



Neutrinos Theory

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1 kton water + Gd guided by KamLAND

KamLAND 1kton LS, 2000 PMT, 33% cov
(1.4 yr, 90%)

Supernova (8.5 kpc) : 300 events

Relic Supernova ($E > 6\text{MeV}$) ~ 0.4 events/yr

	bck	React	Geov
Bin1	125.2 (24.6)	2.3	6.3
Bin2	24.6 (19.8)	19.5	7.4
Bin3	12.9 (9.8)	35.1	3.9
Bin4	5.9 (3.9)	36.1	2.8

1 kton water + Gd guided by KamLAND

Gd-1kton, 2000 PMT, 33% cov
(1.4 yr, 90%)

Supernova (8.5 kpc) : 300 events

Relic Supernova ($E > 5\text{MeV}$) ~ 0.5 events/yr

	bck	React	Geov
Bin1	12.2 (24.6)	~ 0	~ 7
Bin2	24.6 (19.8)	~ 1	~ 8.5
Bin3	12.9 (9.8)	~ 2	~ 4.5
Bin4	5.9 (3.9)	~ 2	~ 3

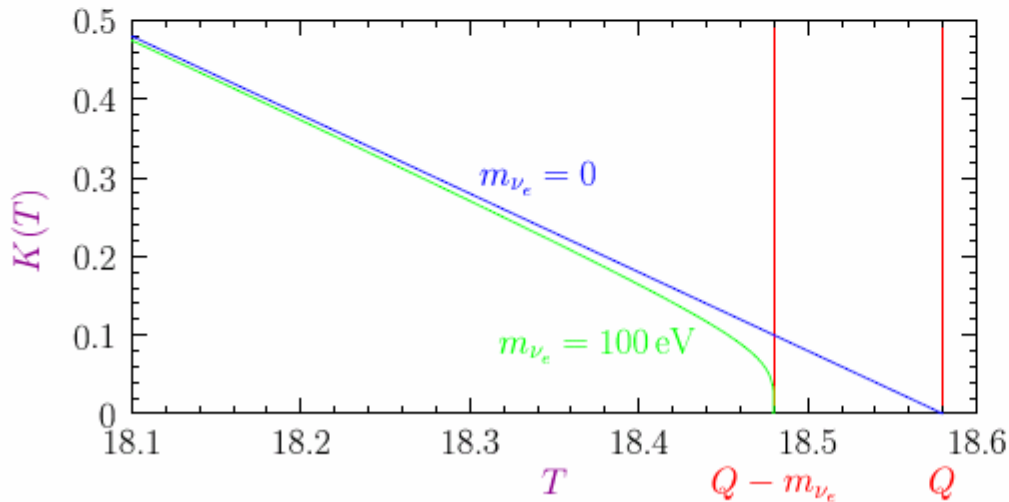
Neutrino mass : Direct methods

Tritium β Decay: ${}^3\text{H} \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$

$$\frac{d\Gamma}{dT} = \frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE (Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}$$

$$Q = M_{{}^3\text{H}} - M_{{}^3\text{He}} - m_e = 18.58 \text{ keV}$$

Kurie plot:
$$K(T) = \sqrt{\frac{d\Gamma/dT}{\frac{(\cos\vartheta_C G_F)^2}{2\pi^3} |\mathcal{M}|^2 F(E) pE}} = [(Q - T) \sqrt{(Q - T)^2 - m_{\nu_e}^2}]^{1/2}$$



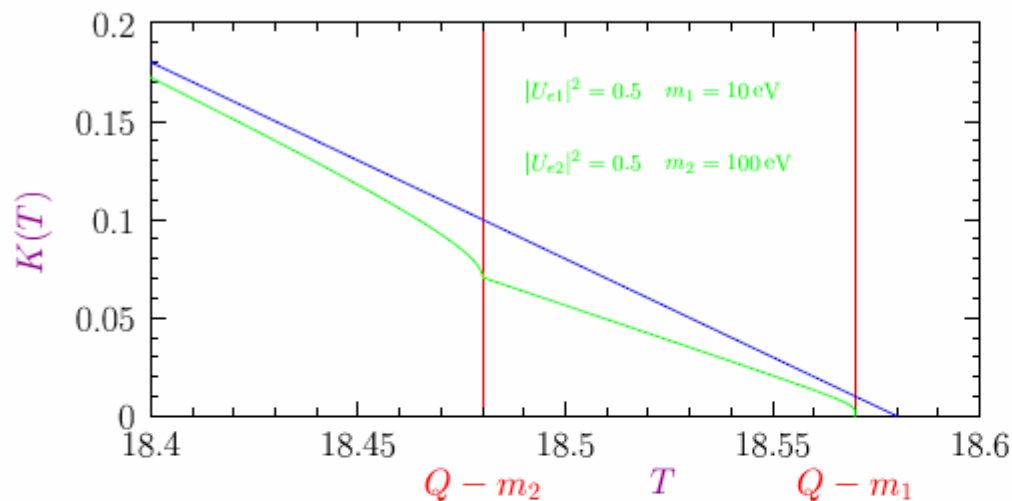
$$m_{\nu_e} < 2.2 \text{ eV} \quad (95\% \text{ C.L.})$$

[Mainz, Troitsk, hep-ex/0210050]

Future: KATRIN [hep-ex/0109033]

sensitivity: $m_{\nu_e} \gtrsim 0.3 \text{ eV}$

Neutrino Mixing $\implies K(T) = \left[(Q - T) \sum_k |U_{ek}|^2 \sqrt{(Q - T)^2 - m_k^2} \right]^{1/2}$



analysis of data is
different from the
no-mixing case:
 $2N - 1$ parameters
 $\left(\sum_k |U_{ek}|^2 = 1 \right)$

if experiment is not sensitive to masses ($m_k \ll Q - T$) \implies effective mass

$$m_\beta^2 = \sum_k |U_{ek}|^2 m_k^2$$

$$\begin{aligned} K^2 &= (Q - T)^2 \sum_k |U_{ek}|^2 \sqrt{1 - \frac{m_k^2}{(Q - T)^2}} \simeq (Q - T)^2 \sum_k |U_{ek}|^2 \left[1 - \frac{1}{2} \frac{m_k^2}{(Q - T)^2} \right] \\ &= (Q - T)^2 \left[1 - \frac{1}{2} \frac{m_\beta^2}{(Q - T)^2} \right] \simeq (Q - T) \sqrt{(Q - T)^2 - m_\beta^2} \end{aligned}$$

Plan

- I. Neutrinos as Dirac Particles
- II. Making Neutrinos
- III. Neutrinos as Majorana Particles

Massive Neutrinos : Dirac

$$\begin{array}{ccc}
 \nu(p, h) & \longleftrightarrow & \bar{\nu}(p, -h) \\
 \uparrow & \text{CPT} & \uparrow \\
 \text{Boost,} & & \text{Boost,} \\
 \text{if } m \neq 0 & & \text{if } m \neq 0 \\
 \downarrow & & \downarrow \\
 \nu(p, -h) & \longleftrightarrow & \bar{\nu}(p, h) \\
 & \text{CPT} &
 \end{array}$$

Chirality vs Helicity

$$\nu_{Li} \propto \nu_i(p, h) + O(m_i / E) \nu_i(p, -h)$$

$$\bar{\nu}_{Li} \propto \bar{\nu}_i(p, -h) + O(m_i / E) \bar{\nu}_i(p, h)$$

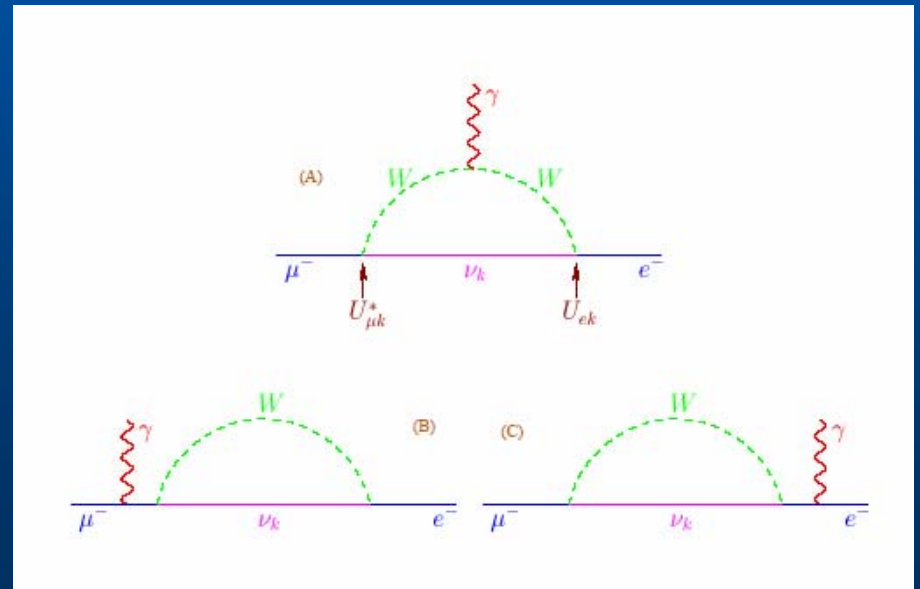
$$m_i \bar{\nu}_i \nu_i = \frac{y \nu}{\sqrt{2}} (\bar{\nu}_R \nu_L + \bar{\nu}_L \nu_R)$$

Example : $\mu \rightarrow e \gamma$

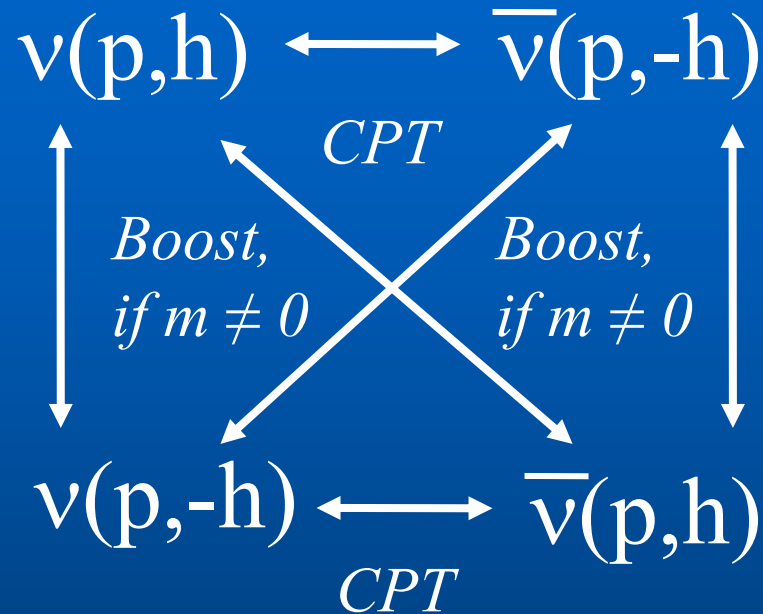
$\sum U_{\mu k}^* U_{ek} = 0 \Rightarrow GIM$ mechanism

$$\Gamma = \frac{G_F^2 m_\mu^5}{192 \pi^3} \frac{3\alpha}{32 \pi} \left| \sum U_{\mu k}^* U_{ek} \frac{m_j^2}{m_W^2} \right|^2 < 10^{-25} \frac{G_F^2 m_\mu^5}{192 \pi^3}$$

$$\Gamma_{\text{exp}} < 10^{-11} \frac{G_F^2 m_\mu^5}{192 \pi^3}$$

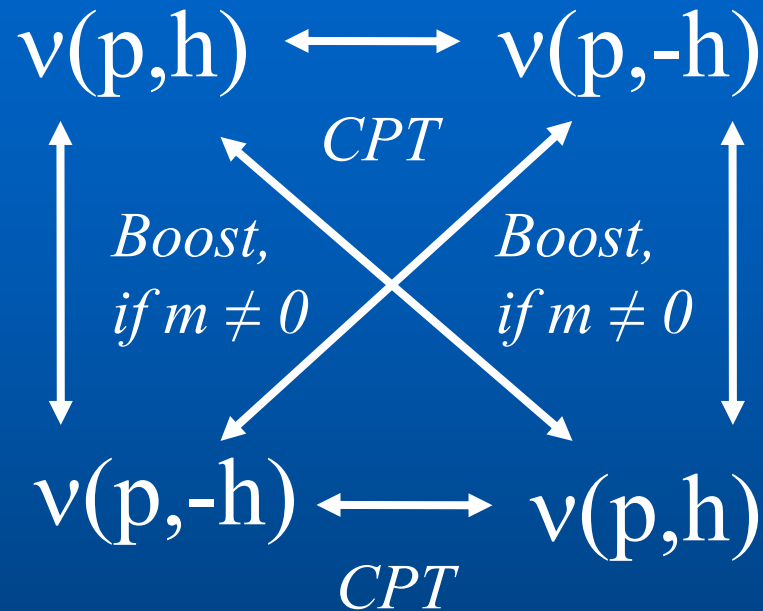


Massive Neutrinos : Majorana



$$\begin{aligned}
 \mathcal{L}_\nu = & i \bar{\nu}_i \gamma^\mu \partial_\mu \nu_i - \frac{g}{2 \cos \theta_W} \bar{\nu}_i \gamma^\mu Z_\mu^0 \nu_i \\
 & - \frac{g}{2 \cos \theta_W} \bar{l}_i \gamma^\mu U_{ij} W_\mu^- \nu_i + \text{h.c.} \\
 & + m_i \bar{\nu}_i \nu_i + m_{Ri} \bar{\nu}_{Ri}^c \nu_{Ri} + \text{h.c.} + \dots
 \end{aligned}$$

Massive Neutrinos : Majorana



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 & + m_i \bar{\nu}_i \nu_i + m_{Ri} \bar{\nu}_{Ri}^c \nu_{Ri} + \text{h.c.} + \dots
 \end{aligned}$$

Parameters : Majorana

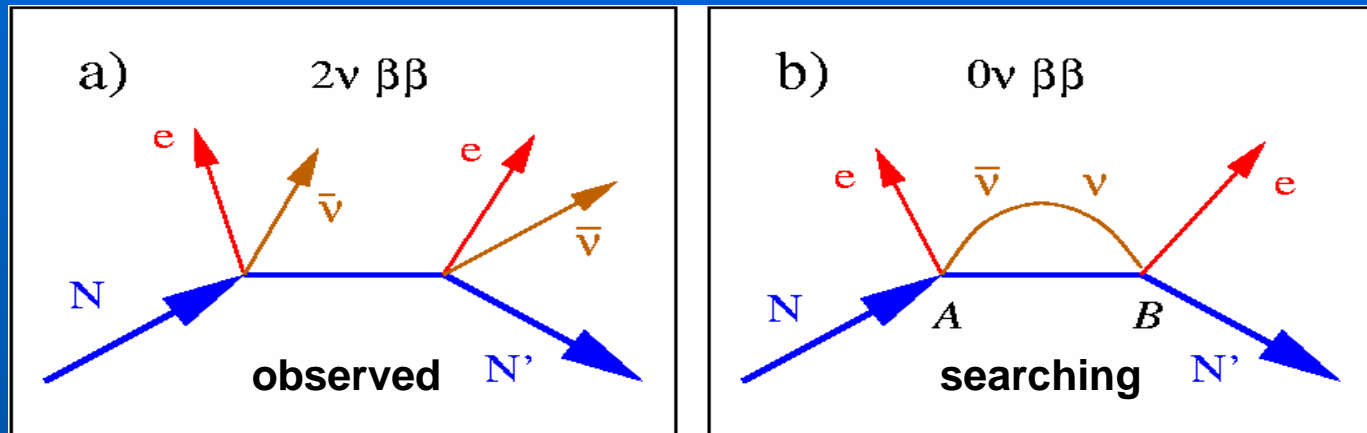
$$U_{\alpha i} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{+i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{+i\lambda_{21}} & 0 \\ 0 & 0 & e^{+i\lambda_{31}} \end{pmatrix}$$

Majorana phases only relevant in processes involving
Lepton Number Violation ($\nu_\alpha - \bar{\nu}_\beta$)

Previous discussion on neutrino osc. is valid for Majorana !

Helicity suppressed by smallness of neutrino masses

Majorana vs Dirac : $0\nu\beta\beta$ decay



$0\nu\beta\beta: nn \rightarrow ppe^-e^-$ (without neutrinos)

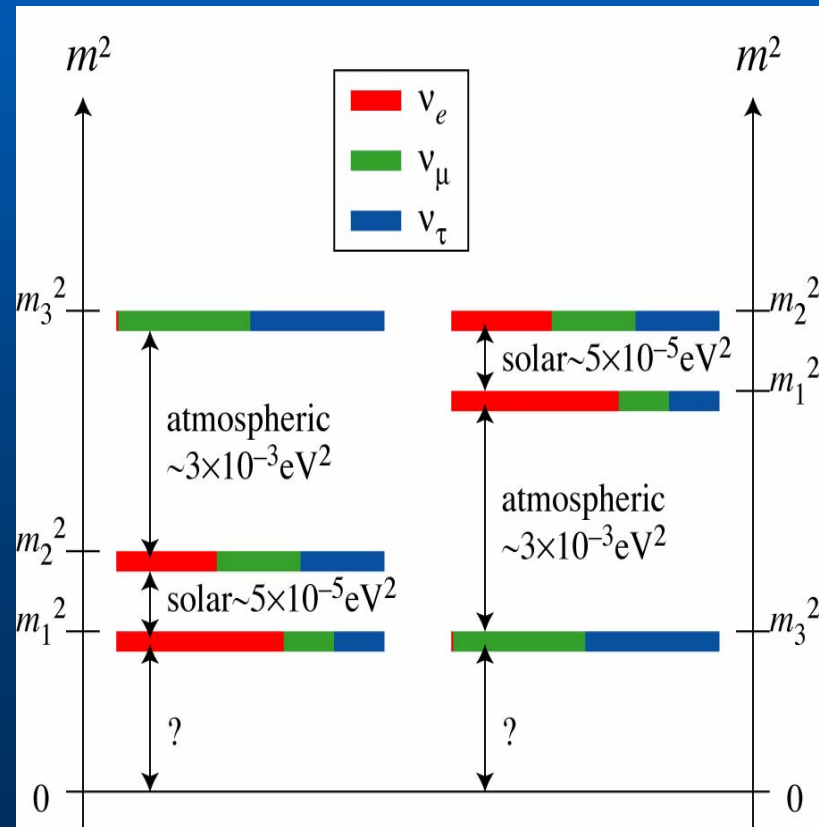
$$|\langle m_{ee}^{\nu} \rangle| = \frac{m_e}{(T_{1/2} F_N)^{1/2}}$$

$$F_N = G^{0\nu} |M^{0\nu}_f - (g_A/g_V)^2 M^{0\nu}_{GT}|^2$$

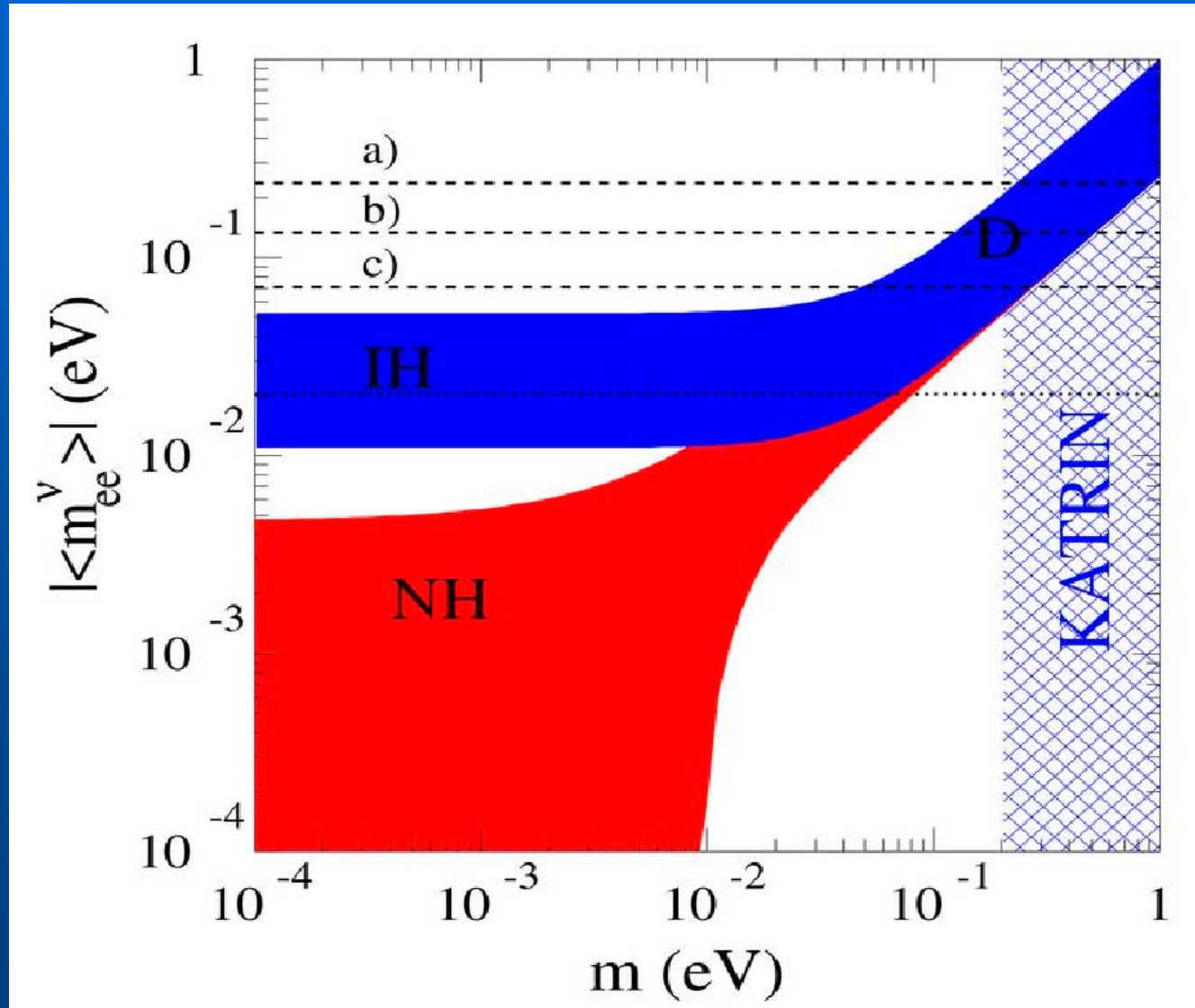
Connection with neutrino parameters :

$$|\langle m^{\nu}_{ee} \rangle| = |\sum m_j U_{ej}^2| = |\sum m_j |U_{ej}|^2 e^{i\phi_j}|$$

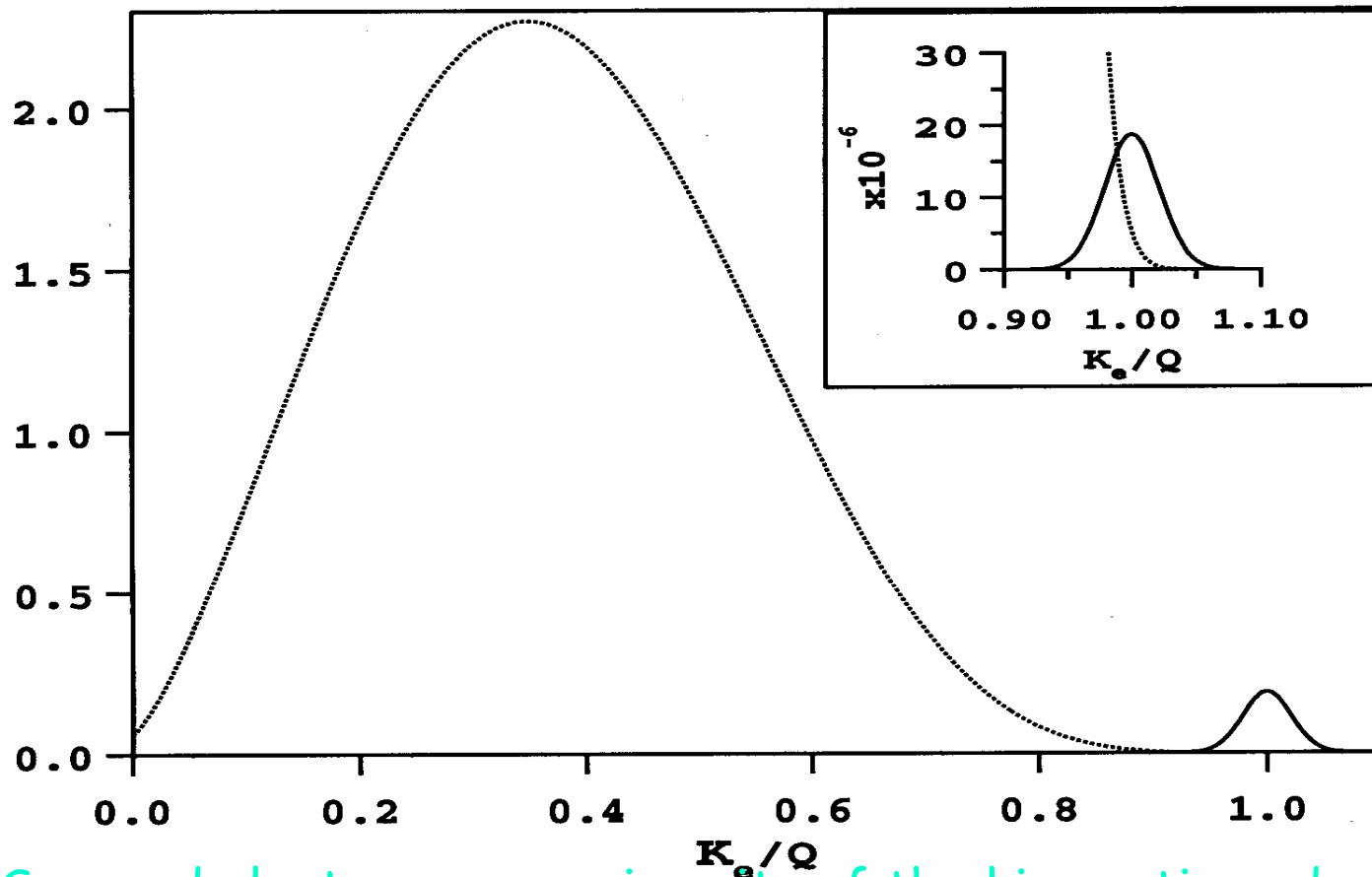
Normal Hierarchy
 Inverted Hierarchy
 Degenerate



From Osc. Data :



Signal : Energy Resolution required



Summed electron energy in units of the kinematic endpoint (Q)

nuclear matrix elements?

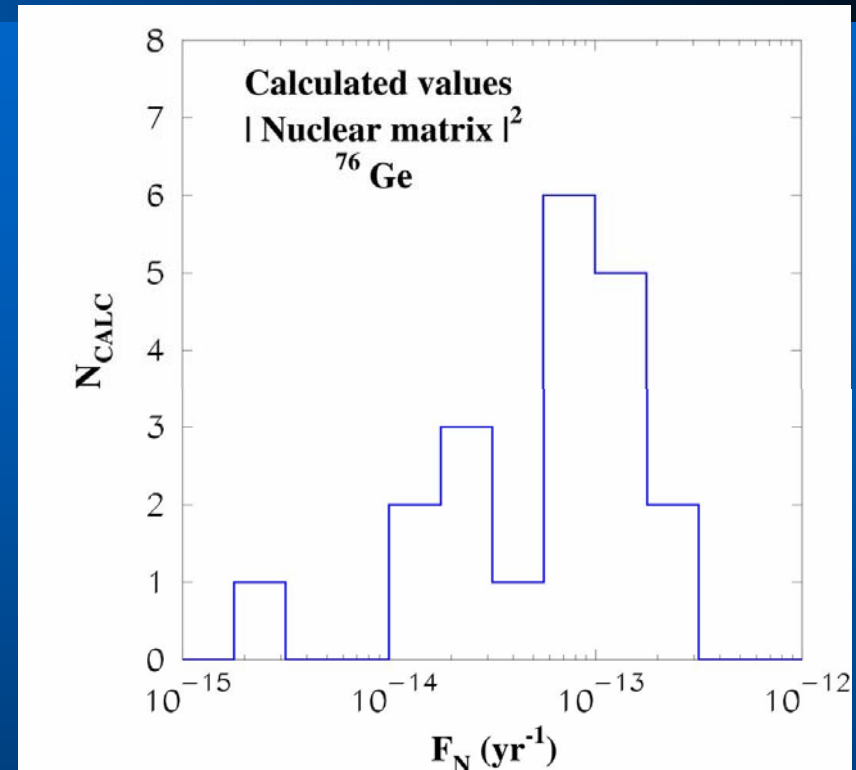
Nuclear Physics methods :

QRPA, SM

Next Generation experiments :

Observed : **Neutrinos are Majorana**

Not observed : **Hard to extract neutrino parameters**, and
exclude Dirac.

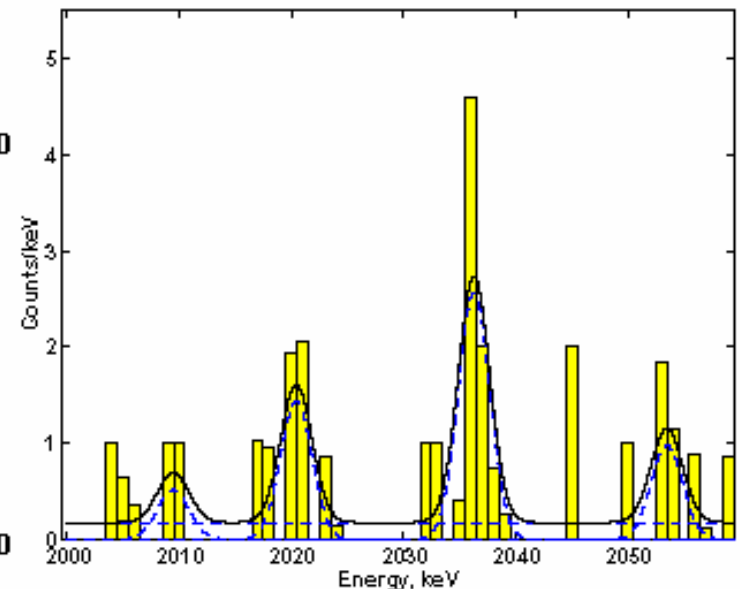
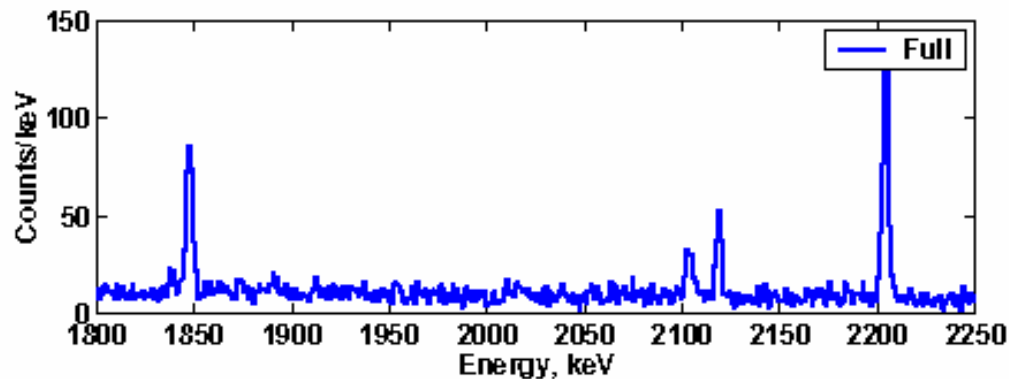
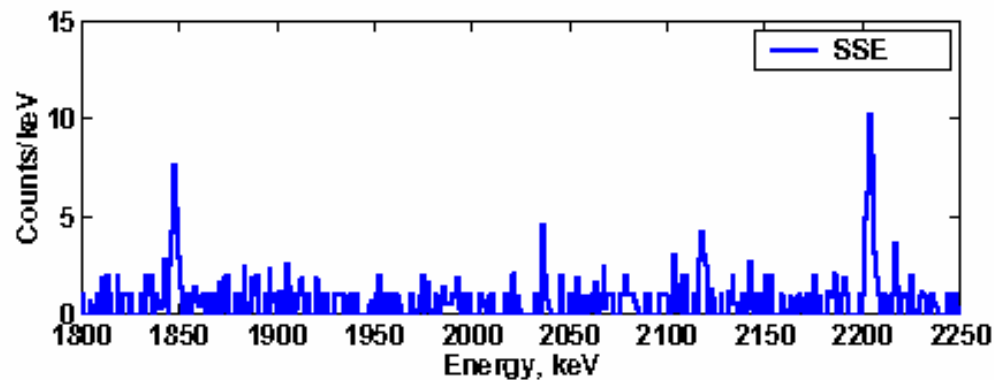
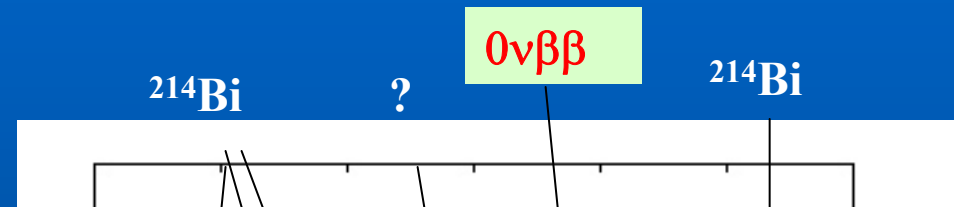


Positive signal ?

Klapdor et al (part of HM)

28.8 (6.9) \rightarrow 3-4 σ

Bckg ~ 60 ev.



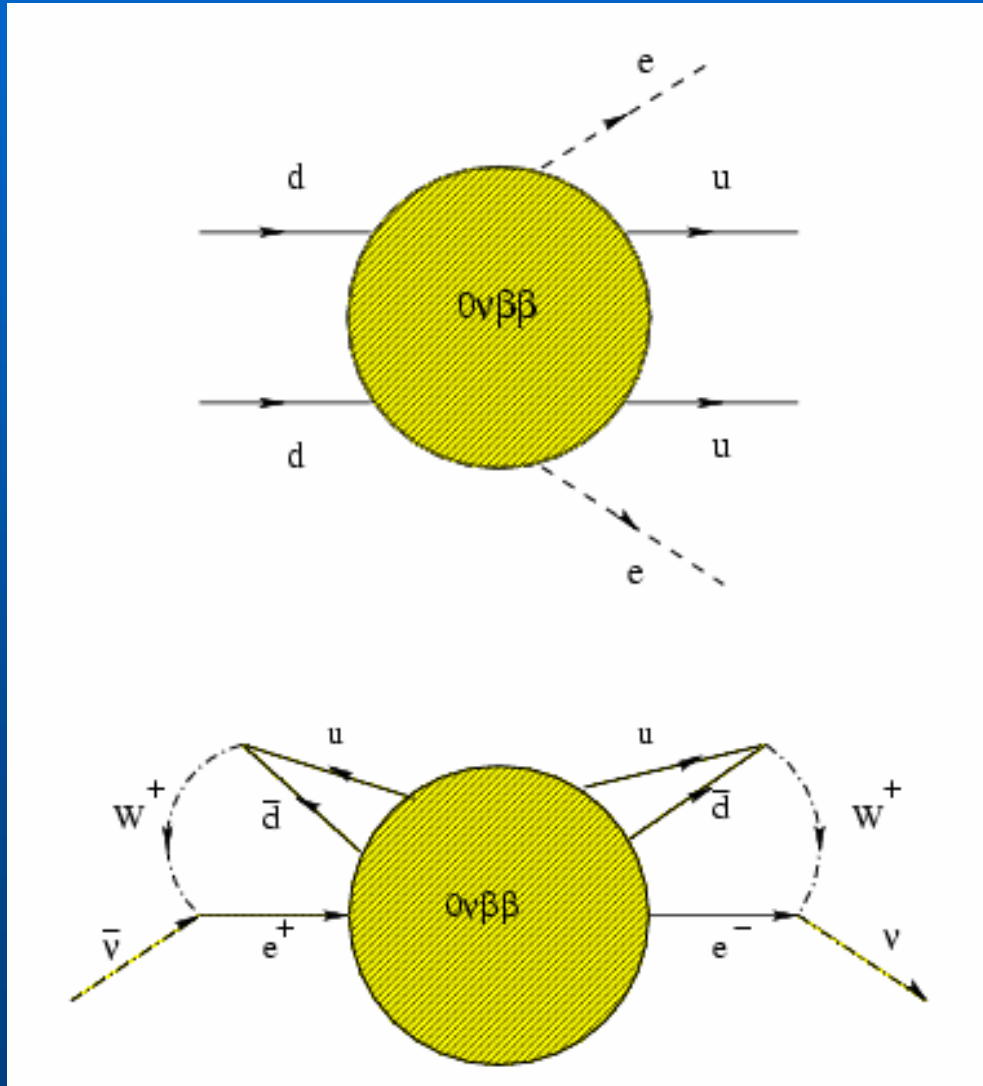
Present Limits :

Candidate nucleus	Detector type	(kg yr)	Present $T_{1/2}^{0\nu\beta\beta}$ (yr)	$\langle m \rangle$ (eV)
^{48}Ca	Ge diode	~30	$>9.5 \cdot 10^{21}$ (76%CL)	$<0.39^{+0.17}_{-0.28}$
^{76}Ge			$>1.9 \cdot 10^{25}$ (90%CL)	
^{82}Se			$>9.5 \cdot 10^{21}$ (90%CL)	
^{100}Mo			$>5.5 \cdot 10^{22}$ (90%CL)	
^{116}Cd			$>7.0 \cdot 10^{22}$ (90%CL)	
^{128}Te	TeO ₂ cryo	~3	$>1.1 \cdot 10^{23}$ (90%CL)	$<1.1 - 2.6$
^{130}Te	TeO ₂ cryo	~3	$>2.1 \cdot 10^{23}$ (90%CL)	
^{136}Xe	Xe scint	~10	$>1.2 \cdot 10^{24}$ (90%CL)	
^{150}Nd			$>1.2 \cdot 10^{21}$ (90%CL)	
^{160}Gd			$>1.3 \cdot 10^{21}$ (90%CL)	

Projected/proposed

Experiment	Nucleus	Detector	$T^{0\nu}$ (y)	$\langle m_\nu \rangle$ eV
CUORE	^{130}Te	.77 t of TeO_2 bolometers (nat)	7×10^{26}	.014-.091
EXO	^{136}Xe	10 t Xe TPC + Ba tagging	1×10^{28}	.013-.037
Gertha	^{76}Ge	1 t Ge diodes in LN	1×10^{28}	.013-.050
Majorana	^{76}Ge	1 t Ge diodes	4×10^{27}	.021-.070
MOON	^{100}Mo	34 t nat.Mo sheets/plastic sc.	1×10^{27}	.014-.057
DCBA	^{150}Nd	20 kg Nd-tracking	2×10^{25}	.035-.055
CAMEO	^{116}Cd	1 t CdWO_4 in liquid scintillator	$> 10^{26}$.053-.24
COBRA	^{116}Cd , ^{130}Te	10 kg of CdTe semiconductors	1×10^{24}	.5-2.
Candles	^{48}Ca	Tons of CaF_2 in liq. scint.	1×10^{26}	.15-.26
GSO	^{116}Cd	2 t $\text{Gd}_2\text{SiO}_5:\text{Ce}$ scint in liq scint	2×10^{26}	.038-.172
Xmass	^{136}Xe	1 t of liquid Xe	3×10^{26}	.086-.252

Theorem $0\nu\beta\beta \leftrightarrow$ Majorana

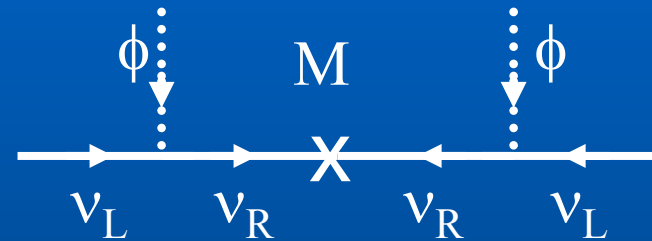


in gauge theories
with SSB

Seesaw mechanism

$$\mathcal{L}_\nu \prec m_i \bar{\nu}_i \nu_i + m_{Ri} \bar{\nu}_{Ri}^c \nu_{Ri} + \text{h.c.} + \dots$$

Integrating out the heavy field



$m_R \gg \langle \phi \rangle$:

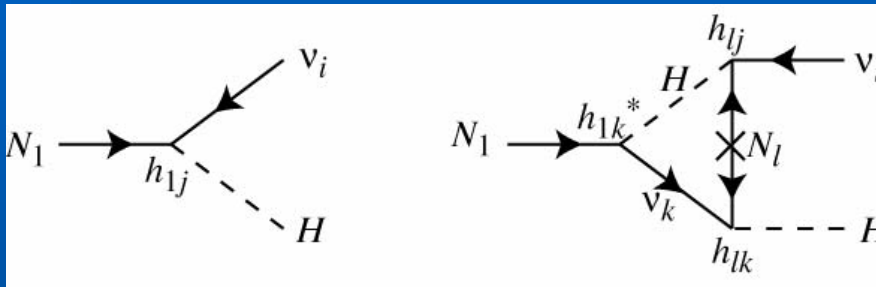
$$\mathcal{L}_{eff} \prec \frac{m_i^2}{4m_{Ri}} \bar{\nu}_{Li}^c \nu_{Li}$$

Equivalently, diagonalize the mass matrix in ν_L - ν_R basis

$$\begin{bmatrix} 0 & m \\ m & m_R \end{bmatrix}$$

Leptogenesis

Generate L asymmetry from direct CP violation in right handed decay



Net effect (2 families needed):

$$\varepsilon = \frac{\Gamma(N_1 \rightarrow \nu_i H) - \Gamma(N_1 \rightarrow \bar{\nu}_i H)}{\Gamma(N_1 \rightarrow \nu_i H) + \Gamma(N_1 \rightarrow \bar{\nu}_i H)} \sim \frac{1}{8\pi} \frac{\text{Im}(h_{13} h_{13} h_{33}^* h_{33}^*)}{|h_{13}|^2} \frac{M_1}{M_3}$$

Convert L into B via anomaly

→ **Matter- Antimatter asymmetry**

Summary

Neutrinos oscillate, refract, decohere,...

→ test sources and explore mixing matrix

The field is open with a well defined program:

- Complete the mixing matrix : θ_{13} and δ_{CP}
- $0\nu\beta\beta$: Dirac vs Majorana
- Test sources (Solar luminosity with neutrinos,...)

Expand to discovery regions :

- Low energies : solar, (s. DM searches), terrestrial, SN, ...
- High energies : hadronic acceleration sources, CR connection

Be ready for surprises