Weak electric and magnetic dipole moments of the τ lepton from azimuthal asymmetries

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Measurements of the weak electric dipole moment d_τ^w and, for the first time, the weak magnetic dipole moment a_τ^w of the τ lepton using L3 detector at LEP are presented. Azimuthal asymmetries for $\tau \to \pi\nu$ and $\tau \to \rho\nu$ are used to obtain these measurements. Observed asymmetries are consistent with zero, and the limits set on d_τ^w and a_τ^w are $|d_\tau^w| < 8.56 \times 10^{-17} e \cdot cm$ and $|a_\tau^w| < 0.014$ at 95% C. L.

1. Theoretical Motivation

The third family τ lepton can provide a window to new physics beyond the Standard Model. With the present level of accuracy in the knowledge of the intrinsic properties of the τ lepton is still not possible to exclude deviations from the Standard Model predictions. In particular, the weak electric and magnetic dipole moments, d_τ^w and a_τ^w, are fundamental magnitudes that can be measured from e^+e^- collisions at the Z^0 peak, using the τ decay products as spin analyzers. Besides, in some extensions of the Standard Model, the weak dipole moments are enhanced with the mass of the corresponding fermion, what makes the τ lepton a suitable framework for studying these properties.

In fact, it can be shown [1] that the normal component (to the collision plane) and the transverse component (within the collision plane) of the single τ polarization give access to d_τ^w and a_τ^w. Some appropiate asymmetries can be defined for measuring these quantities.

The weak dipole moments are quantum corrections to the tree level matrix elements. In the Standard Model, the leading value for the weak magnetic dipole moment comes from one loop corrections. But the weak electric dipole moment is a CP-violating quantity and it takes a non-zero value through the CKM mechanism, and then only at a very high order in the coupling constant [2,3]. These values are clearly far from the present sensitivity, therefore any observation of non-vanishing values for these properties would indicate new physics beyond the Standard Model.

To use the angular distributions of the decay products of the τ, the reconstruction of the τ flight direction is needed. The information about the τ direction is lost when any of both τ's decays to leptons. Then, the decays of the τ to hadrons (ρ or π) must be used to do the measurement of the weak dipole moments.

1.1. Asymmetries

These dipole moments can be studied computing its values in the Standard Model or using the effective lagrangian approach. Here the second method will be used. The weak electric dipole moment d_τ^w will be sensible to new physics and the weak magnetic dipole moment a_τ^w will be sensible to new physics and also to radiative corrections [2].

The following effective interaction lagrangian density [1] is considered:

$$L = -\frac{i}{2}\frac{eF_3^w}{2m_\tau}\bar{\psi}\sigma^{\mu\nu}\gamma_5\psi Z_{\mu\nu} + \frac{1}{2}\frac{eF_2^w}{2m_\tau}\bar{\psi}\sigma^{\mu\nu}\psi Z_{\mu\nu} \quad (1)$$

where $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$, e is the proton charge,

$$d_\tau^w = \frac{eF_3^w}{2m_\tau}$$

is the weak electric dipole moment and

$$a_\tau^w = F_2^w$$

is the weak magnetic dipole moment.

To obtain the cross section for the process $e^+e^- \to \tau^+\tau^- \to h^+\bar{\nu}_\tau h^-\nu_\tau$, with $h = \rho, \pi$ the amplitude from this effective lagrangian must be added to the Standard Model amplitude. The results [1] are:

$$\frac{d\sigma(e^+e^- \to \tau^+\tau^- \to h_1^+\bar{\nu}_\tau h_2^-\nu_\tau)}{d(\cos\theta_{\tau^-})d\phi_{h_2^-}} =$$

$$Br(\tau^- \to h_2^-\nu_\tau)Br(\tau^+ \to h_1^+X)(2\frac{d\sigma^0}{d\Omega_{\tau^-}} +$$

$$\frac{\alpha^2\beta\pi}{128 s_w^3 c_w^3 \Gamma_{Z^0}^2}\alpha_{h_2^-}(X_+\cos\phi_{h_2^-} + Y_-\sin\phi_{h_2^-})) \quad (2)$$

$$\frac{d\sigma(e^+e^- \to \tau^+\tau^- \to h_1^+\bar{\nu}_\tau h_2^-\nu_\tau)}{d(\cos\theta_{\tau^-})d\phi_{h_1^+}} =$$

$$Br(\tau^- \to h_2^-X)Br(\tau^+ \to h_1^+\bar{\nu}_\tau)(2\frac{d\sigma^0}{d\Omega_{\tau^-}} +$$

$$\frac{\alpha^2\beta\pi}{128 s_w^3 c_w^3 \Gamma_{Z^0}^2}\alpha_{h_1^+}(-X_+\cos\phi_{h_1^+} + Y_-\sin\phi_{h_1^+})) \quad (3)$$

The angles θ_τ and ϕ_h are defined in Figure 1, the angle θ_h has been integrated out. The term $\frac{d\sigma^0}{d\Omega_{\tau^-}}$ includes the leading spin independent structure of the cross section. The rest are the linear terms on the spin, X_+ is related to the transverse component of the spin and Y_- is related to the normal component, where:

$$\begin{aligned} X_+ &= a\sin\theta_{\tau^-} \\ &\times \{-[2v^2 + (v^2+a^2)\beta\cos\theta_{\tau^-}]\frac{v}{\gamma s_w c_w} + \\ &+ 2\gamma[2v^2(2-\beta^2) + (v^2+a^2)) \\ &\quad \beta\cos\theta_{\tau^-}]F_2^w\} \end{aligned} \quad (4)$$

, and

$$\begin{aligned} Y_- &= 2a\gamma\beta\sin\theta_{\tau^-} \\ &\times [2v^2 + (v^2+a^2))\beta\cos\theta_{\tau^-}]F_3^w \end{aligned} \quad (5)$$

With $\gamma = \frac{M_{Z^0}}{2m_\tau}$ and $\beta = \sqrt{1-\frac{1}{\gamma^2}}$; v, a are the vector and axial vector $Z^0\tau^+\tau^-$ couplings. α_h is the polarization analyzer ($\alpha_\rho = 0.47$, $\alpha_\pi = 1$), Γ_{Z^0} is the Z^0 width and s_w, c_w are the weak mixing angles sine and cosine, respectively. The spin correlations give no contribution to the angular distributions (2) and (3).

For extracting F_2^w and F_3^w from the angular distributions (2) and (3), two elements are needed. The first one is the reconstruction of the τ direction in order to compute the angles. The second one is the identification of some observables (asymmetries) that can isolate the terms of the cross section depending on F_2^w and F_3^w.

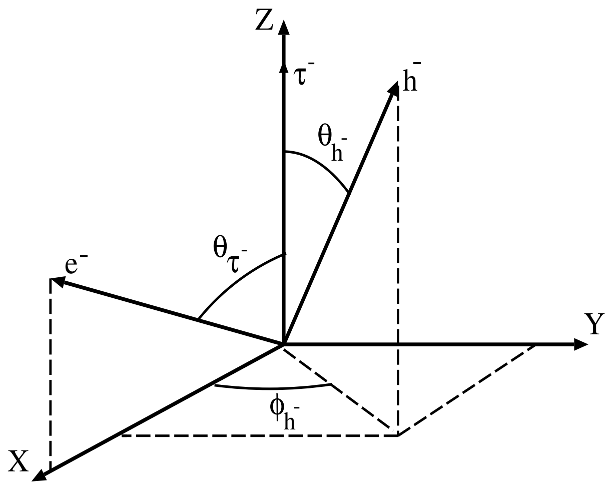

Figure 1. Reference system for the reaction $e^+e^- \to \tau^+\tau^- \to h_1^+\bar{\nu}_\tau h_2^-\nu_\tau$.

These asymmetries can be defined as follows [1].

$$A_{cc}^{\pm} = \frac{\sigma_{cc}^{\pm}(+) - \sigma_{cc}^{\pm}(-)}{\sigma_{cc}^{\pm}(+) + \sigma_{cc}^{\pm}(-)} \quad (6)$$

where

$$\sigma_{cc}^{\pm}(+) = (\int_0^1 d(\cos\theta_{\tau^-})\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi_{h^\pm} +$$

$$\int_{-1}^0 d(\cos\theta_{\tau^-})\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\phi_{h^\pm})\frac{d\sigma}{d(\cos\theta_{\tau^-})d\phi_{h^\pm}} \quad (7)$$

$$\sigma_{cc}^{\pm}(-) = (\int_0^1 d(\cos\theta_{\tau^-}) \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} d\phi_{h^\pm} + \int_{-1}^{0} d(\cos\theta_{\tau^-}) \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} d\phi_{h^\pm}) \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^\pm}} \quad (8)$$

and

$$A_{sc}^{\pm} = \frac{\sigma_{sc}^{\pm}(+) - \sigma_{sc}^{\pm}(-)}{\sigma_{sc}^{\pm}(+) + \sigma_{sc}^{\pm}(-)} \quad (9)$$

where

$$\sigma_{sc}^{\pm}(+) = (\int_0^1 d(\cos\theta_{\tau^-}) \int_0^{\pi} d\phi_{h^\pm} + \int_{-1}^{0} d(\cos\theta_{\tau^-}) \int_{\pi}^{2\pi} d\phi_{h^\pm}) \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^\pm}} \quad (10)$$

$$\sigma_{sc}^{\pm}(-) = (\int_0^1 d(\cos\theta_{\tau^-}) \int_{\pi}^{2\pi} d\phi_{h^\pm} + \int_{-1}^{0} d(\cos\theta_{\tau^-}) \int_0^{\pi} d\phi_{h^\pm}) \frac{d\sigma}{d(\cos\theta_{\tau^-}) d\phi_{h^\pm}} \quad (11)$$

$A_{sc}^{\pm}$ isolates the leading term in the weak magnetic dipole moment and $A_{cc}^{\pm}$ isolates the leading term in the weak electric dipole moment. Doing the calculation [1] one finds:

$$\begin{aligned} A_{sc}^{\rho^-} &= A_{sc}^{\rho^+} &= -3.8F_3^w \\ A_{cc}^{\rho^-} &= -A_{cc}^{\rho^+} &= 0.0003 + 5.0F_2^w \end{aligned} \quad (12)$$

$$\begin{aligned} A_{sc}^{\pi^-} &= A_{sc}^{\pi^+} &= -10.6F_3^w \\ A_{cc}^{\pi^-} &= -A_{cc}^{\pi^+} &= 0.0007 + 10.6F_2^w \end{aligned} \quad (13)$$

These are the relations among the real asymmetries and F_2^w and F_3^w. But the detector imperfections make the measured asymmetries different of the real ones. The effects of the detector and the reconstruction of the τ flight direction must be taken into account to compare with the experimental results. This will be done in the next sections.

2. Reconstruction of the τ flight direction

Detectors at present e^+e^- colliders are not able to measure the flight direction of the τ leptons. But it is possible to reconstruct it up to a twofold ambiguity [5] for the hadronic decay channels ($h = \pi,\ \rho$), see Figure 2.

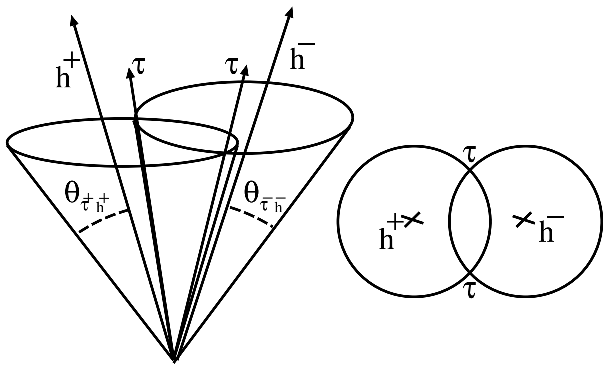

Figure 2. Geometric view of the τ flight direction reconstruction and its twofold ambiguity.

The inputs are the measured momentum and energy of the hadron, $E_{h^\pm}, \vec{p}_{h^\pm}$. The hypotheses used in the reconstruction are two. The τ's are back to back and the energy of each τ is $M_{Z^0}/2$ (at LEP. In other experiments the energy should be one half of the total energy), and the τ neutrinos are massless. Using the fourmomentum conservation , $\cos(\theta_{h^\pm\tau^\pm})$ can be computed, where $\theta_{h^\pm\tau^\pm}$ are the angles between the hadron $h^\pm$ and the $\tau^\pm$. the result is:

$$\cos(\theta_{h^\pm\tau^\pm}) = \frac{M_{Z^0}E_{h^\pm} - m_\tau^2 - m_{h^\pm}^2}{\sqrt{(M_{Z^0}^2 - 4m_\tau^2)(E_{h^\pm}^2 - m_{h^\pm}^2)}} \quad (14)$$

Now, the τ flight direction can be easily written in the basis $\{\hat{n}_1, \hat{n}_2, \hat{n}_3\}$, with these unit vectors defined in the following way:

$$\begin{aligned}\hat{n}_1 &= \hat{n}_2 \times \hat{n}_3 = \frac{1}{\sin(\Delta\theta)}(\hat{p}_{h^-} - \hat{p}_{h^+}\cos(\Delta\theta)) \\ \hat{n}_2 &= \frac{1}{\sin(\Delta\theta)}(\hat{p}_{h^-} \times \hat{p}_{h^+)} \\ \hat{n}_3 &= -\hat{p}_{h^+}\end{aligned}$$

Where $\cos(\Delta\theta)) = \hat{p}_{h^-} \cdot \hat{p}_{h^+}$, and $\hat{p_{h^\pm}}$ are unit vectors along the $h^\pm$ momenta.

If the τ direction is written $\hat{e}_\tau = a\hat{n}_1 + b\hat{n}_2 + c\hat{n}_3$, then the coefficients a, b, c can be obtained from the following expressions

$$\cos(\theta_{h^-\tau^-}) = (a\hat{n}_1 + b\hat{n}_2 + c\hat{n}_3) \cdot \hat{p}_{h^-}$$

$$\cos(\theta_{h^+\tau^+}) = (a\hat{n}_1 + b\hat{n}_2 + c\hat{n}_3) \cdot \hat{n}_3$$

The values for a, b, c are, using the normalization condition for b,

$$\begin{aligned}a &= \frac{1}{\sin(\Delta\theta)}(\cos(\theta_{h^-\tau^-}) + \cos(\theta_{h^+\tau^+})\cos(\Delta\theta) \\ c &= \cos(\theta_{h^+\tau^+}) \\ b &= \pm\sqrt{1 - a^2 - c^2} \qquad (15)\end{aligned}$$

The ambiguity in the reconstruction is reflected in the sign of the parameter b. In the present study, to solve this ambiguity, the + sign has been chosen for b in all the cases. This introduces some biases and systematic effects, which have to be studied and corrected in the analysis of the weak dipole moments.

3. Selection of the Sample

The initial sample is the data collected by the detector L3 at LEP from 1991 to 1994. The L3 detector is described in detail in Ref. [7]. The selection of the τ pairs is performed as in Ref. [8], in the barrel region $|\cos(\theta_{thrust})| < 0.7$ with an additional cut refered to the b parameter of the τ direction which will be described in the next section.

The efficiencies and purities of the selected sample in the fiducial volume are quoted in Table 1. All the backgrounds in the sample come from other τ decays.

Table 1
Efficiencies and purities for the selected sample $e^+e^- \to \tau^+\tau^- \to h^+\bar{\nu}_\tau h^-\nu_\tau$, ($h = \rho,\ \pi$) in the fiducial volume.

Mode	Efficiency	Purity	N. of Events
$\rho^-\rho^+$	0.243	0.768	1814
$\rho^-\pi^+$	0.183	0.750	822
$\pi^-\rho^+$	0.188	0.756	913
$\pi^-\pi^+$	0.313	0.699	537

4. Systematic effects

The effects of the photon radiation are neglected in the reconstruction of the τ flight direction mentioned above. On the other hand, the ρ's and π's energies and directions are reconstructed as described in Ref. [8], and the finite resolution of the detector must be taken into account. These are the two main experimental difficulties to be overcome.

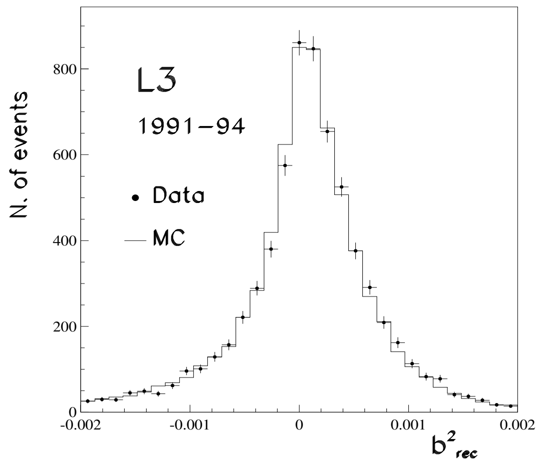

Figure 3. Distribution of b^2 for data (dots) and MonteCarlo (histogram). Good agreement is observed.

The consequences of these problems come to the b parameter, and events with $a^2 + c^2 > 1$ are found, what means that b should be imaginary. Then, an aditional requirement in b is applied: $b^2 > 0$. The final efficiency in the $\vec{p}_\tau$ reconstruction is $\sim 50\%$. The effects of the radiation , resolution and efficiency have been simulated in detail in the MonteCarlo [6], and a good agreement is observed between MonteCarlo and data distributions of b, as can be seen in Figure 3.

For the reconstructed sample, the MonteCarlo and the data are also in good agreement, see Figure 4. The consequences of the reconstruction method are therefore studied in the MonteCarlo.

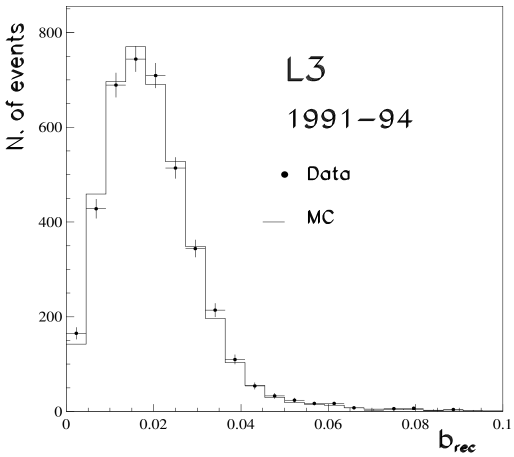

Figure 4. Distribution of the b parameter for data (dots) and MonteCarlo (histogram) for the well reconstructed events ($b^2 > 0$). Good agreement is observed.

As result of the choice of the + sign for b, the $\phi_{h^\pm}$ angle reconstructed is wrong for one half of the events, leading to asymmetric resolution functions, see Figure 5. The effects of the detector imperfections and reconstrcution of the $\vec{p}_\tau$ are taken into account by these resolution functions. The functions are parametrized as a sum of three gaussians and obtained from the MonteCarlo that includes the complete detector simulation. Then, the expressions of the cross sections must be convoluted with these resolution functions to obtain in this way the relations among the measured asymmetries and the parameters F_2^w and F_3^w.

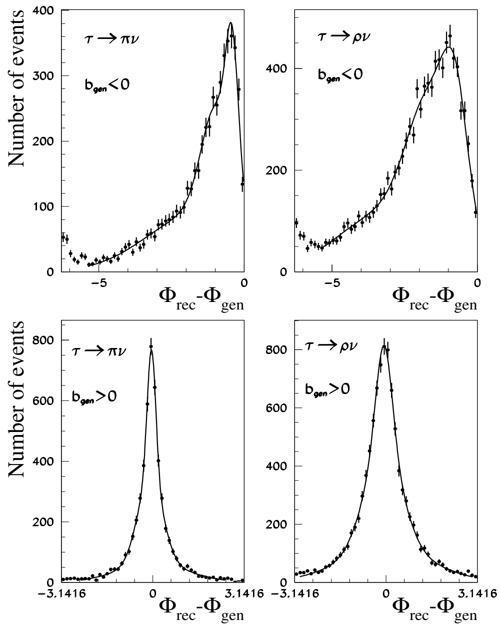

Figure 5. Resolution functions for the reconstructed $\phi_{h^\pm}$ angle. Dots are the full simulated MonteCarlo and curves are the parametrization used in the analysis.

Doing the calculation, one finds for the $\tau \to \rho\nu$ channel,

$$\begin{aligned} A_{sc}^{\rho^-} &= A_{sc}^{\rho^+} = & 0.0001 + 1.59F_2^w - 2.53F_3^w \\ A_{cc}^{\rho^-} &= -A_{cc}^{\rho^+} = & 0.0002 + 2.54F_2^w + 1.59F_3^w \end{aligned} \tag{16}$$

$$\begin{aligned} A_{sc}^{\pi^-} &= A_{sc}^{\pi^+} = & 0.0002 + 2.59F_2^w - 7.41F_3^w \\ A_{cc}^{\pi^-} &= -A_{cc}^{\pi^+} = & 0.0005 + 7.41F_2^w + 2.59F_3^w \end{aligned} \tag{17}$$

The asymmetric resolution functions produce contributions to the measured asymmetries both from F_2^w and from F_3^w. The sensitivity is reduced, but the parameters can still be extracted from the data sample.

The whole method has been checked introducing large values of F_2^w and F_3^w in the MonteCarlo and reweighting the events according to the cross section dependence. The values of F_2^w and F_3^w were recovered after the complete procedure.

4.1. Systematic Errors

The main sources of systematic errors for this analysis are the finite MonteCarlo statistics, the errors associated to the selection of the sample (cuts, efficiencies, purities) and the error coming from the reconstruction of the τ flight direction. The values found are listed in Table 2.

The first error comes from the finite MonteCarlo set used for the analysis and yields to an uncertainty in the asymmetries of 0.006 in the ρ channel and 0.008 in the π channel.

The second is due to the selection procedure. It has been computed varying the cuts in a wide range and looking at the induced variations in the asymmetries. The efficiencies and purities have been also varied according to their errors and their influence in the asymmetries has been included in this error.

The error in the reconstruction of the τ flight direction is by far the most important one. It has been computed using the parametrization of the resolution functions. Their variances have been computed and taken as the errors associated to $\phi_{h^\pm}$ due to the reconstruction method. Then, this error has been propagated from the angle to the cross sections and to the asymmetries.

Table 2
Systematic errors in the measured asymmetries.

Source	ρ channel	π channel
Finite MC statistics	0.006	0.008
Selection effects	0.004	0.004
Reconstruction method	0.034	0.062

5. Results

The final measured asymmetries , after reconstruction and background corrections are given below, where the first error is statistical and the second is systematic.

$$\begin{aligned} A_{cc}^{\rho^-} &= -0.001 \pm 0.021 \pm 0.035 \\ A_{sc}^{\rho^-} &= 0.035 \pm 0.021 \pm 0.035 \end{aligned} \tag{18}$$

$$\begin{aligned} A_{cc}^{\rho^+} &= 0.033 \pm 0.020 \pm 0.035 \\ A_{sc}^{\rho^+} &= -0.023 \pm 0.020 \pm 0.035 \end{aligned} \tag{19}$$

$$\begin{aligned} A_{cc}^{\pi^-} &= -0.015 \pm 0.029 \pm 0.063 \\ A_{sc}^{\pi^-} &= 0.017 \pm 0.029 \pm 0.063 \end{aligned} \tag{20}$$

$$\begin{aligned} A_{cc}^{\pi^+} &= 0.037 \pm 0.029 \pm 0.063 \\ A_{sc}^{\pi^+} &= 0.029 \pm 0.029 \pm 0.063 \end{aligned} \tag{21}$$

From these values and using the expressions (16) and (17), four values for F_2^w and four values for F_3^w are obtained, because $\rho^\pm$ and $\pi^\pm$ decays are treated separately. Combining them with uncorrelated systematic errors, the following measurement can be given for the weak dipole moments:

$$\begin{aligned} d_\tau^w &= (-2.19 \pm 1.28 \pm 3.66) \times 10^{-17} e \cdot cm \\ a_\tau^w &= 0.0025 \pm 0.0023 \pm 0.0066 \end{aligned} \tag{22}$$

The first error is statistical and the second is systematic. These values are compatible with zero, so, it is possible to set upper limits for the absolute values of the weak dipole moments. These limits are

$$\begin{aligned} |d_\tau^w| &< 8.56 \times 10^{-17} e \cdot cm \\ |a_\tau^w| &< 0.014 \end{aligned} \tag{23}$$

at 95% C. L.

6. Conclusions

The angular distributions of $e^+e^- \rightarrow \tau^+\tau^-$ events collected at L3 have been used to measure the weak electric dipole moment and, for the first time, the weak magnetic dipole moment of the τ lepton. The obtained values are:

$$\begin{aligned} d_\tau^w &= (-2.19 \pm 1.28 \pm 3.66) \times 10^{-17} e \cdot cm \\ a_\tau^w &= 0.0025 \pm 0.0023 \pm 0.0066 \end{aligned} \quad (24)$$

From these values, the following limits at 95% C. L. can be set:

$$\begin{aligned} |d_\tau^w| &< 8.56 \times 10^{-17} e \cdot cm \\ |a_\tau^w| &< 0.014 \end{aligned} \quad (25)$$

The ambiguity in the τ flight direction reconstruction reduces the sensitivity to the anomalous couplings, but it does not make impossible the study of these properties.

This is the first direct measurement of the weak magnetic dipole moment a_τ^w. The limit on the weak electric dipole moment d_τ^w is not competitive with that obtained by the CP-odd tensor method [9,10], but some improvements are expected. In this analysis, the Silicon Microvertex Detector of L3 has not been used, and its inclusion will improve the sensitivity and the limits on the weak dipole moment.

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