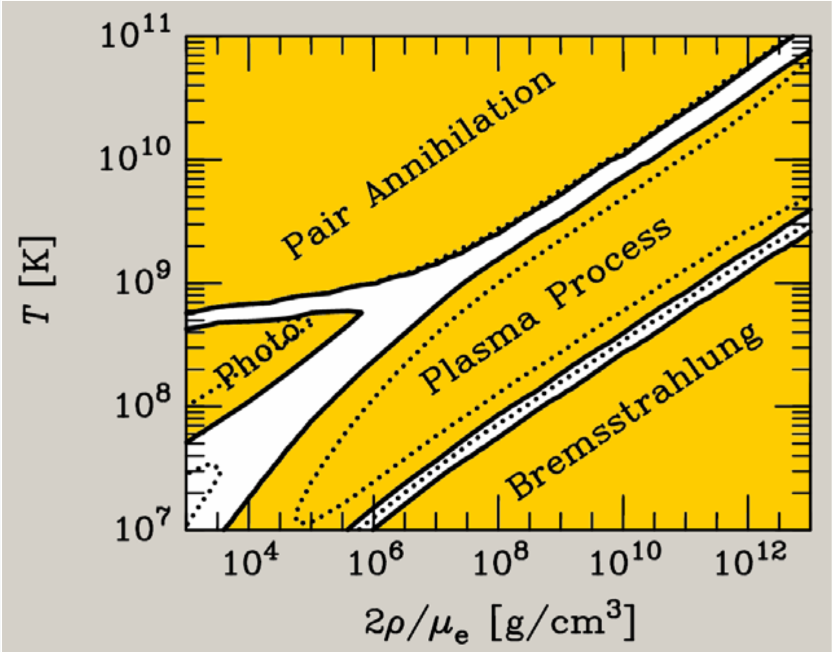
Plasmon decay



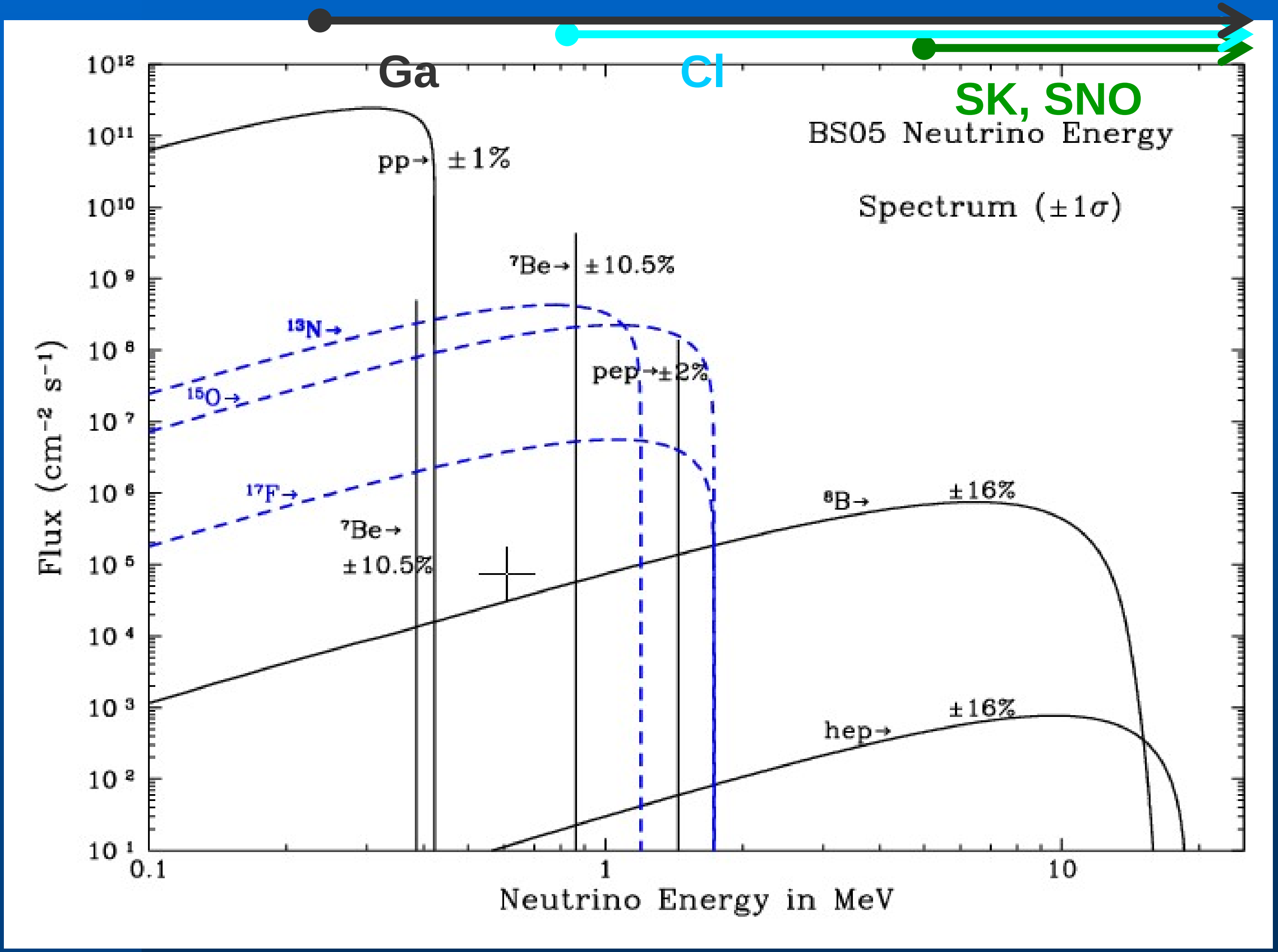
Pair annihilation



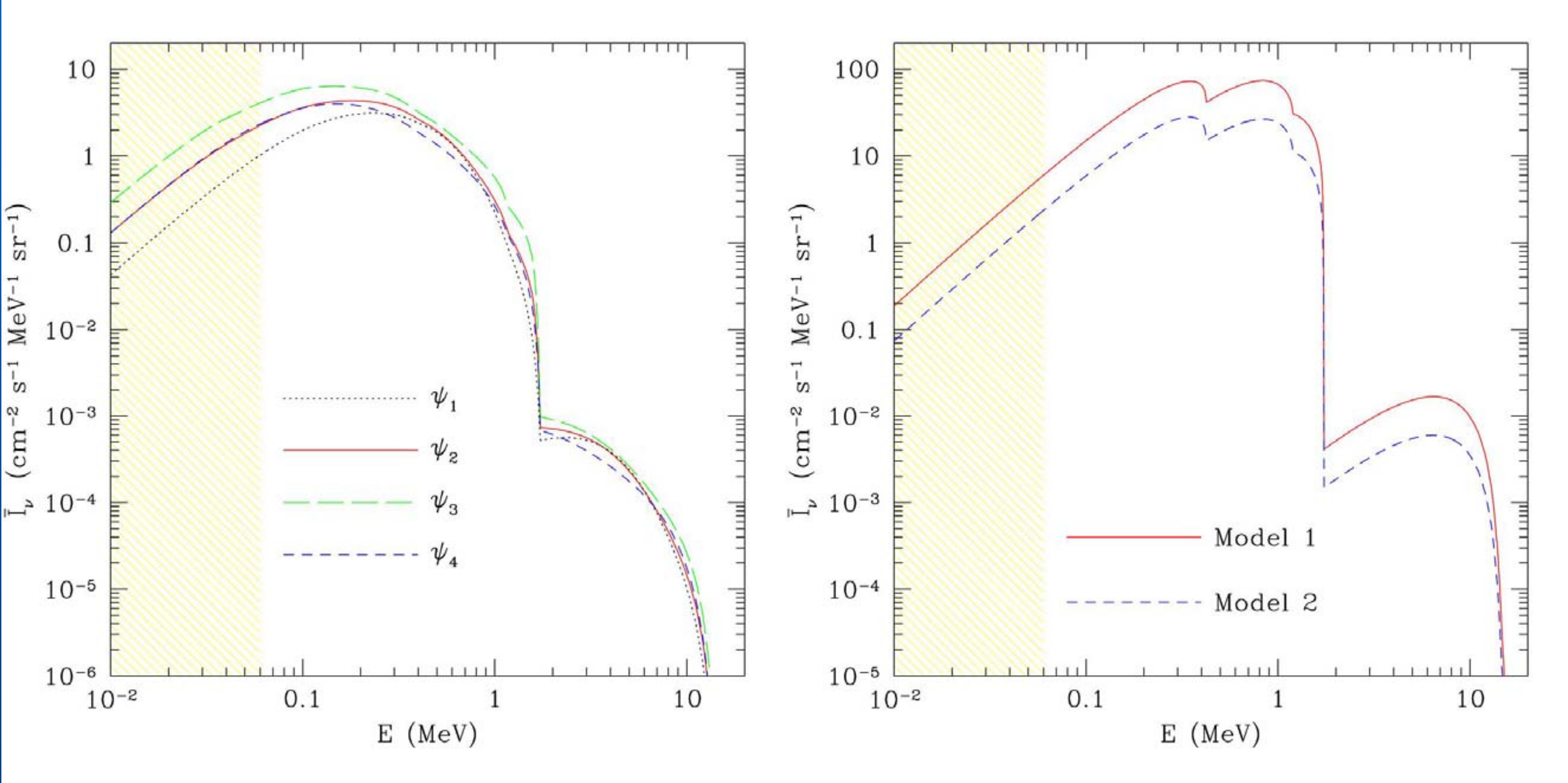
Bremsstrahlung



Solar (thermonuclear) Neutrinos

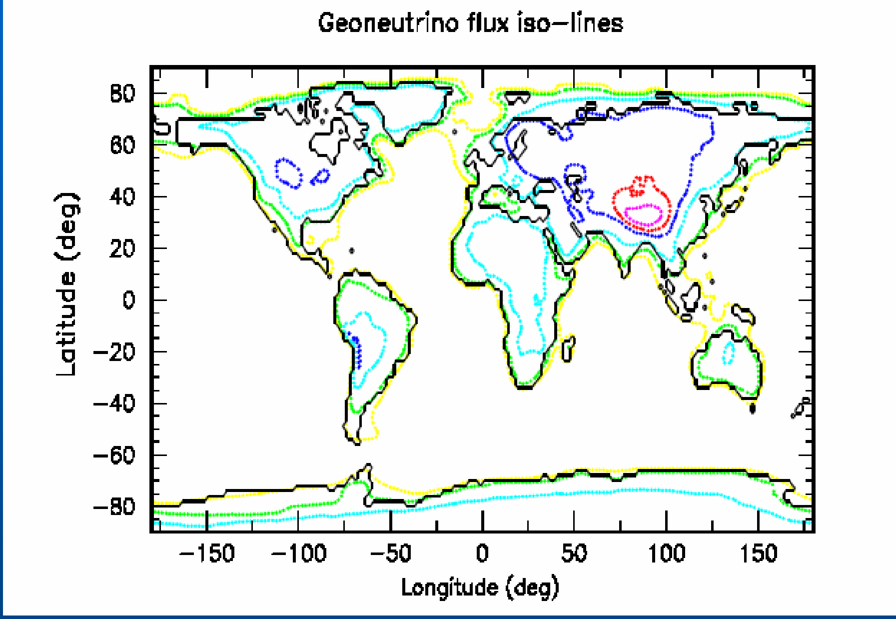


Thermonuclear Neutrinos

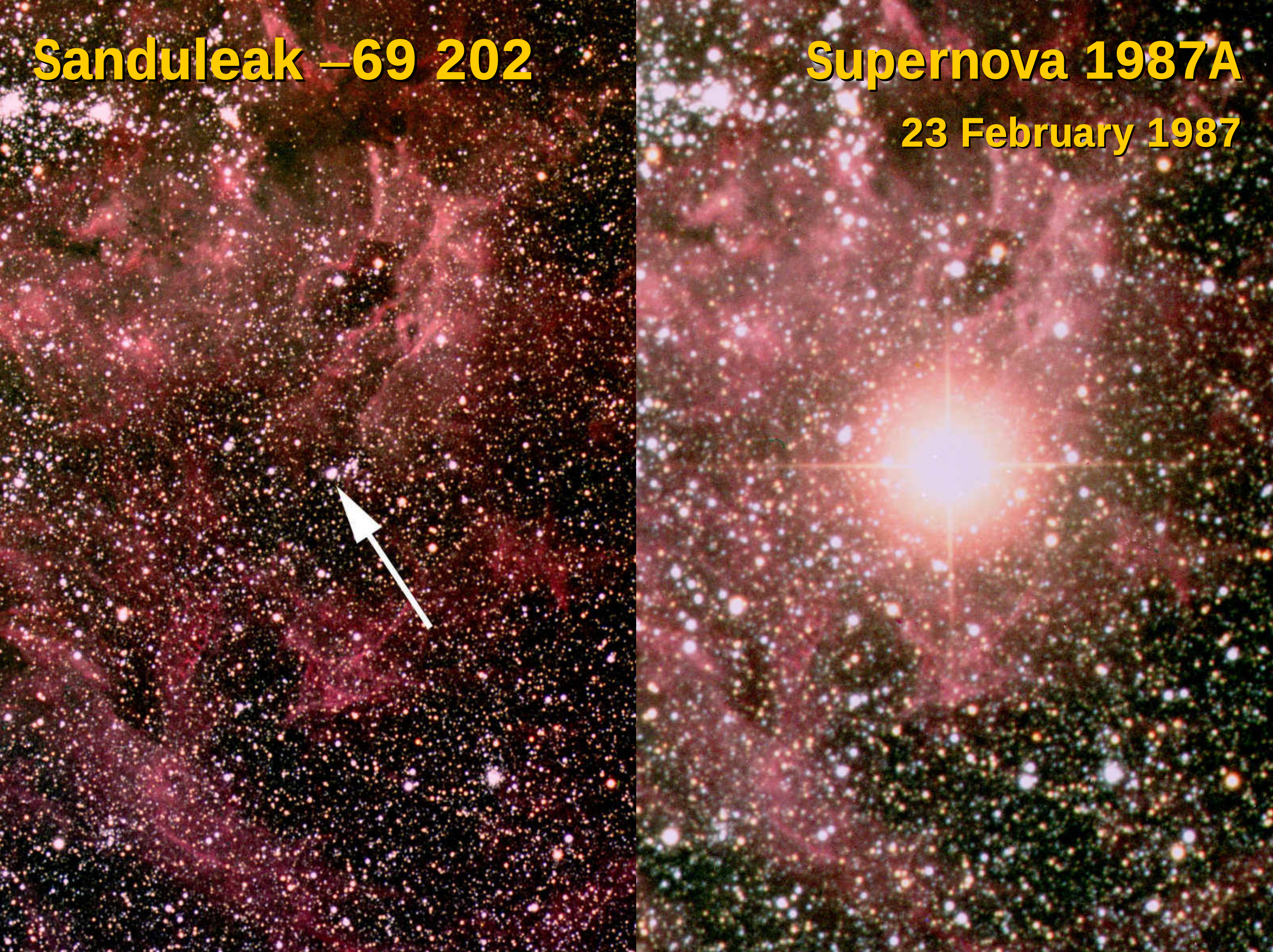


Geoneutrinos

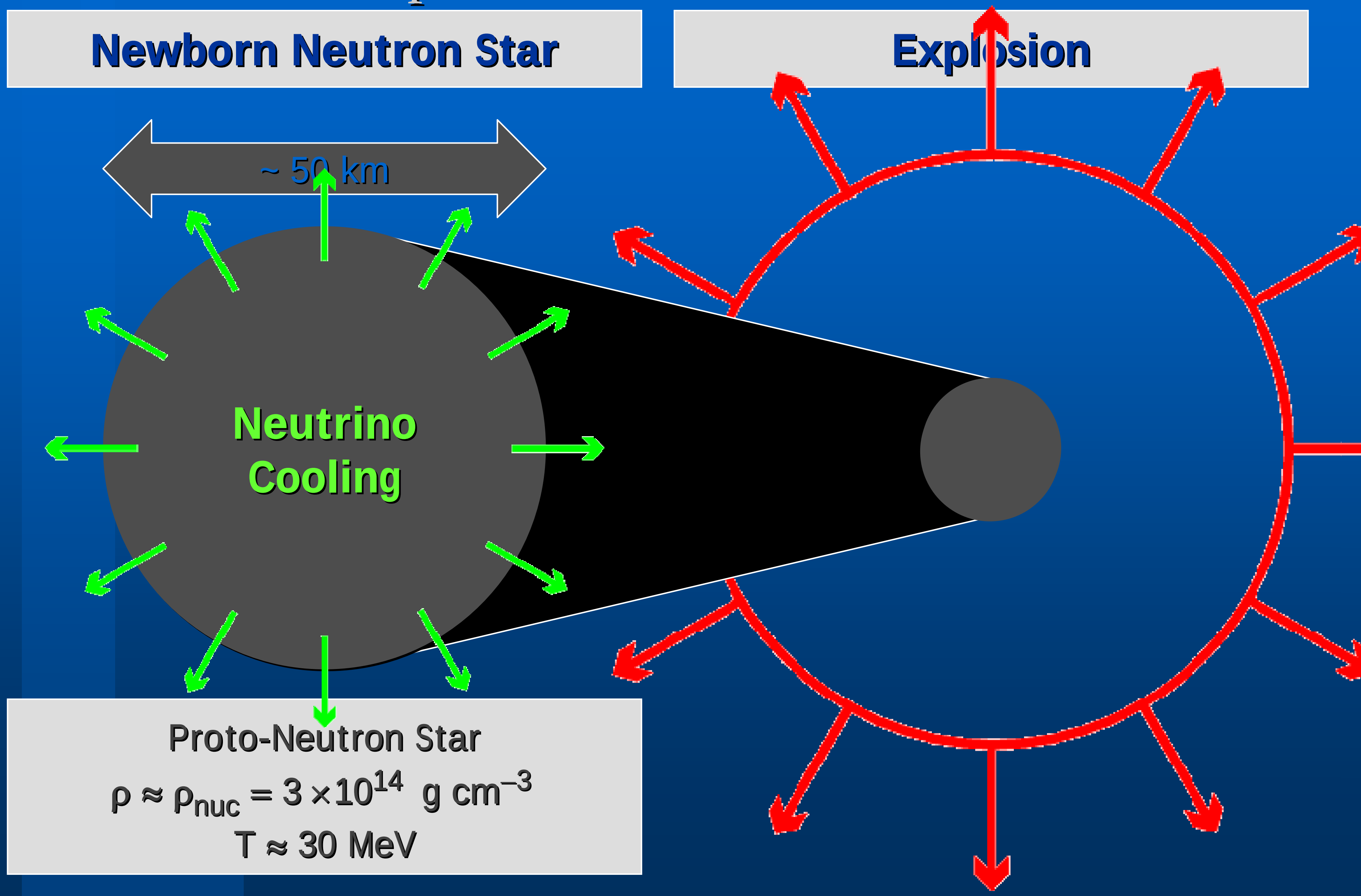
Decay	Q [MeV]	$\tau_{1/2}$ [10^9 yr]	E_{max} [MeV]	ε_H [W/kg]	$\varepsilon_{\bar{\nu}}$ [$kg^{-1}s^{-1}$]
$^{238}U \rightarrow ^{206}Pb + 8\,^4He + 6e + 6\bar{\nu}$	51.7	4.47	3.26	$0.95\cdot10^{-4}$	$7.41\cdot10^{7}$
$^{232}Th \rightarrow ^{208}Pb + 6\,^4He + 4e + 4\bar{\nu}$	42.8	14.0	2.25	$0.27\cdot10^{-4}$	$1.63\cdot10^{7}$
$^{40}K \rightarrow ^{40}Ca + e + \bar{\nu}$	1.32	1.28	1.31	$0.36\cdot10^{-8}$	$2.69\cdot10^{4}$



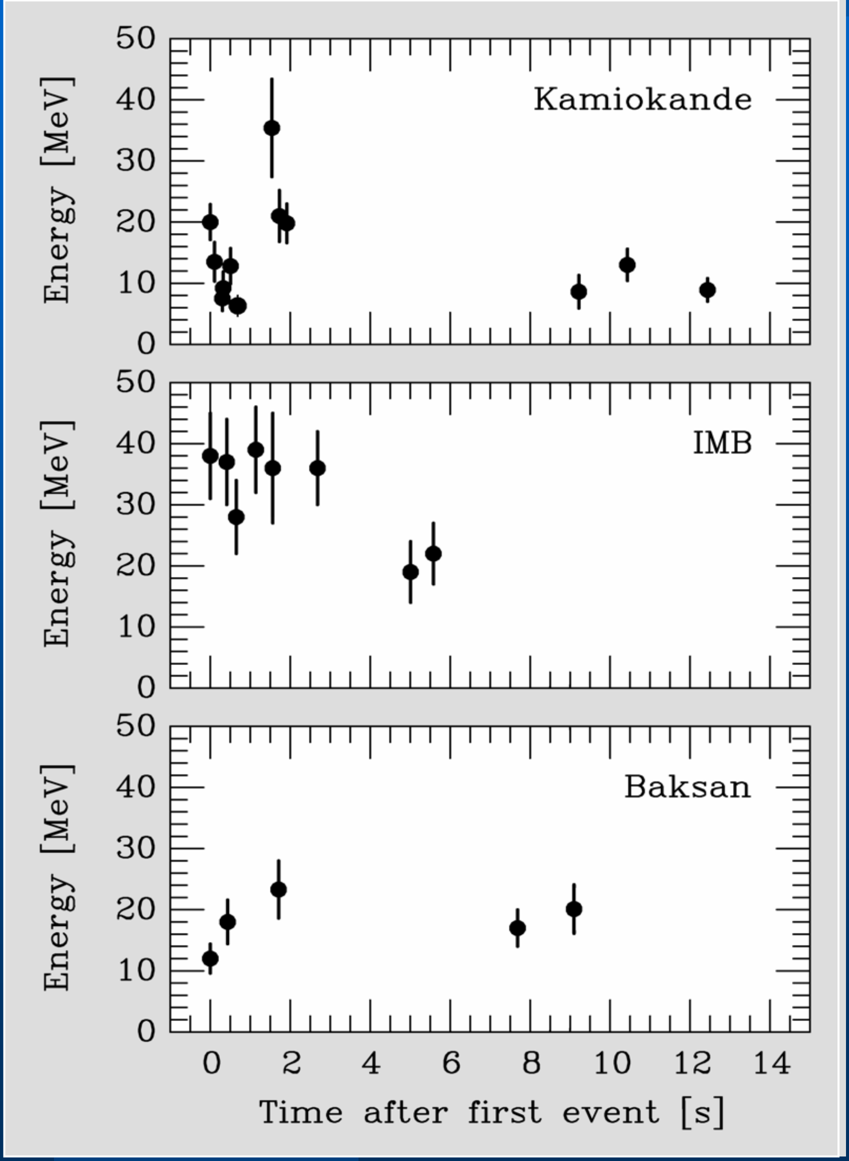
Site	U (10^6 $cm^{-2}s^{-1}$)	Th (10^6 $cm^{-2}s^{-1}$)
Kamioka	3.7	3.2
GS	4.2	3.7


Sanduleak –69 202
Supernova 1987A
23 February 1987

Stellar Collapse and Supernova Explosion



Neutrino Signal of Supernova 1987A



Kamiokande (Japan)
Water Cherenkov detector
Clock uncertainty ±1 min

Irvine-Michigan-Brookhaven (US)
Water Cherenkov detector
Clock uncertainty ±50 ms

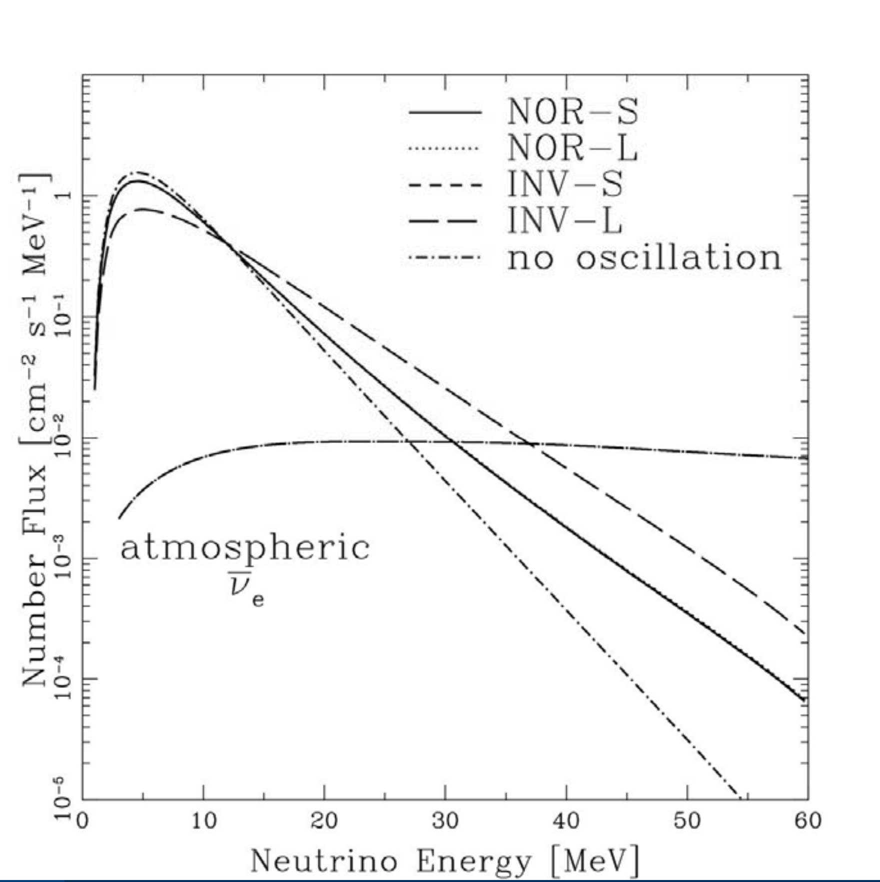
Baksan Scintillator Telescope
(Soviet Union)
Clock uncertainty +2/-54 s

Within clock uncertainties, signals are contemporaneous

Limits on Supernova Relic Neutrinos

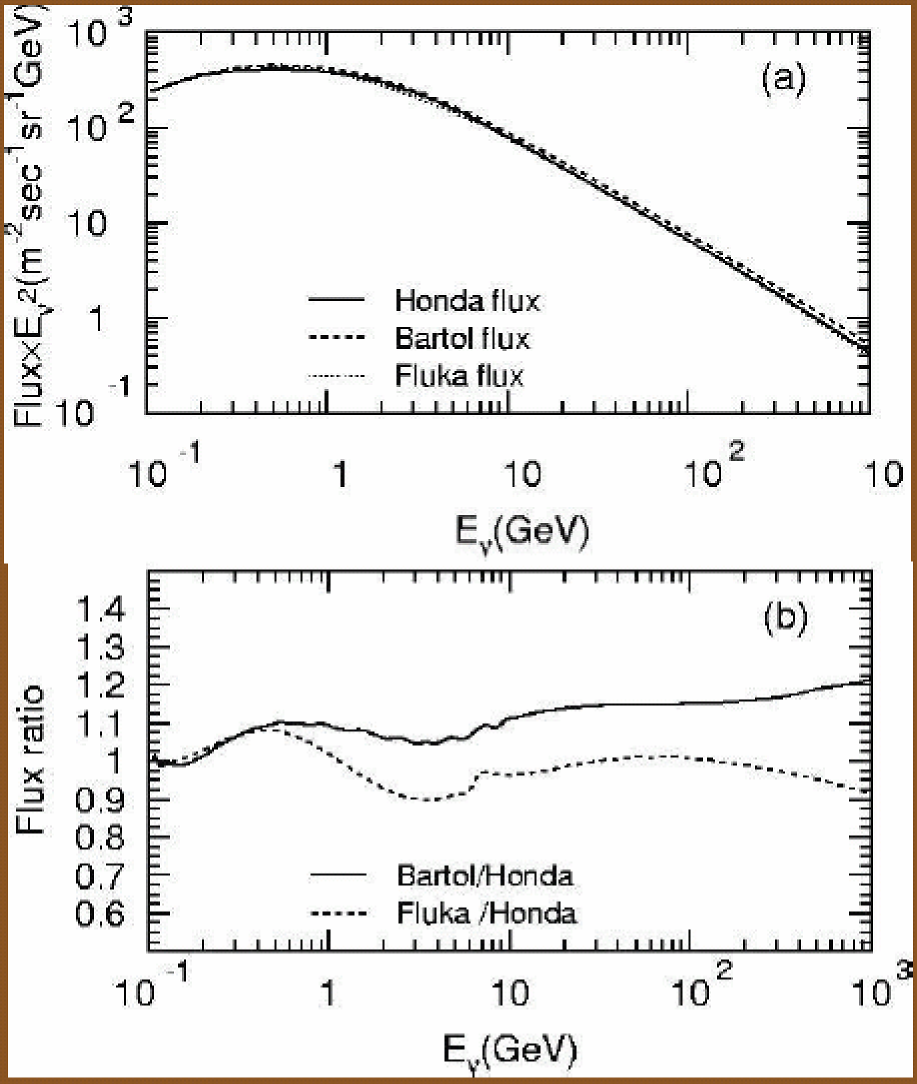
Super-Kamiokande :

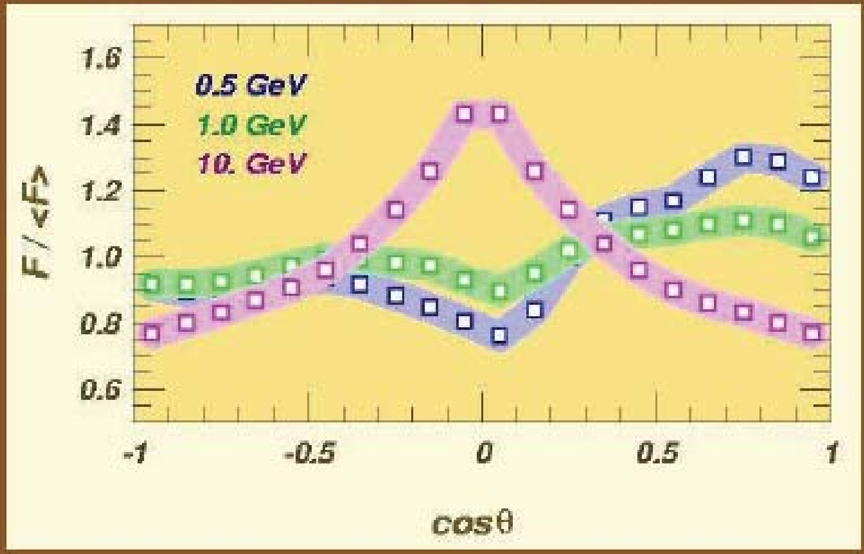
$$\Phi_{\bar{\nu}_e} \leq 1.2\ \mathrm{cm}^{-2}\mathrm{s}^{-1} \quad (90\%\mathrm{CL}) \quad > 19.3\ \mathrm{MeV}$$

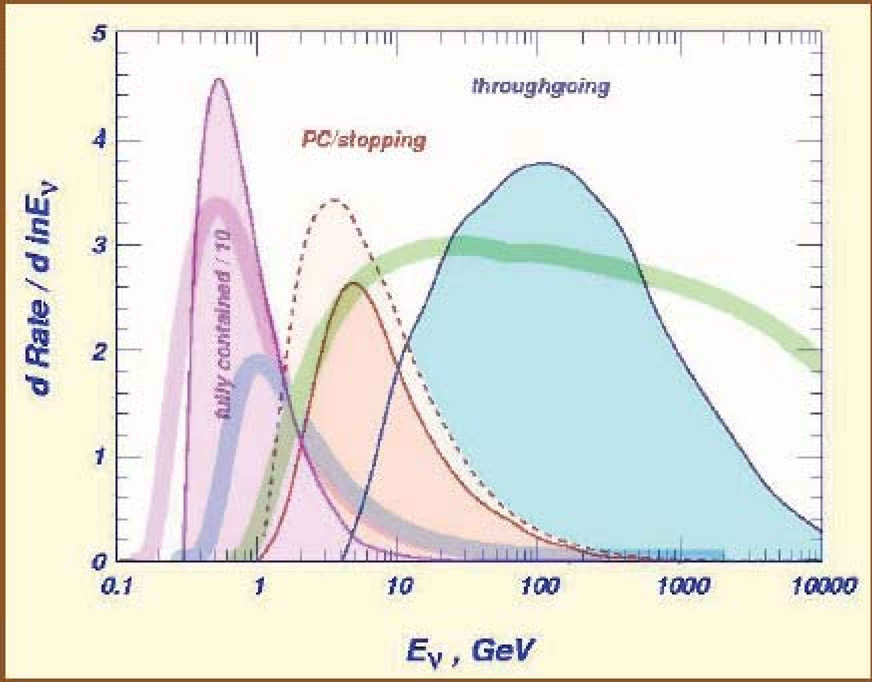


Ando et al (2004)

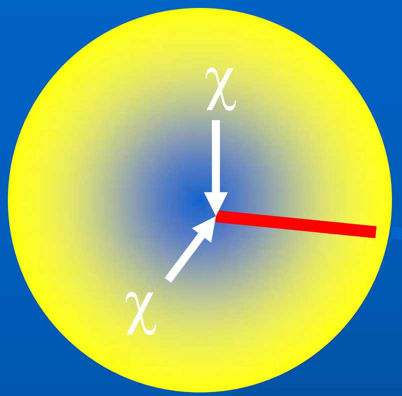
Atmospheric Neutrinos



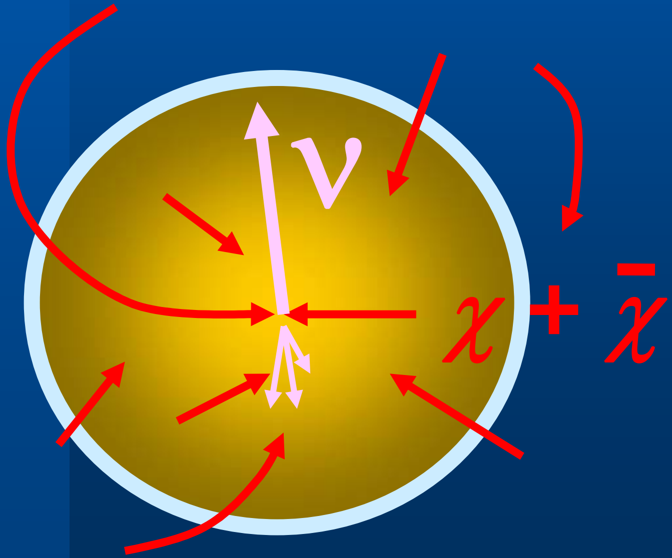




Undiscovered neutrino sources

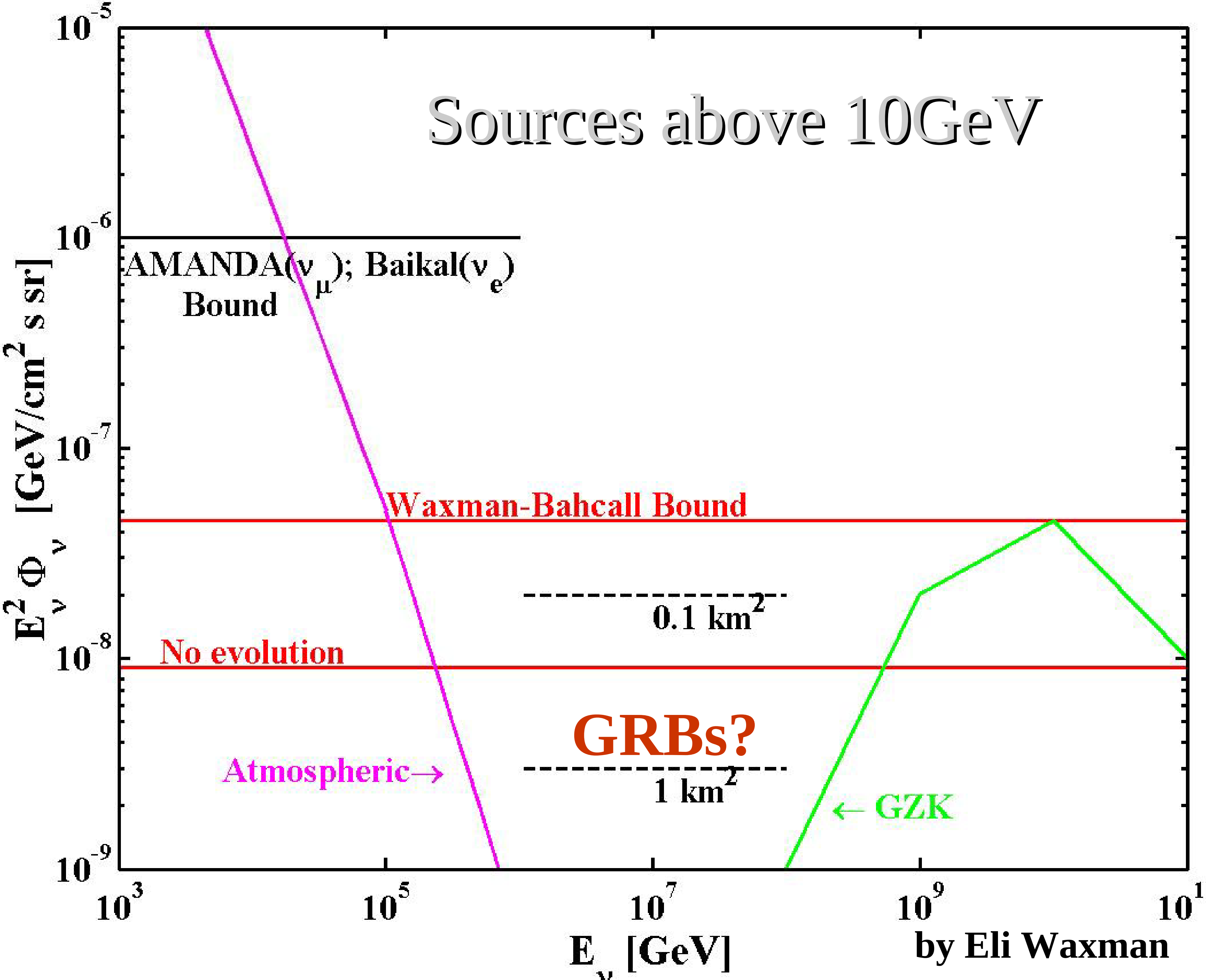


Neutrinos from the Sun



Neutrinos from the Earth

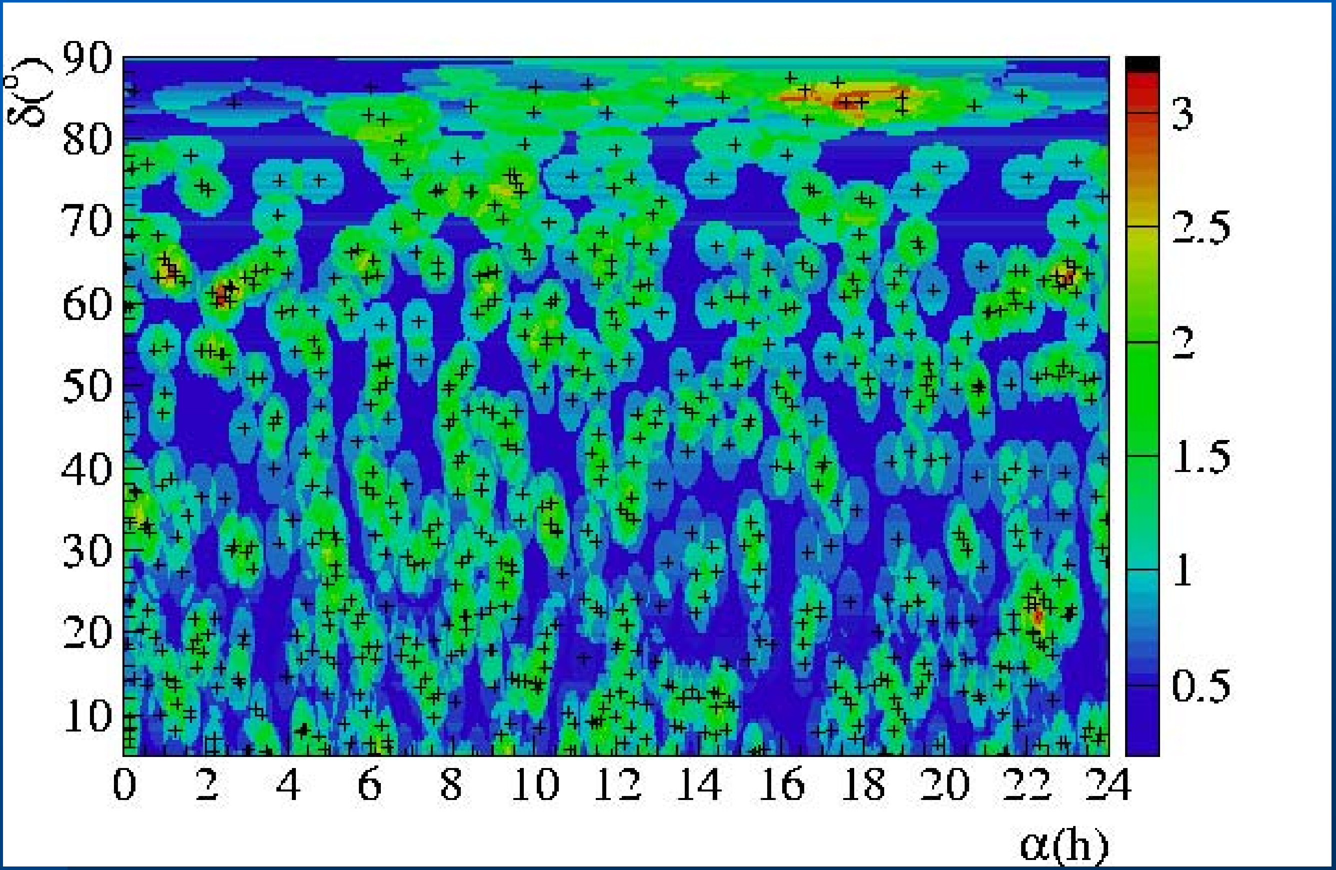
Sources above 10GeV



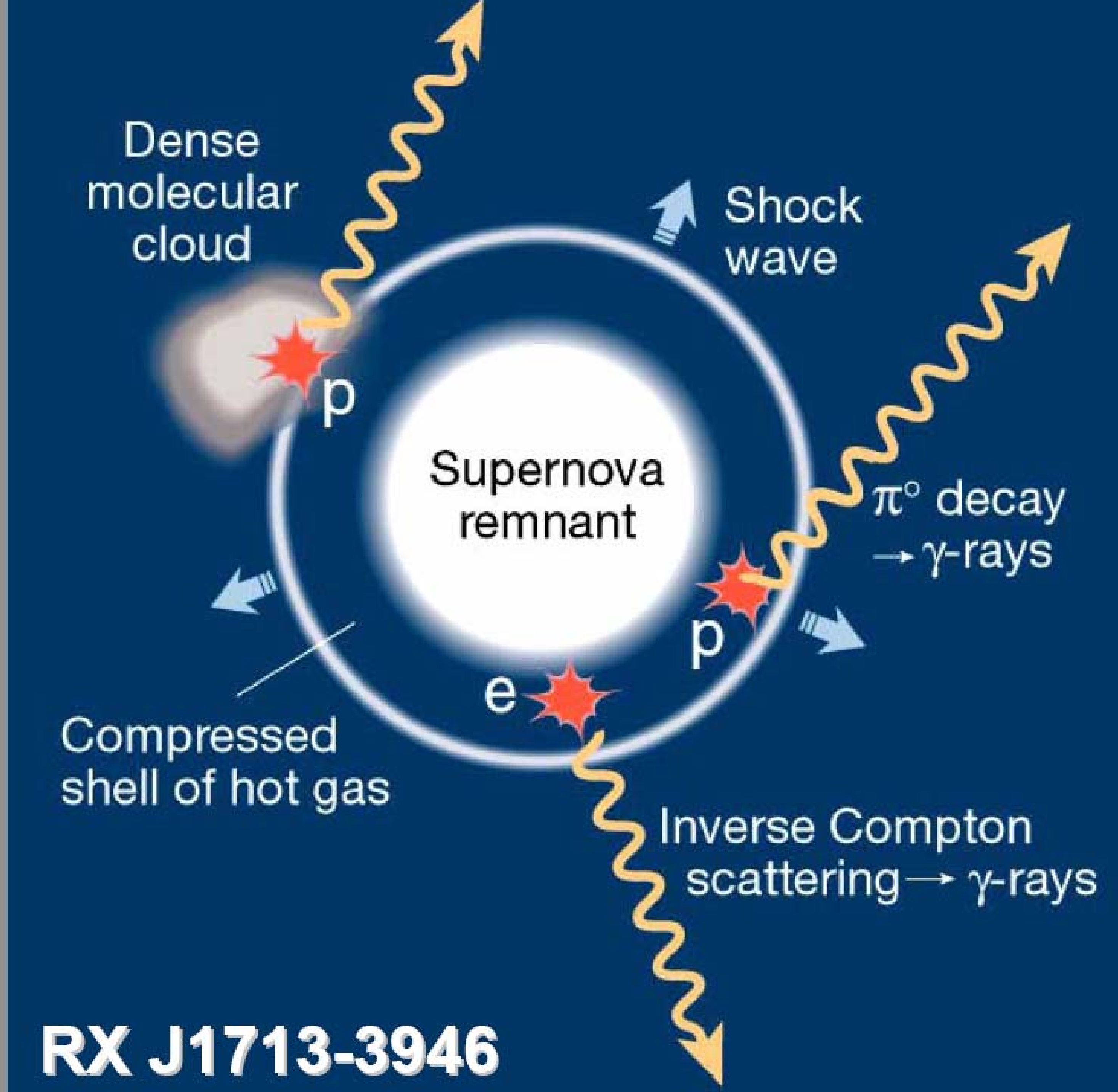
by Eli Waxman

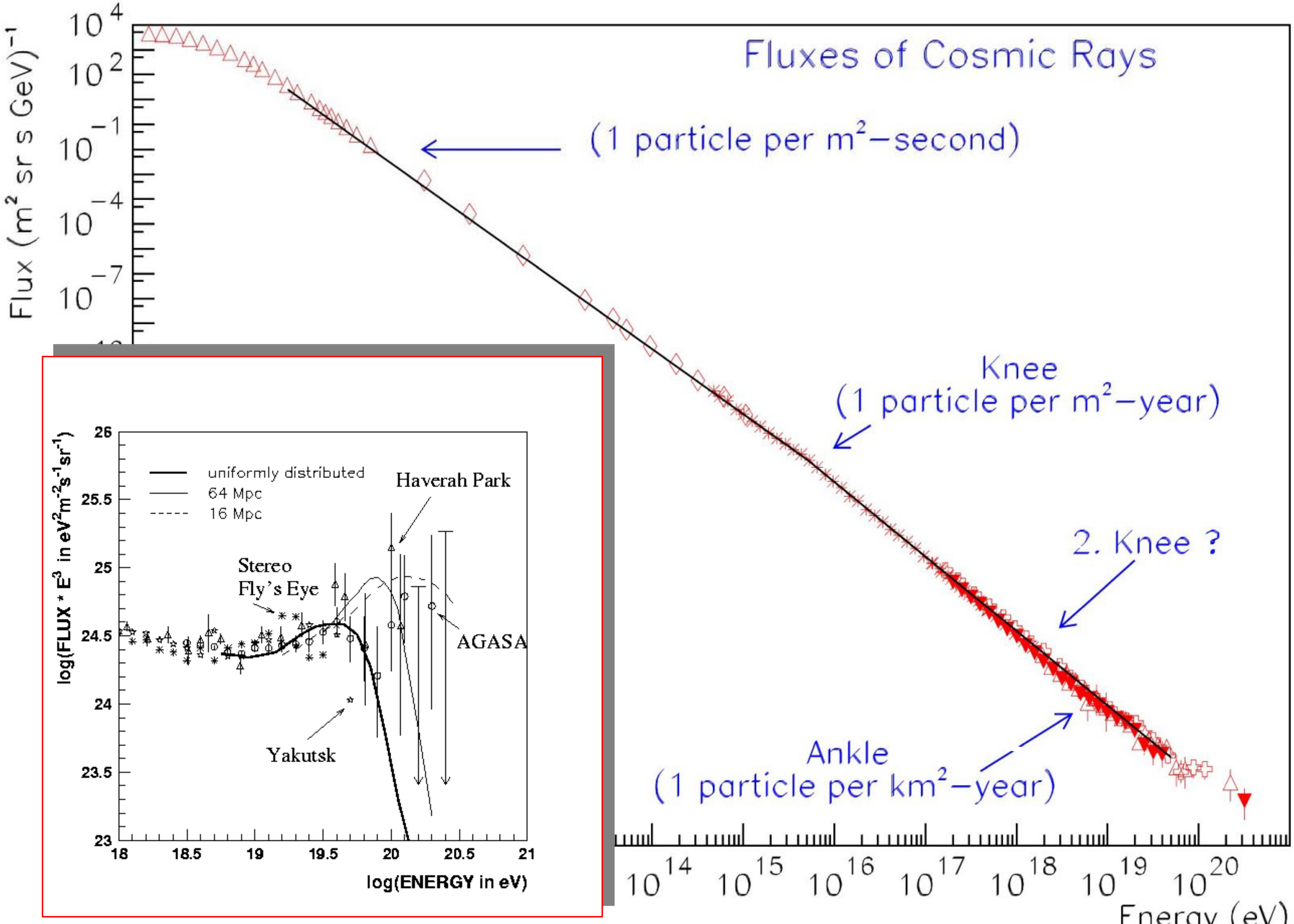
Sources above 10GeV

90% CL upper limits in 10^{-8} $cm^{-2}s^{-1}$ for E^{-2} spectrum from AMANDA II (2000-2002) :



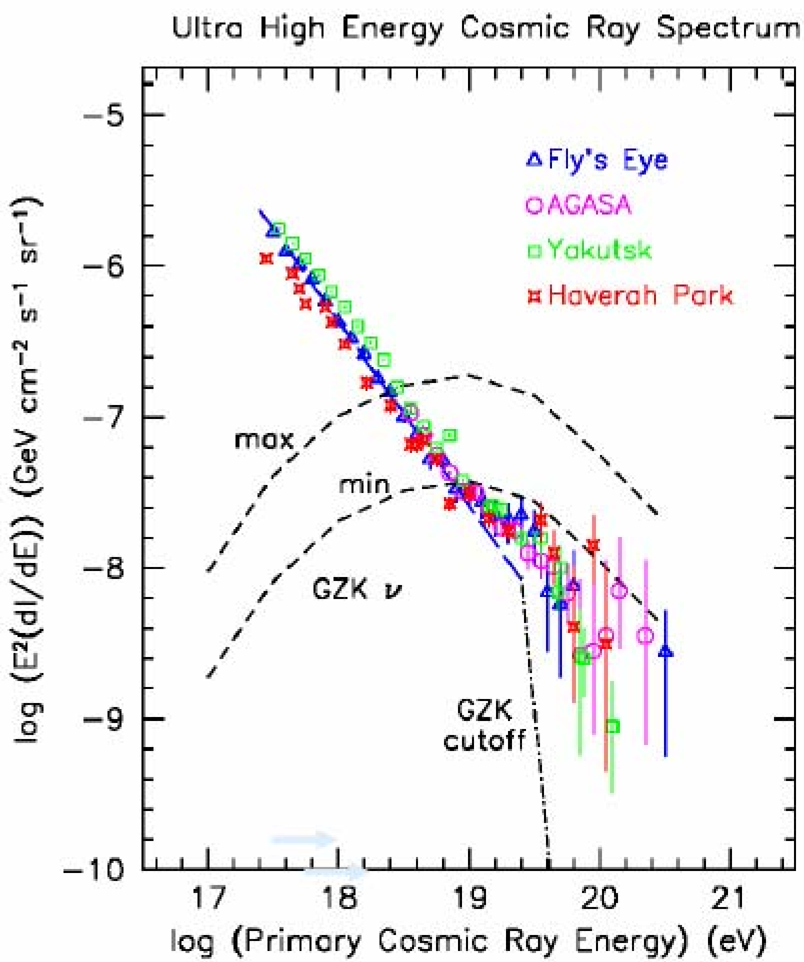
Supernova Beam Dump





Fluxes of Cosmic Rays
(1 particle per m²−second)
Knee
(1 particle per m²−year)
2. Knee ?
Ankle
(1 particle per km²−year)
Flux (m² sr s GeV)⁻¹
10⁴
10²
10⁻¹
10⁻⁴
10⁻⁷
10¹⁴ 10¹⁵ 10¹⁶ 10¹⁷ 10¹⁸ 10¹⁹ 10²⁰
Energy (eV)
uniformly distributed
64 Mpc
16 Mpc
Haverah Park
Stereo
Fly's Eye
AGASA
Yakutsk
log(FLUX * E³ in eV²m⁻²s⁻¹sr⁻¹)
26
25.5
25
24.5
24
23.5
23
18
18.5
19
19.5
20
20.5
21
log(ENERGY in eV)

GZK Cosmic Rays & Neutrinos



- cosmogenic neutrinos are "guaranteed"
- 0.1– few events per year in IceCube

$$p + \gamma_{CMB} \rightarrow \pi + n$$

Flavor ratio for far π sources

We start with $1:2:0$

Decoherence: wave packets separate ν_i

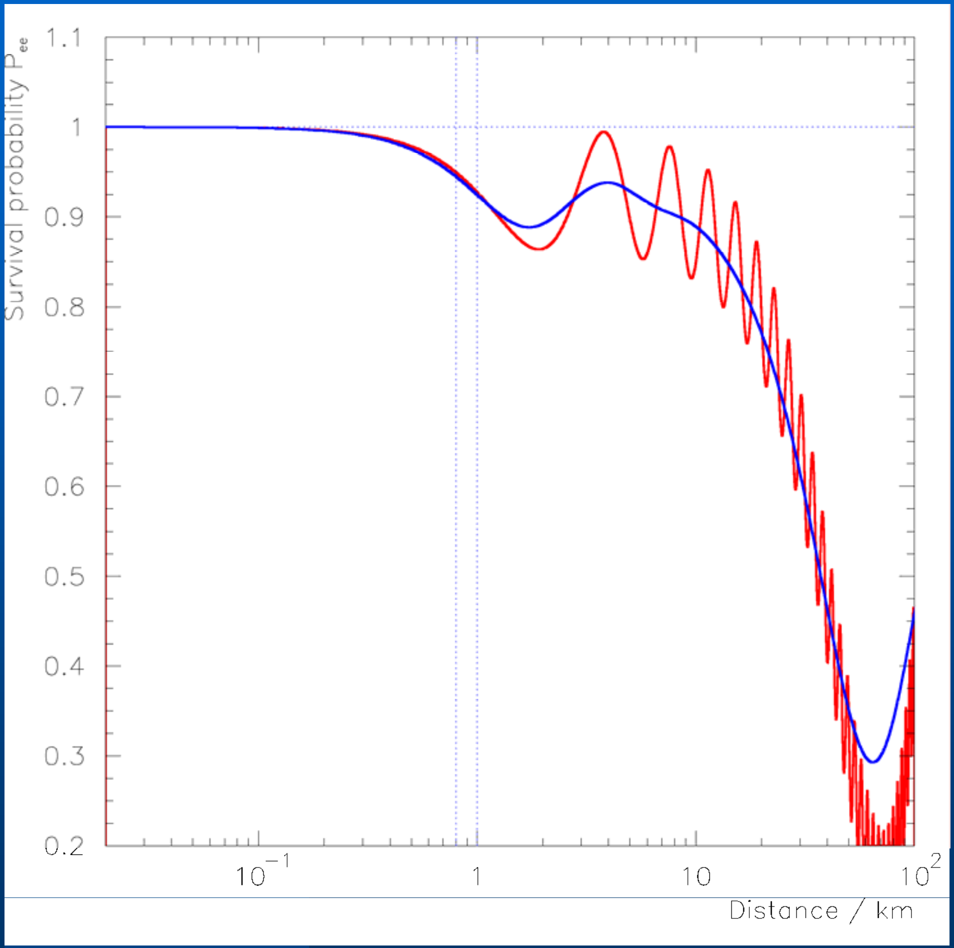
$$\phi(\nu_i) \propto |U_{ei}|^2 + 2|U_{\mu i}|^2$$

$$\left[\theta_{23} = \frac{\pi}{2}, \theta_{13} = 0\right] \quad \propto (\cos^2\theta_{12} + \sin^2\theta_{12}, \sin^2\theta_{12} + \cos^2\theta_{12}, 1)$$

We end with $1:1:1$ in mass basis (present precision ~10%)

$\Rightarrow$ We end with $1:1:1$ in flavor basis

Man-made Neutrino Sources : Reactors



Goal : Measure θ_{13}

$$P_{ee} \approx 1 - \sin^2 2\theta_{13} \sin \frac{1.27\,\Delta m_{13}^2 L}{E_{\bar{\nu}}} - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin \frac{1.27\,\Delta m_{12}^2 L}{E_{\bar{\nu}}}$$

Man-made Neutrino Sources : Accelerator

- full numerical simulation
- $\Delta = \Delta m^2_{31} L/4E$
- qualitative understanding $\Rightarrow$ expand in $\alpha = \Delta m^2_{21}/\Delta m^2_{31}$ and $\sin^2 2\theta_{13}$
- matter effects $\hat{A} = A/\Delta m^2_{31} = 2VE/\Delta m^2_{31}; \; V = \sqrt{2} G_F n_e$

$$P(\nu_\mu \to \nu_\mu) \approx 1 - \cos^2\theta_{13} \sin^2 2\theta_{23} \sin^2\Delta + 2\,\alpha\, \cos^2\theta_{13} \cos^2\theta_{12} \sin^2 2\theta_{23} \Delta \cos\Delta$$

$$\begin{aligned}
P(\nu_e \to \nu_\mu) \approx\; & \sin^2 2\theta_{13} \sin^2\theta_{23} \frac{\sin^2((1-\hat{A})\Delta)}{(1-\hat{A})^2} \\
\pm\; & \sin\delta_{CP}\; \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \sin(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
+\; & \cos\delta_{CP}\, \alpha \sin 2\theta_{12} \cos\theta_{13} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\Delta) \frac{\sin(\hat{A}\Delta)\sin((1-\hat{A})\Delta)}{\hat{A}(1-\hat{A})} \\
+\; & \alpha^2 \sin^2 2\theta_{12} \cos^2\theta_{23} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\end{aligned}$$